ENGINEERING TRIPOS   PART IB

Monday 2 June 2003   2 to 4

Paper 2

STRUCTURES

Answer not more than ***four*** *questions, which may be taken from either section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

1 The weightless frame shown in Fig. 1 is unstressed when unloaded. All members have flexural rigidity EI, are axially rigid, and can be considered to behave elastically. The columns are fixed to the ground at F and G. B is a pin-joint and C is a rigid connection.

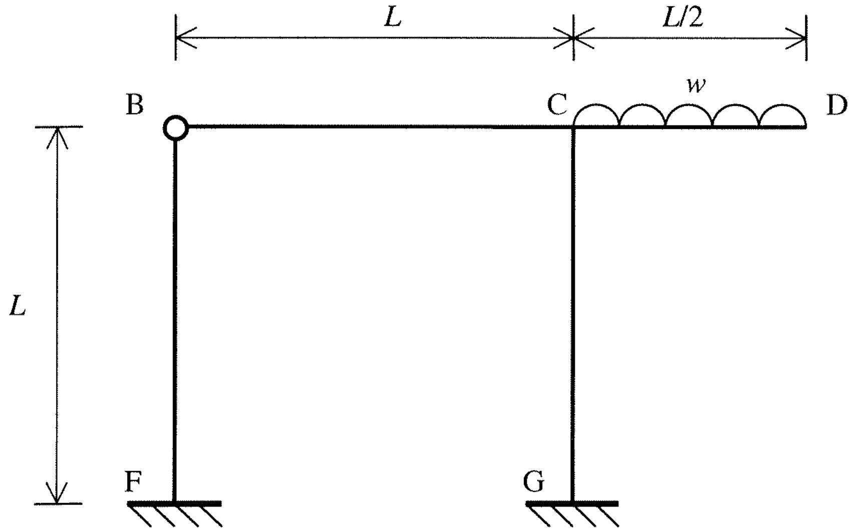

Fig. 1

(a) Determine the number of redundancies in the structure. [2]

(b) A uniformly distributed vertical load of w per unit length is applied to section CD of beam BCD.

(i) By replacing the pin at B with a cut, or otherwise, show that the magnitudes of the vertical and horizontal internal forces at B are $V = 15wL/184$ and $H = 3wL/92$ respectively. [12]

(ii) Determine the reactions at the supports F and G. [6]

2 A 60° strain gauge rosette is used to measure the in-plane stresses at a critical location on a pressure vessel made from a mild steel with a Young's modulus of 210 GPa, a Poisson's ratio of 0.3 and a uniaxial yield stress of 230 N/mm^2. The rosette consists of three strain gauges orientated at 0°, 60° and 120° which measure the strains ε_0, ε_{60} and ε_{120} respectively.

(a) If the rosette is orientated so that the 0° direction is aligned with the *x*-axis show that the normal strain ε_{yy} at the same point is given by:

$$\varepsilon_{yy} = \frac{-\varepsilon_0 + 2\varepsilon_{60} + 2\varepsilon_{120}}{3}$$

and therefore obtain γ_{xy} as a function of ε_0, ε_{60} and ε_{120}. [6]

(b) At normal working pressure the strain gauge measurements were as follows:

$$\varepsilon_0 = -200 \times 10^{-6} \qquad \varepsilon_{60} = 250 \times 10^{-6} \qquad \varepsilon_{120} = 75 \times 10^{-6}.$$

(i) Obtain the normal strains ε_{xx}, ε_{yy} and the shear strain γ_{xy} at the same point. [3]

(ii) Obtain the stress state σ_{xx}, σ_{yy} and τ_{xy} for this location. State any assumptions you have made. [3]

(iii) Draw a two dimensional Mohr's circle of stress for this point and hence obtain the principal stresses and their directions. [5]

(iv) Using von Mises' criterion, determine the factor of safety. [3]

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3 The pin-jointed truss shown in Fig. 3 has five light members OP, PQ, QR, SP and SQ. It is unstressed when unloaded.

(a) How many redundancies does this structure have? [2]

(b) A vertical force W is applied downwards at joint Q as shown.

(i) Find a particular equilibrium solution for this applied load. [3]

(ii) Find a state of self-stress that could exist in the structure. [3]

(iii) Hence find the forces in each of the members of the truss. Each member has cross-sectional area A, Young's modulus E and all behaviour is linear elastic. [9]

(iv) Find the horizontal displacement of node P under the applied load. [3]

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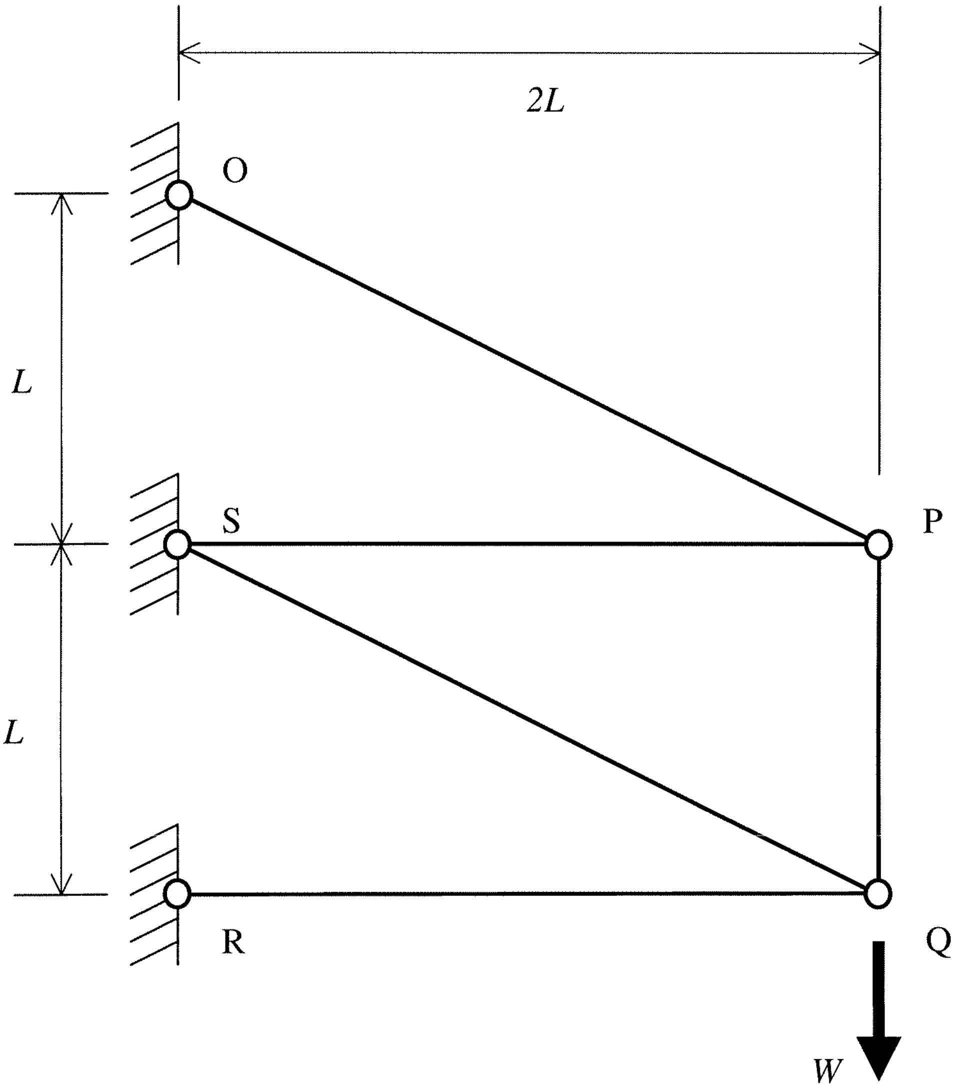

Fig. 3

SECTION B

4 An 8 m high steel column, made from a thin-walled rectangular hollow section, is welded rigidly to a horizontal base plate as shown in Fig. 4(a). A rigid diaphragm plate at the top of the column ensures that local effects can be ignored. The cross-section of the column is shown in Fig. 4(b). A 3 m length of 5 mm diameter steel wire is attached to Q at one corner of the column at the top. The wire lies in the horizontal plane at 45 degrees to the x and y axes. A force F is applied to the wire such that the wire elongates by 27.36 mm. A second force, $P = 1000$ kN, is applied vertically upwards at the centre of the top diaphragm plate. The self-weight of the column is negligible.

(a) Find the tension in the wire. [2]

(b) Calculate the displacement in both the x and y directions of the point O, located at the origin of the x-y axes at the centre of the top diaphragm plate. [4]

(c) Determine the angle through which the top diaphragm plate at B rotates about the longitudinal z-axis of the column due to the applied loads. [4]

(d) Find the location and magnitude of the maximum longitudinal bending stress in the cross-section at the base of the column. [4]

(e) At this same cross-section determine the shear stress at R, located at the centre of the flange of the column section as shown in Fig. 4(b), due to the force F in the wire. [6]

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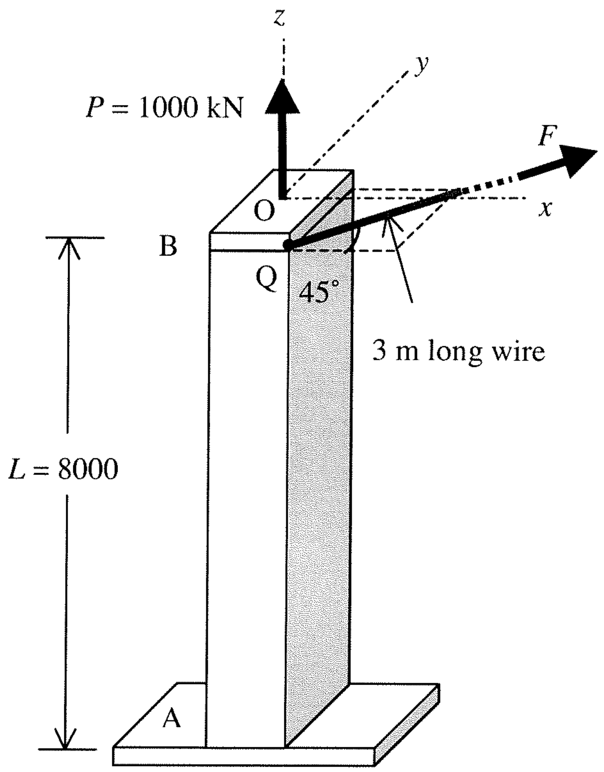
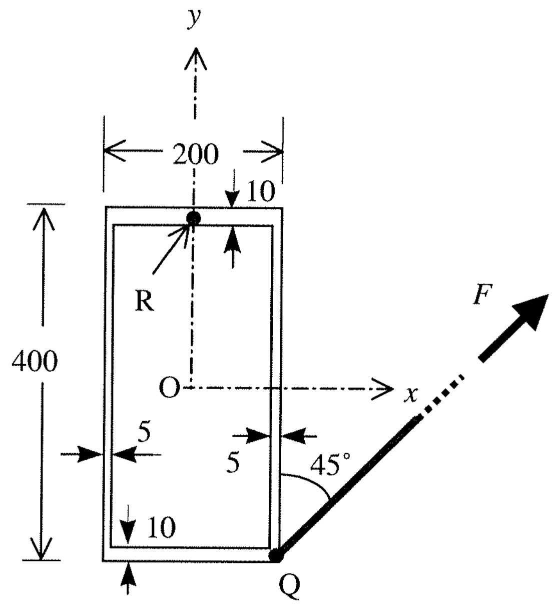


(a) Column



(b) Cross-section

Not to scale

Dimensions in mm unless indicated otherwise

Fig. 4

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5 The rectangular reinforced concrete bridge slab ABCD shown in Fig. 5 is fully fixed along abutments BC and AD. The bridge is required to carry a single point load of magnitude P, at mid-span as shown. The slab is reinforced isotropically in the top and bottom so that its moment capacity in sagging is m per unit width and in hogging m' per unit width. The self-weight of the slab may be ignored.

(a) What property of the slab is of fundamental importance in deciding whether or not yield-line analysis is appropriate? [2]

(b) Consider the pyramid-shaped failure mechanism shown in Fig. 5(a) which is comprised of four rigid planar triangular regions of slab (n = 4). The position of point load P is fixed whereas the positions of the yield-lines EF and GH are defined by the parameter α.

(i) For this mechanism show that the collapse load is given by [5]

$$P = 2(m + m')\left(4\alpha + \frac{1}{\alpha}\right);$$

(ii) hence find the least upper bound value for P in terms of the moment capacities m and m' for this mechanism. [2]

(c) An alternative hexagonal-shaped mechanism, shown in Fig. 5(b), is now considered. This mechanism has six equal sized, rigid, planar triangular regions of slab (n = 6). Derive an expression for the least upper bound value of the collapse load, P in terms of the moment capacities m and m' for this mechanism. [4]

(d) A third failure mechanism is postulated as shown in Fig. 5(c). This consists of a circular fan pattern with a large number of equal sized, rigid, planar fan sectors (n = large). The fan radius r is half the span, i.e. $r = L/2$. Derive an expression for the least upper bound value of the collapse load P in terms of the moment capacities m and m' for this mechanism. [5]

(e) What is your best estimate of the collapse load P of this bridge? [2]

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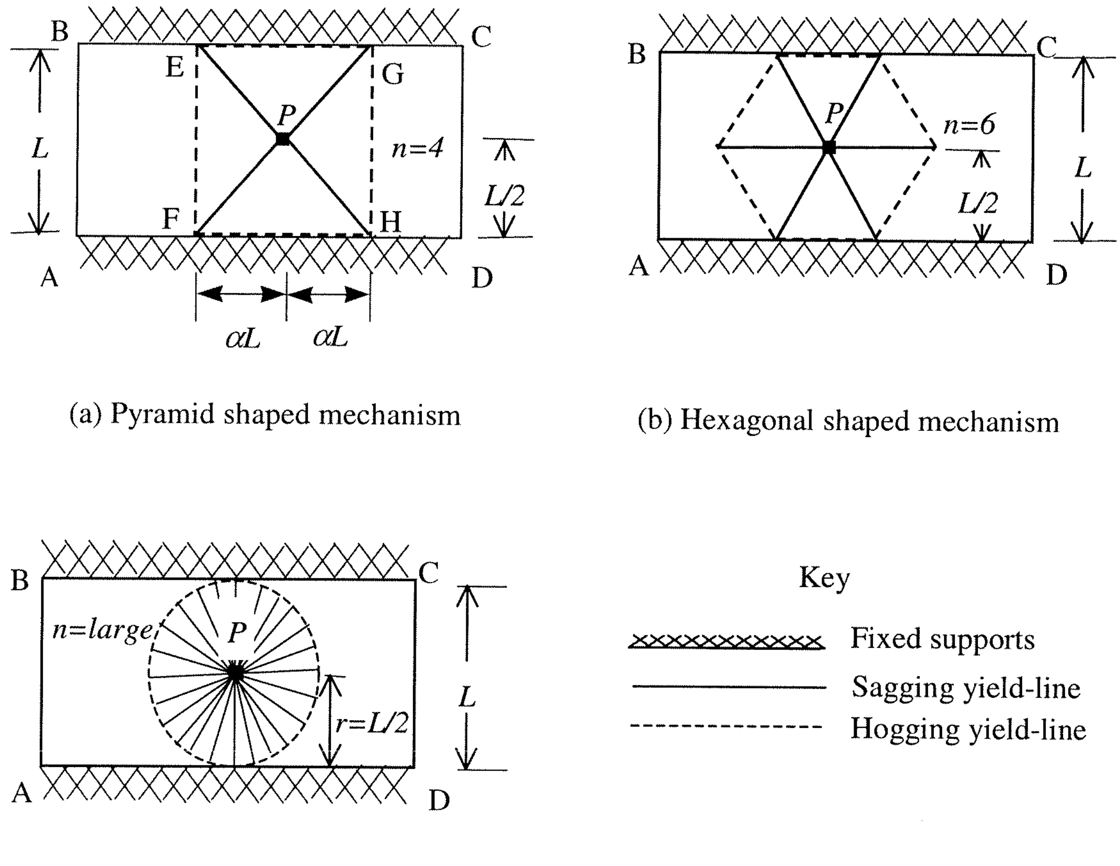

(a) Pyramid shaped mechanism

(b) Hexagonal shaped mechanism

(c) Circular fan shaped mechanism

Fig. 5

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6 (a) There are three basic principles which are fundamental to plastic structural analysis. These are (1) equilibrium, (2) the material law (also referred to as the yield criterion) and (3) compatibility. State which, if any, of these three principles are invoked when undertaking each of the following:

(i) Lower Bound plastic analysis; [2]

(ii) Upper Bound plastic analysis. [2]

(b) A portal frame is shown in Fig. 6. All the members have the same cross-section (Universal Beams UB127 × 76 × 13) with fully plastic moment M_p. The frame span is L. A vertical load of total magnitude W is uniformly distributed across the inclined roof member over the full span, L. A horizontal point load, H, is applied at D. You may assume that any plastic hinges will only form at the base connections (A, E), the corners (B, D) or at C, which is midway along member BD. The yield stress of the steel is $\sigma_y = 350$ N/mm^2.

(i) Sketch three feasible collapse mechanisms, clearly identifying the location of the plastic hinges on your drawing. [3]

(ii) Plot to scale an interaction diagram relating WL/M_p and HL/M_p for the mechanisms identified in (i) above. [10]

(iii) If $L = 5$ m and $W = H$ calculate the magnitude of the least upper bound estimate of collapse load for this structure. [3]

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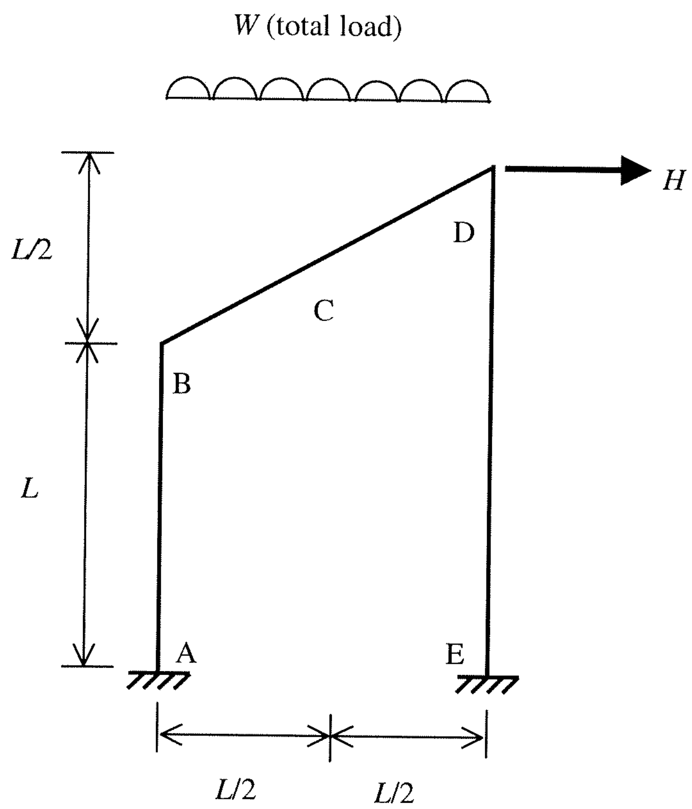

Not to scale

Fig. 6

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