ENGINEERING TRIPOS PART IB

Thursday 5 June 2003 2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than ***four*** *questions.*

Answer at least **one** *question from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

(TURN OVER

SECTION A

*Answer at least **one** question from this section*

1 Figure 1 shows a feedback control system where $K(s)$, $G(s)$ and $H(s)$ are the transfer functions of the controller, the plant and the transducer respectively.

(a) Derive an expression for the controlled output $\bar{y}(s)$ in terms of the inputs $\bar{r}(s)$, $\bar{d}_i(s)$ and $\bar{d}_o(s)$ in its simplest form. [6]

(b) The transfer function of the plant is given by

$$G(s) = \frac{100}{s(s^2 + 20s + 100)}$$

and $H(s) = 1$.

Determine for a controller with $K(s) = K_p$, the steady state value of $e(t)$ in each of the following cases assuming that the closed-loop system is stable:

(i) $r(t) = 0$, $d_i(t) = u(t)$ and $d_o(t) = 0$;

(ii) $r(t) = 0$, $d_i(t) = 0$ and $d_o(t) = t$;

where $u(t)$ denotes the unit step function. [6]

(c) How should the controller be modified to reject the effects of constant disturbances as in (b) part (i). For your modified controller, calculate the steady state error for the disturbances given in (b) part (i) and (ii). [4]

(d) For the original controller in (b) and for $r(t) = 0$, $d_i(t) = 0$ and $d_o(t) = \sin 10t$, show that the amplitude of $y(t)$ can be kept below 2 if $K_p \leq 10$ and that the system is unstable for $K_p \geq 20$. [4]

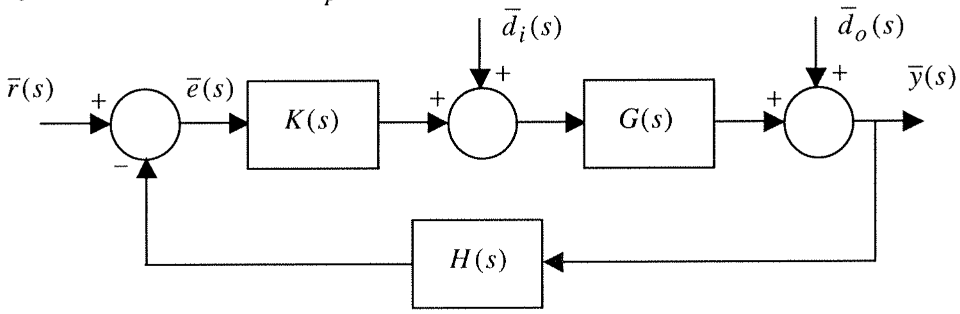


Fig. 1

2 A frequency synthesiser is implemented using a phase locked voltage controlled oscillator (VCO) as shown in Fig. 2. The instantaneous rate of change of the phase $\phi(t)$ at the output of the VCO in response to its input voltage $v_c(t)$ is given by

$$\frac{\mathrm{d}\phi(t)}{\mathrm{d}t} = K_o v_c(t)$$

where K_o is the gain of the VCO. The phase error detector output is given by

$$e(t) = K\left[\theta(t) - \frac{\phi(t)}{N}\right]$$

where K is the gain of the phase detector and $\theta(t)$ is an input disturbance.

(a) If the Loop Filter has the transfer function $F(s) = 1/(1+\tau s)$, where τ is the time constant, show that the transfer function between $\bar{\theta}(s)$ and $\bar{v}_c(s)$ is

$$v_c(s) = \frac{sK_a}{\left(\frac{K_a\tau}{K}\right)s^2 + \left(\frac{K_a}{K}\right)s + 1}\bar{\theta}(s)$$

where $K_a = N/K_o$. [6]

(b) If K=1 and N=100, select K_o and τ to achieve a damping factor of $1/\sqrt{2}$ and a natural frequency of 2 rad/s. [6]

(c) For the parameters in (b), sketch $v_c(t)$ in response to the disturbance

$$\theta(t) = \begin{cases} 0 & t < 0 \\ \left(1/\sqrt{2}\right)t & t \geq 0 \end{cases}$$

[8]

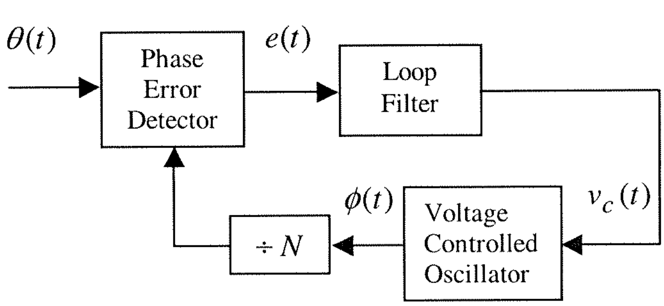


Fig. 2

(TURN OVER

3 (a) Figure 3 shows the Bode diagram for an asymptotically stable linear system with a transfer function of the form

$$G(s) = \frac{as}{\left(\frac{s^2}{\omega_n^2} + 0.04s + 1\right)(1 + sT_1)(1 + sT_2)}$$

Estimate the values of a, T_1, T_2 and ω_n. [8]

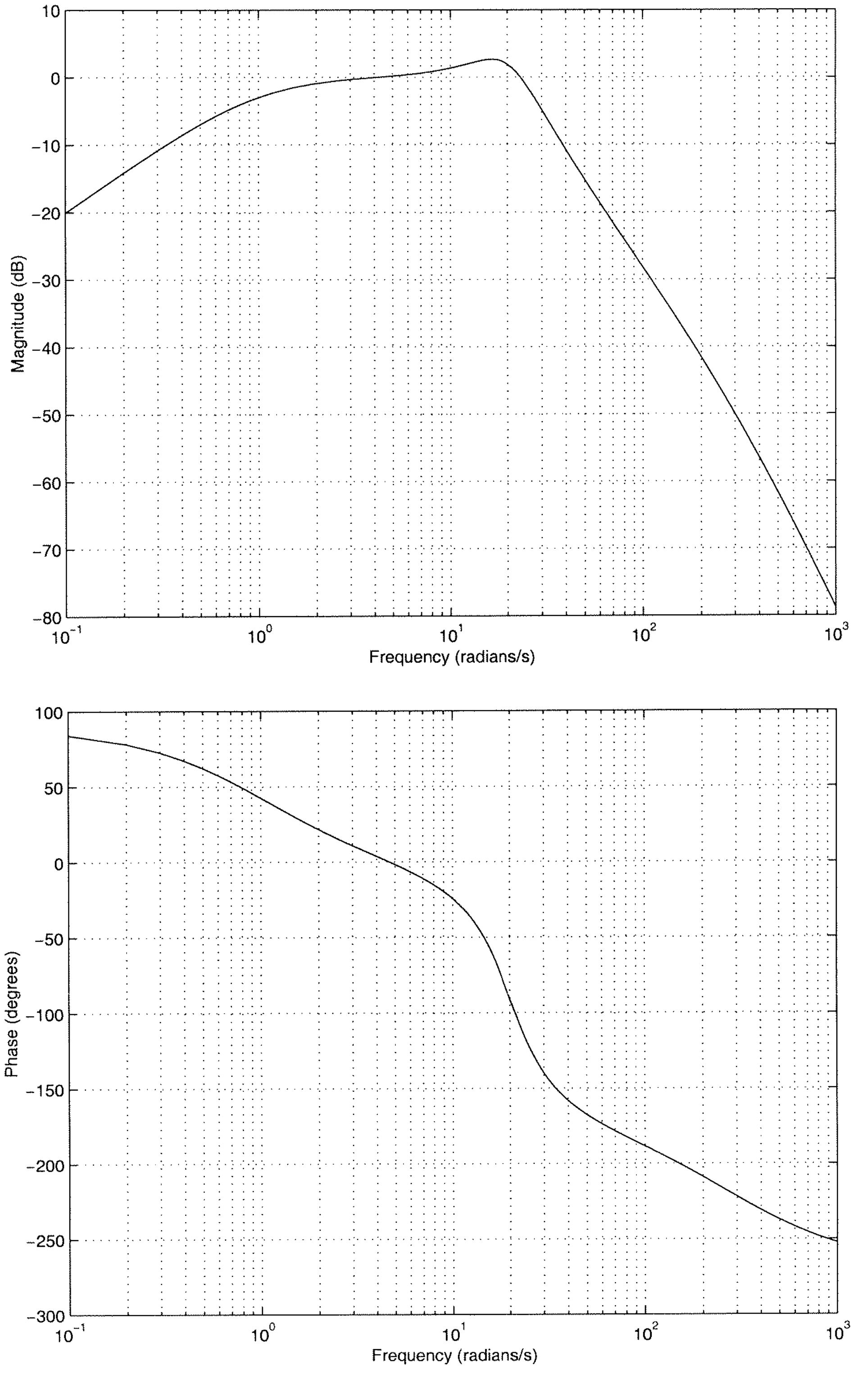
(b) Given a physical system, briefly describe how the information needed to plot its Bode diagram may be determined experimentally. Mention any difficulties which may arise. [6]

(c) Explain what is meant by the gain margin (GM) and the phase margin (PM) of a feedback system. The system with the Bode diagram given in Fig. 3 is controlled by a proportional negative feedback controller with a gain of 7. Graphically determine the GM and PM for the closed loop system and comment on its time domain performance.

[6]

Note: The Bode Diagram of $G(s)$ is given in Fig. 3. An extra copy is provided on a separate sheet and it should be handed in with your answer if constructions are made on it.

(cont.



Magnitude (dB)
10
0
-10
-20
-30
-40
-50
-60
-70
-80
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (radians/s)
Phase (degrees)
100
50
0
-50
-100
-150
-200
-250
-300
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (radians/s)

Fig. 3

4 (a) State the Nyquist stability criterion as it applies to the system of Fig. 4 where $K(s)$ and $G(s)$ are transfer functions of asymptotically stable systems. [4]

(b) Consider the feedback system of Fig. 4 with $K(s) = K$. The Nyquist diagram of $G(s)$ is given in Fig. 5. What is the maximum positive value of K for which the closed loop system in Fig. 4 is stable? What is the closed loop frequency of oscillation if K takes this maximum positive value? [6]

(c) The controller transfer function $K(s)$ is now modified so that it includes the effect of a pure time delay evident in the real system. Consequently

$$K(s) = Ke^{-4s}.$$

For K=1 sketch the Nyquist diagram of $K(s)G(s)$ and find the maximum positive value of K for which the closed loop system is stable. [6]

(d) Sketch against frequency the magnitude of the closed loop frequency responses (from r to y) for the feedback systems in (b) and (c) with $K = 1$. Briefly comment on the step responses of the two closed loop systems. [4]

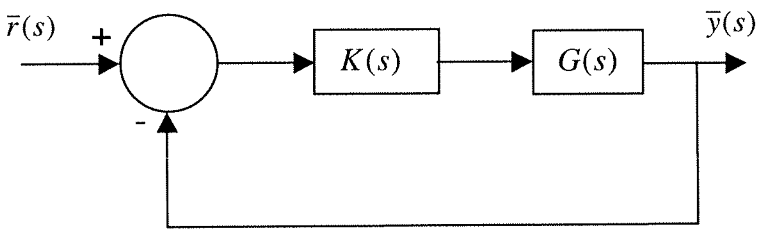


Fig. 4

Note: An extra copy of Fig. 5 is supplied on a separate sheet. This may be annotated with your constructions and handed in with your answer to Question 4.

(cont.

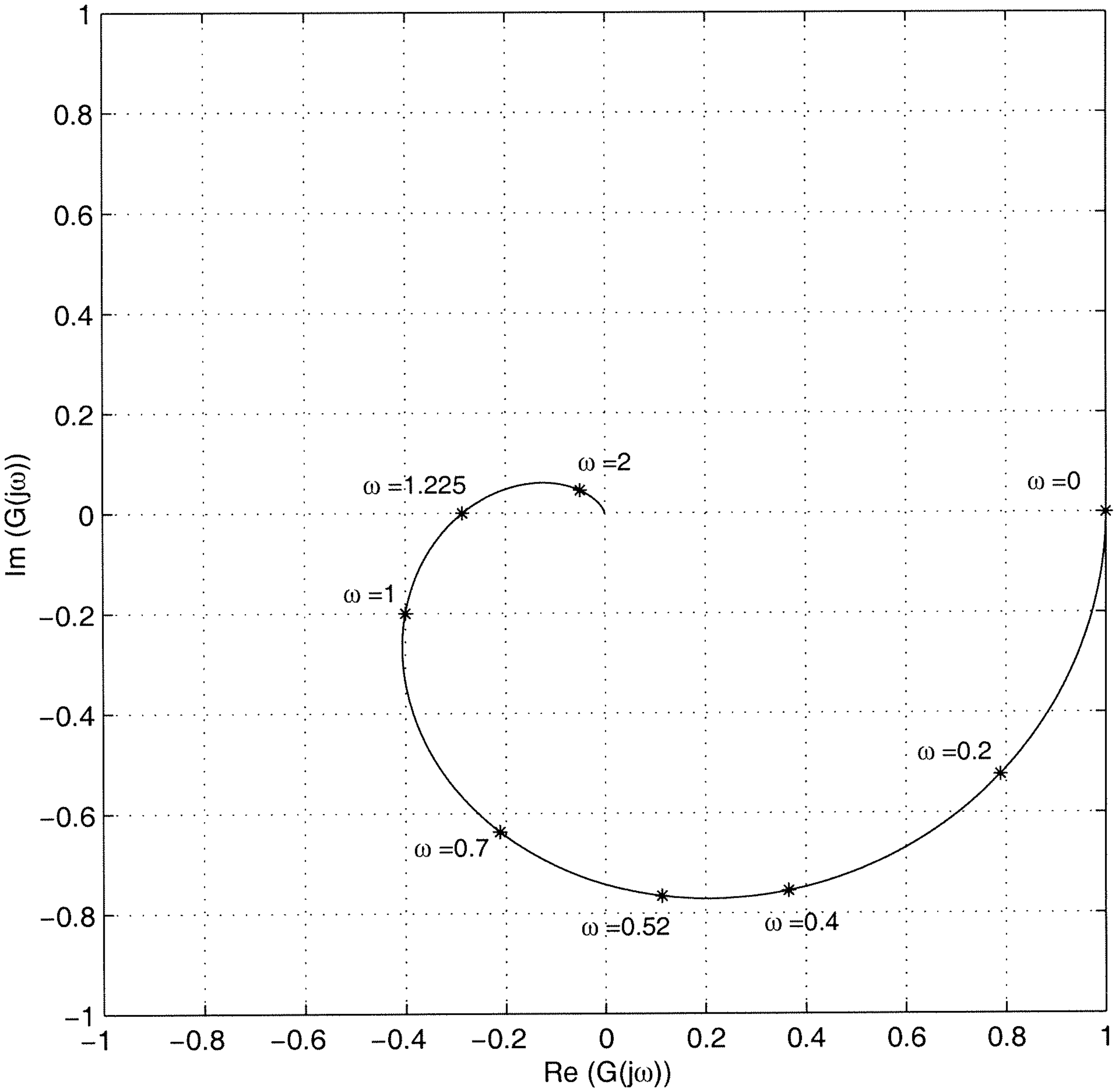


Fig. 5

(TURN OVER

SECTION B

*Answer at least **one** question from this section*

5 (a) With reference to the digitisation of analogue signals, explain what is meant by *aliasing distortion* and *quantisation distortion*. Illustrate your answers with appropriate sketches. [8]

(b) It is required to digitise a telephone speech signal containing frequencies up to 3400 Hz. If a sampling frequency of 8 kHz is to be used, what is the maximum transition bandwidth of the required lowpass filter as a percentage of its passband bandwidth? [4]

(c) Show that the rms noise voltage introduced by an ideal uniform quantiser with a step size δV is given by

$$\delta V / \sqrt{12} \ .$$

State any assumptions made. [4]

(d) Calculate the minimum resolution (in bits) of the Analogue to Digital Converter (ADC) required to achieve a signal to noise ratio (SNR) of at least 35 dB for a zero mean input signal for which the rms amplitude is 0.5774 times its peak amplitude. Assume that the input signal is scaled to occupy the full input range of the ADC.

If the input signal is assumed to be a speech signal, how could the ADC be modified in order to maintain a more-or-less constant SNR as a function of the input signal power? [4]

6 The UK standard for analogue TV is 625 lines/frame, 25 frames/s and 2:1 interlacing with an aspect ratio of 4:3 .

(a) If the transmitted image for the described TV system consists of a single vertical line, one pixel wide on a black background, show using the Fourier series analysis of periodic waveforms presented in the Electrical and Information Data Book that the Fourier series representation for the resulting TV signal is

$$y(t) = 1.2\times10^{-3}\left[1+2\sum_{n=1}^{\infty}\frac{\sin(1.2\times10^{-3}n\pi)}{1.2\times10^{-3}n\pi}\cos(2\pi\times15.625\times10^{3}nt)\right]$$

assuming that white is transmitted as 1 V and black as 0 V. Ignore synchronization pulses and blanking periods and assume equal horizontal and vertical resolution. [8]

(b) Show that the bandwidth of the signal $y(t)$ in (a) is approximately 5.78 MHz if the bandwidth is defined as being equal to the frequency where the power of the harmonic components fall to 3 dB less than that of the fundamental component.

Note: sinc(1.392)=0.707 [5]

(c) Calculate the approximate transmitted signal bandwidths if the following modulation schemes are used

(i) Amplitude Modulation (AM);
(ii) Frequency Modulation (FM) with a modulation index of three.

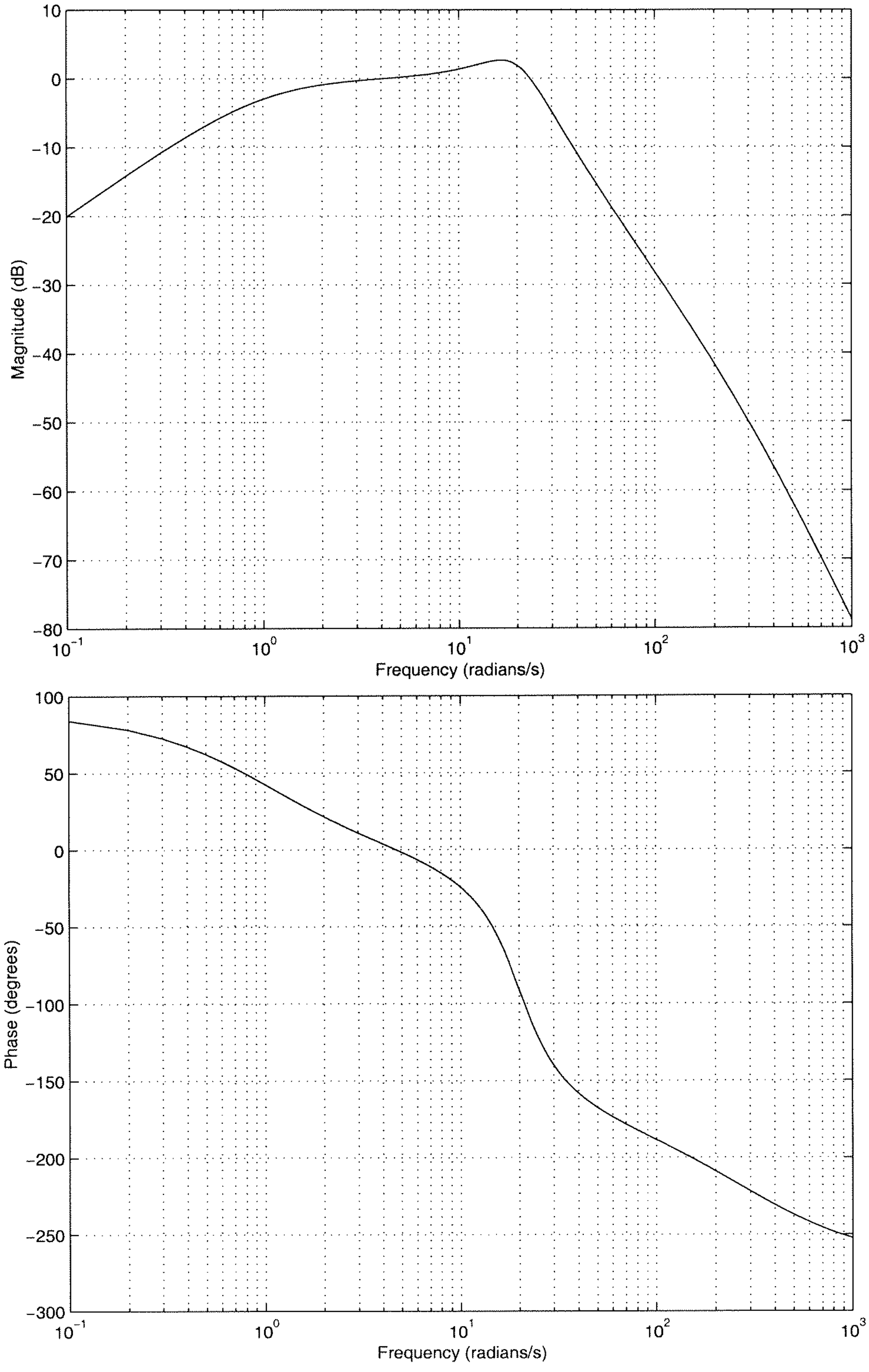
Why might the apparently wasteful FM scheme be chosen in preference to the AM scheme in certain circumstances?

Briefly describe the modulation scheme employed in terrestrial analogue TV systems in order to reduce the transmitted signal bandwidth. [7]

END OF PAPER

ENGINEERING TRIPOS PART IB, Paper 6, 5th June 2003,   Candidate number:

Extra copy of Fig. 3 which may be annotated and handed in with your answer to Question 3.



Extra copy of Fig. 5 which may be annotated and handed in with your answer to Question 4