ENGINEERING TRIPOS PART IB 2004

Paper 1 Mechanics

Answers:

1. (a) I/m, $3I/ma$ (b) It/m $a - gt^2/2$ $3It/ma$
2. (a) ω clockwise (b) ω^2 clockwise (c) $4mL^2\omega^2$
3. (b) (i) V/a (ii) V/R (c) $\dfrac{7MV^2}{5mg}$
4. (b) $mg(r - a\cos\theta)$ $\tfrac{1}{2}(I_G + m(r-a)^2)\dot{\theta}^2$ (c) $0.807\sqrt{(g/r)}$
5. (a) $\sqrt{\dfrac{3g(1-\cos\theta)}{l}}$ clockwise $\dfrac{3g\sin\theta}{2l}$ clockwise

   (b) $\dfrac{3mg}{2}$ $\dfrac{mg}{4}$ (c) $\dfrac{mgl}{32}$
6. (a) 12.5g, 37.5g, both at $\theta = 210°$

   (b) two 10g weights 102.6° apart to implement 12.5g
   two 20g weights 40.8° apart to implement 37.5g

   (c) 10g

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Paper 2 : Structures

ANSWERS

Section A

1)

(b) (i) $\mathbf{t_0} = W\begin{bmatrix}1 & 1 & 0 & 0 & 0 & 0\end{bmatrix}^T$

(ii) $\mathbf{s_1} = \begin{bmatrix}-1 & 0 & 1 & 0 & -\sqrt{2} & 0\end{bmatrix}^T \qquad \mathbf{s_2} = \begin{bmatrix}1 & 1 & 0 & 1 & 0 & -\sqrt{2}\end{bmatrix}^T$

(iii) $\mathbf{t_0} = W\begin{bmatrix}0.540 & 0.681 & 0.141 & -0.319 & -0.199 & 0.451\end{bmatrix}^T$

2)

(a) 2 redundancies

(b) (i) At A: Vertical reaction = $17W/32$ Anti-clockwise moment = $WL/32$

At D: Vertical reaction = $15W/32$ No moment

(ii) Anti-clockwise rotation of C = $\dfrac{7WL^2}{192EI}$

Horizontal displacement of C = $\dfrac{WL^3}{64EI}$ (no vertical displacement)

(iii) Maximum bending moment is the sagging moment that occurs in BC at a point $17L/32$ from B with magnitude $\dfrac{225}{2048}WL$

3)

(c) Bending moment about Oxx = 4.38 kNm

Shear force in direction Oy = -14.1 kN

Torque about Ozz = 0.624 kNm

Section B

4. a) OK, b) OK, c) 4306 cm^3

5. a) 317.9 cm3 , b) $W = 5Yt^2/4L^2$

6. b) Slip circle: $4V - H = 8\pi\ bk$

Sliding block $\sqrt{3}V + H = 12\ bk$

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Paper 3 MATERIALS

Numerical Solutions

2. (b) $Q \approx 100$ kJ/mol
   (c) Length change = 45 μm

3. (c) Silica glass

5. (c) $Q = 106$ kJ/mol

Engineering Tripos Part IB Paper 4

Answers

1. (b) 62.4°C 625.5 kJ (c) 1774 kJ

2. (b) 733.5 K 594.2 m/s 39.2 % (c) 256.8 kJ/kg 57.5%

3. (b) 12.8 267°C (c) -296.1 J/kgK

4. (a) $V_3 = V_2\left[1 - \left(\frac{d_i}{d_o}\right)^2\right]$ (b) $p_2 - p_3 = -\rho V_2^2\left[1 - \left(\frac{d_i}{d_o}\right)^2\right]\left(\frac{d_i}{d_o}\right)^2$

(c) $Q\Delta p_0 = \frac{\pi}{8}(d_o^2 - d_i^2)\,\rho V_2^3\left(\frac{d_i}{d_o}\right)^4$ (d) $Q\Delta p_0 = \frac{\pi}{8}(d_o^2 - d_i^2)\,\rho V_2 V_\theta^2$

5. (a) 2.529 m/s (c) 0.2, 0.0069, depending on definition of denominator for c_D
(d) 60%, 7.2 kN (e) 9.61 kN

Part IB Paper 5 2004

Answers

Q1 470 ohm, 463 ohm, 17.2 volt, 0.55 volt, 490 ohm

Q2 68 dB, 1.04 kohm, 1.19 kohm, 21.2 volt, 21.6 volt

Q3 1.5 kA, 42 km, 58 kV

Q4 310 rad/s, 31 rad/s, 33 kV, 520 MVA

Q5 0.016, 94 + j39 ohm, 10 A, 100 Nm

Q6 3×10^8 m/s, 6×10^6 m, 950 ohm, 2.4×10^8 m/s, 24 m, 47 ohm, 1.9 V, 0.9, 3.5 V

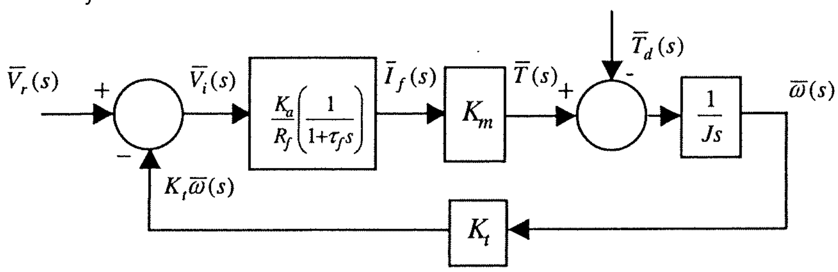
Q7 380 ohm, 1.6×10^{-5} m, 7 mT, 10^6 V/m

Engineering Triops Part 1B
Paper 6. Information Engineering, June 2004

Answers

1. By far the most popular question and also the best answered. Some candidates experienced difficulties applying the final value theorem in part (b). Some candidates also had trouble applying the second order differential equation information in the Mechanics data book for the solution of part (c).
a)
b) (i) $-1/K$
(ii) $-bK_I/K$
c) $K = 56.25$, $K_I = 9$

2. The least popular question. Some candidates had difficulty determining the system block diagram which compromised their answers to the remainder of part (a). Difficulties again emerged when making use of the second order differential equation information in the Mechanics data book required for the solution of part (c).
a) $\dfrac{K_a K_m K_t}{R_f Js(1+\tau_f s)}$



b) $K_a = 5J/K_m K_t$

c) $\dfrac{1}{K_t}\left(\alpha - \dfrac{R_f}{K_m K_a}\right)$

3. The bookwork in part (a) was generally well answered. Even so, quite a number of candidates could not correctly account for the non-unity gain proportional controller when calculating the gain and phase margins. Most candidates proved capable of plotting the Bode diagram of the controller required in part (c). However, relatively few managed to correctly calculate the new gain and phase margins.
a) See notes.
b) GM = 5.3 dB, PM = 20°
c) GM=17.3dB, PM=63°. For the proportional controller the PM and GM are quite low so we can expect some overshoot and oscillation in response to a step change on the input. This will be reduced when the compensator is included owing to the raised GM and PM.

4. The bookwork element in part (a) of this question was well answered. A number of candidates had difficulty correctly calculating the gain margin and the maximum amplitude of the sensitivity function required in part (b). Surprisingly, few could correctly plot the closed loop frequency required in part (c) using the supplied Nyquist diagram.
a) See notes
b) (i) $k_p = 0.514$, GM = 5.51 (14.8 dB), (ii) $|S(jw)|_{max} = 1.74$
c) The peak value of the CLTF is about 1.3 and this occurs at about 0.6 rad/s. The CLTF does not rise significantly above unity, hence the step-response should not be too oscillatory, i.e., it will be quite well damped.

5. This question was attempted by about half of the candidates. Many attempted to try to obtain the formula given in part (a) using incorrect arguments. In part (b), despite using the right methodology, few candidates managed to obtain the final correct numerical value. Part (c) needed fewer calculations and consequently the candidates performed better.
a) See notes
b) 0.8156 km
c) 52 kHz

6. This was the most popular question in Section B. Parts (a), (b) and the final part of (d) were based on examples questions and were quite well answered. Part (c) was more difficult and, even amongst candidates who performed well in the remainder of the question, a significant number had trouble determining the correct answer. Part (d) was answered reasonably well but many candidates did not justify their answers.
a) See notes
b) Max envelope +/- 1.7V, min envelope +/- 0.3V. Max value of b = 1V, otherwise overmodulation will occur, giving rise to severe distortion on the demodulated signal.
c) Upper cut-off freq = $\omega_c+\omega_m$, lower cut-off freq = $\omega_c-\omega_m$. Modulation index = $b/2$.
d) Max efficiency of DSB-AM is 1/3. Double Sideband Suppressed Carrier (DSB-SC) AM and Single Sideband (SSB) AM.

Engineering Tripos Part IB, 2004
Paper 7: Mathematical Methods – Answers

1 (b) $\frac{1}{8}$ (c) $-\frac{3}{8}$

3 (b)

$$\begin{aligned} r^2\ddot{R} + r\dot{R} - \lambda R &= 0 \\ \ddot{X} + \lambda X &= 0 \end{aligned}$$

where λ is a positive or negative constant.
(c) $\alpha = \pm\beta$
(d) $T = 12\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} \left(\frac{r}{A}\right)^n \sin n\theta$

4 (a)

$$\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}, \quad \mathbf{U} = \begin{bmatrix} 4 & 2 & 0 & 3 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

A has rank 3.
(b) A basis for the nullspace of **A** is:

$$\begin{bmatrix} -3/4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The column space is spanned by the first three columns of **A**.
(c) Yes. All solutions given by:

$$\begin{bmatrix} 0 \\ 1/2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -3/4 \\ 0 \\ 0 \\ 1 \end{bmatrix} \alpha$$

5 (a) **A** has rank 2.
(b) $\hat{\boldsymbol{\theta}} = \begin{bmatrix} 3.82 \\ 8.89 \end{bmatrix}$, $E = 0.53$.

6 (b) $\frac{1}{\pi}$ (c) $\frac{2}{\sqrt{\pi}}$

7 (b) $\frac{1}{1 + 0.5e^{-i\omega T}}$ approximately

8 (b) $\frac{4}{9}$ (b) $\frac{1}{4}$ (b) $\frac{11}{12}$

ENGINEERING TRIPOS PART IB
Friday 4 June 2004
Paper 8 SELECTED TOPICS

Answers

SECTION A *Business Economics*

No numerical answer.

SECTION B *Civil and Structural Engineering*

3 Essay

4 (b) FoS = 1.96
(c) (i) FoS = 1.06
(ii) FoS = 0.62

5 (a) Max BM = 500 kNm
(b) d = 650 mm (with 50 mm cover)
(c) Max BM due to self weight = 89.7 kNm
(d) $A_s = 3157\ mm^2$
(e) shear reinforcement needed $v_s = 1.52\ N/mm^2$; S = 450 mm.

(TURN OVER

SECTION C *Mechanical Engineering*

6 (a) $\bar{p} = 2\sqrt{3}\ \dfrac{t}{\ell}$

(b) $\sigma_1 = \dfrac{\bar{p}}{3}\ \sigma_Y$

(c) $\sigma_2 = \dfrac{\bar{p}^2}{6}\ \sigma_Y$

8 (b) (i) $P = \dfrac{2bt^2}{\ell}\sigma_Y + 2bc\tau_C$

(ii) $P = 4\dfrac{btc}{\ell}\sigma_Y$

SECTION D *Aerothermal Engineering*

9 (a) 38.5 kNm

(b) 3 stages, $V_x = 187.2$ m/s, $\Delta h_o / U^2 = 1.51$

(c) $\alpha_2 = 70^0$, $\alpha_{2r} = 42.7^0$, $\alpha_{3r} = -61.2^0$

10 (b) 900.3K, 3.07 bar

(c) 395.3 m/s

(d) $\beta = 6.06$, $F_N / \dot{m}_c = 1025\,\mathrm{N/kgs^{-1}}$, $\eta_p = 77.5\%$

11 (a) 600 - 700 m^2

(b) $L \approx W$, $F_N = D + W \sin\theta$

(c) $F_N = 263$ kN

(d) 243 kN per engine

(TURN OVER

SECTION E *Electrical Engineering*

12 (b) ~ 74 (dependent upon assumptions made)

(c) (i) 1.86×10^{-11}s

(ii) Use metal with lower resistivity
Use lower k dielectric

13 (b) $8.5 \times 10^{28} m^{-3}$, $4.4 \times 10^{-3}\ m^2V^{-1}s^{-1}$

(c) .0847 $ohm^{-1}\ m^{-1}$

(e) 4×10^{-6} m
32 ps

14 (c) (i) $2.5 \times 10^{19} m^{-3}$

(ii) 2×10^{-6}m

(iii) 8×10^{-4}V (low because of very low N value used)

(iv) 8×10^{3}V - very much smaller than breakdown fields.

SECTION F *Information Engineering*

No numerical answer.

SECTION G *Biological and Medical Engineering*

19 (a) (ii) 3.3 Hz
(iii) $f \propto m^{-1/6}$
(b) 8.3 cm/s

(TURN OVER

SECTION H *Manufacturing, Management and Design*

21 No numerical answer.

22 No numerical answer.

23 The EVA for Option A is £180k, and for Option B £120k.