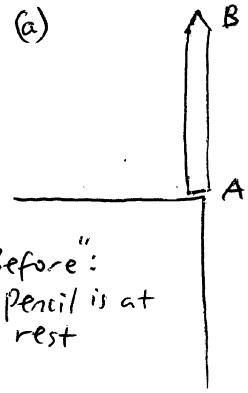
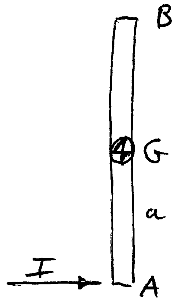
Part IB Mechanics 2004 Engineering Tripos

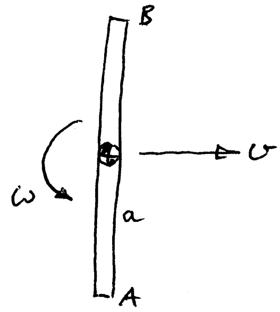
1. (a)



"Before": pencil is at rest



"During": The impulse is the largest force – friction and gravity can be neglected



"After": motion due to impulse
Consider change in moment of momentum
Take moments about G

$\therefore \quad I a = \frac{1}{3} m a^2 \omega$ ①

and change in linear momentum

$I = m v$ ②

① and ② give

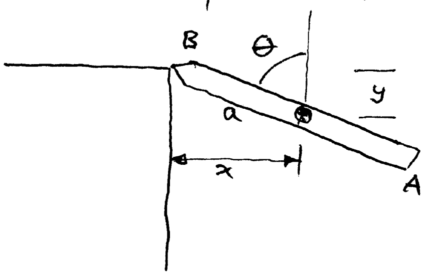
$$v = \dot{x} = \frac{I}{m}$$
$$\omega = \dot{\theta} = \frac{3I}{ma}$$
ans.

b/ Integrate these to give $\underline{\underline{x = \frac{I}{m} t}}$ and $\underline{\underline{\theta = \frac{3I}{ma} t}}$

So $\underline{\underline{x = \frac{a\theta}{3}}}$ ③

Also from projectile motion $\underline{\underline{y = a - \frac{1}{2} g t^2}}$ ④

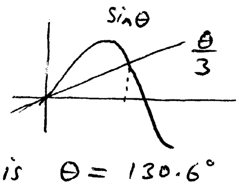
c/ When the pencil hits the table $x = a \sin\theta$ and $y = -a\cos\theta$



Using ③ and ④

$\therefore \quad \frac{a\theta}{3} = a \sin\theta$

$\therefore \quad \sin\theta = \frac{\theta}{3}$



whose solution is $\theta = 130.6°$ or 2.279 rad

and $a - \frac{1}{2} g t^2 = -a\cos\theta$

$\therefore \quad t^2 = \frac{2a}{g}(1 + \cos\theta) \quad \therefore \underline{\underline{t = 0.836\sqrt{\frac{a}{g}}}}$

Value of impulse $I_0 = m\dot{x} = m\frac{x}{t} = m\frac{a\theta}{3t}$

$$= m \frac{2.279\, a}{3 \times 0.836}\sqrt{\frac{g}{a}} = \underline{\underline{0.908\, m\sqrt{ga}}}$$

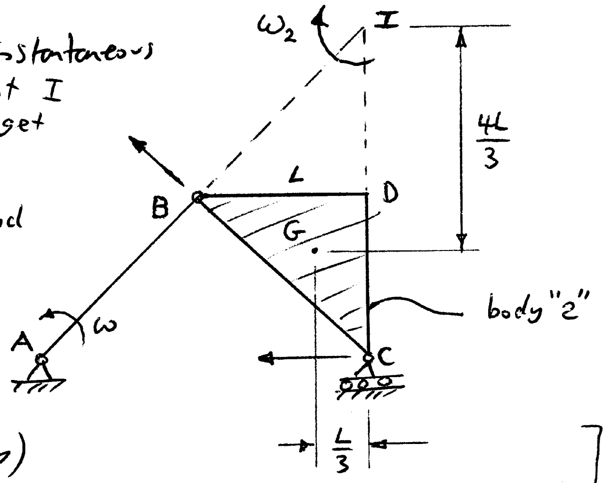
Well done by those that could do it (33 candidates got full marks) but poorly done by the rest. The main problem was inability to apply moment of momentum principles and poor sketches showing θ incorrectly.

2. (a) Velocities are best found using instantaneous centres. The I.C. is found at I and noting that $\overline{AB} = \overline{IB}$ we get

$$\underline{\underline{\omega_2 = \omega \circlearrowleft}}$$

Then noting $\overline{IG}$ is $\frac{4L}{3}$ vertically and $\frac{L}{3}$ horizontally gives

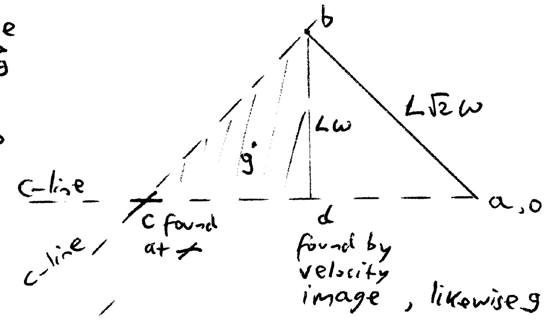
$$\underline{\underline{\underline{v}_G = \left(-\frac{4L}{3}\underline{i} + \frac{L}{3}\underline{j}\right)\omega}}$$



[Alternatively (and most did it this way) Velocity diagram:
$\underline{v}_G$ is determined directly from the diagram as the vector $\vec{og}$

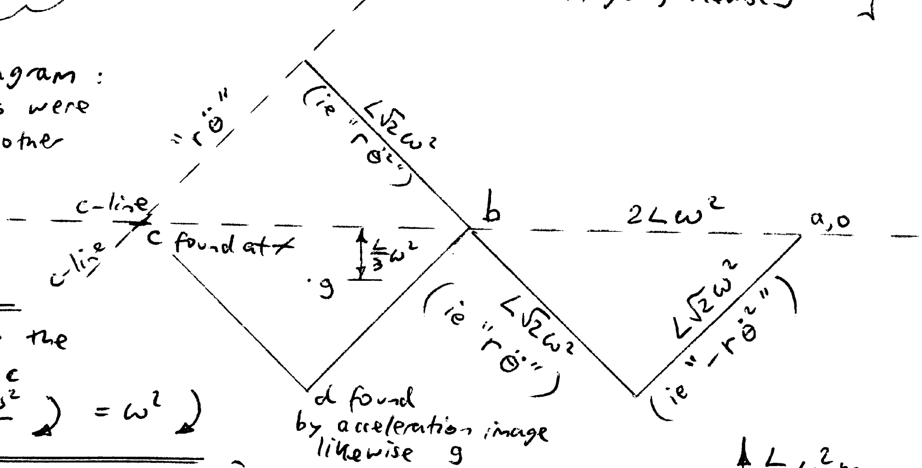
For ω_2, note that $\underline{v}_{BD} = \underline{\omega}_2 \times \underline{r}_{BD}$

hence $\omega_2 = \frac{bd}{BD} \circlearrowright = \omega \circlearrowright$]



Many did not give the sign of ω

(b) Acceleration diagram:
(no correct answers were obtained by any other method.)



From the diagram

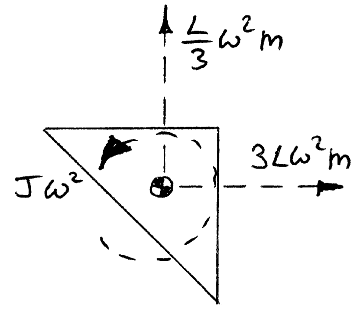
$$\underline{\underline{\underline{a}_G = -3L\omega^2\underline{i} - \frac{L}{3}\omega^2\underline{j}}}$$

and for $\dot{\omega}_2$ consider the "$r\ddot{\theta}$" component of bc

hence $\dot{\omega}_2 = \frac{L\sqrt{2}\omega^2}{L\sqrt{2}} \circlearrowright = \omega^2 \circlearrowright$

Many did not give the sign of $\dot{\omega}_2$

(c) For driving torque use the method of instantaneous power. Use d'Alembert's principle to deal with inertia forces.



d'Alembert forces

$$T\omega + 3L\omega^2 m\left(-\frac{4L}{3}\omega\right) + \frac{L}{3}\omega^2 m\left(\frac{L}{3}\omega\right) + \frac{1}{9}mL^2\omega^2(-\omega) = 0$$

ie net power in $= 0$

$$\therefore \quad T = \left(4 - \frac{1}{9} + \frac{1}{9}\right)mL^2\omega^2$$

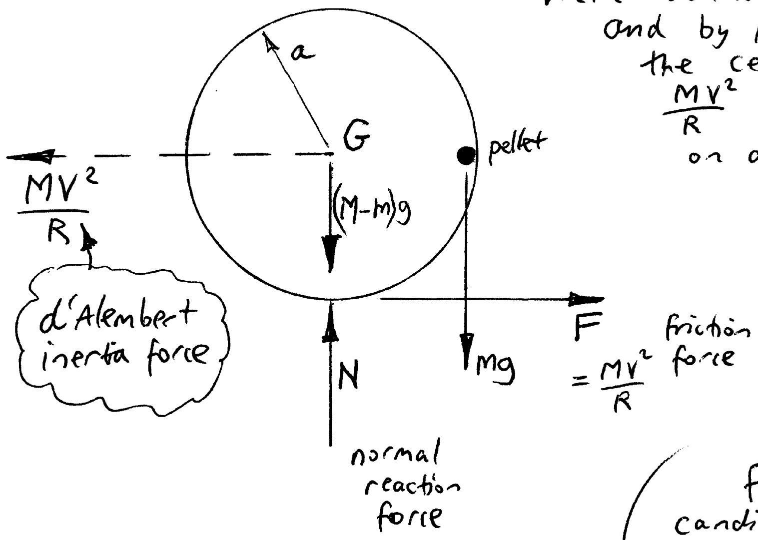
$$J = \frac{1}{9}mL^2$$

$$\underline{\underline{T = 4mL^2\omega^2}}$$

Generally well done. 39 candidates got full marks

Many tried using statics but omitted the reaction at C. Many left out the d'Alembert couple $J\omega^2$

3. (a) Free-body diagram



We're asked to assume $M \gg m$ and $R \gg a$ and by implication to take the centrifugal inertia force as $\frac{MV^2}{R}$ even though the pellet moves on a smaller radius.

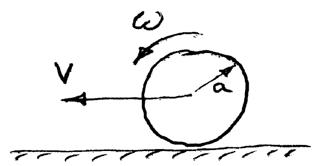
Take moments about G:

$$Q = mga - \frac{MV^2}{R}a$$

(For some reason, many candidates took moments about P or about the contact point with the ground)

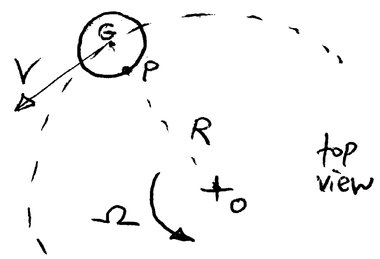
(b) (i) The ball rolls without slip

$\therefore \quad V = a\omega \qquad$ or $\quad \underline{\omega = \frac{V}{a}}$



(ii) The ball moves around the circular path with OPG always radial

$\therefore \quad V = R\Omega \qquad$ or $\quad \underline{\Omega = \frac{V}{R}}$



(c) Gyroscopic Precession :

$$Q = J\omega\Omega$$

$$\therefore \quad mga - \frac{MV^2}{R}a = \tfrac{2}{5}Ma^2 \frac{V}{a}\frac{V}{R}$$

$$\therefore \quad mgR = \left(M + \tfrac{2}{5}M\right)V^2$$

$$\therefore \quad R = \frac{7MV^2}{5mg}$$

If candidates knew about gyro effects then this was an easy question.
25 candidates got full marks which was about $\frac{1}{4}$ of those who attempted the question

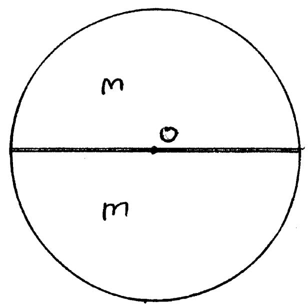
The biggest problem was that many didn't show the friction force F on their diagram and hence couldn't see how $\frac{MV^2}{R}$ could generate a couple about G.

HEMH June 2004

Q4 (a) From databook

$$a = \frac{2}{3} r \cdot \frac{1}{\pi/2} = \frac{4r}{3\pi} = 0.424r$$

Moment of inertia of whole cylinder of mass 2m $= \frac{1}{2}(2m)r^2 = mr^2$



∴, For the half-cylinder:

$$I_O = \frac{1}{2} m r^2$$

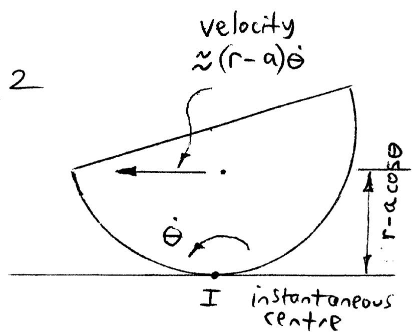
By Parallel axis theorem:

$$I_G = \frac{1}{2} m (r^2 - 2a^2)$$

$$I_G = 0.32\, m r^2$$

COMMENT:
Some students used the wrong databook case (a sphere instead of a cylinder). A few applied the parallel axis theorem incorrectly adding the correction term to the moment of inertia about the center of half-circle rather than subtracting it.

(b) For rocking about the contact point assuming small angle θ.



$$K.E. = \frac{1}{2}\left(I_G + m(r-a)^2\right)\dot{\theta}^2$$

$$P.E. = mg(r - a\cos\theta)$$

COMMENT:
Generally well-done though some students had difficulty in computing the kinetic energy

for rolling about the contact point.

(c) Use Rayleigh's principle

$$\omega_n^2 = \frac{mga}{I_G + m(r-a)^2}$$

$$= \frac{0.424\, mgr}{0.651\, mr^2}$$

$$\omega_n = 0.807\sqrt{\frac{g}{r}}$$

COMMENT: Generally well done.

Q5

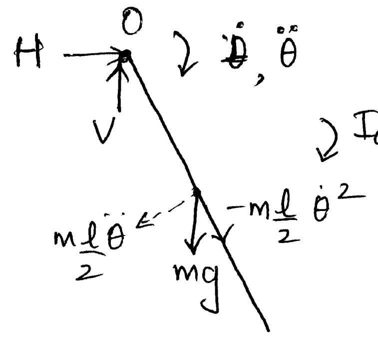
(a) $$I_O = \frac{1}{2} m l^2 + m\left(\frac{l}{2}\right)^2 = \frac{m l^2}{3}$$

Applying conservation of energy:

$$\frac{1}{2} I_O \dot{\theta}^2 = mg\frac{l}{2}(1-\cos\theta)$$

$$\dot{\theta} = \sqrt{\frac{3g(1-\cos\theta)}{l}} \quad \text{clockwise}$$

COMMENT: It is possible to differentiate $\dot{\theta}$ with respect to time to obtain $\ddot{\theta}$. Easier if you notice $\frac{d}{d\theta}\left(\frac{1}{2}\dot{\theta}^2\right) = \ddot{\theta}$. Some students made the mistake of differentiate $\dot{\theta}$ with respect to θ to obtain $\ddot{\theta}$!



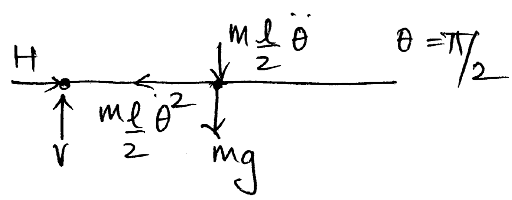
$$\Sigma M_O = 0$$

$$I_O \ddot{\theta} = mg\frac{l}{2}\sin\theta$$

$$\ddot{\theta} = \frac{3g}{2l}\sin\theta \quad \text{clockwise}$$

COMMENT: Some students incorrectly used the small angle approximation $\sin\theta \approx \theta$!

(b) $$V - mg = -m\frac{l}{2}\ddot{\theta}$$



$$V = mg - m\frac{l}{2}\ddot{\theta}$$

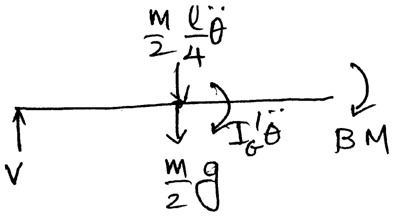
$$V = mg - \frac{3}{4}mg = \frac{mg}{4}$$

$$H = m\frac{l}{2}\dot{\theta}^2 = \frac{3}{2}mg$$

magnitude of force acting on bearing

$$F = mg\sqrt{\frac{1}{16} + \frac{9}{4}} = \frac{\sqrt{37}}{4}mg.$$

(c) $BM = \frac{m}{2}\frac{\ell^2}{48}\ddot{\theta} + \frac{mg}{4}\frac{\ell}{2} - \left(\frac{mg}{2} + \frac{m\ell\ddot{\theta}}{8}\right)\frac{\ell}{4}$



$\ddot{\theta} = \frac{3}{2}\frac{g}{\ell}$ at $\theta = \pi/2$

$I_G' = \frac{m}{2}\frac{\ell^2}{48} = \frac{m\ell^2}{96}$

$\therefore BM = \frac{mg\ell}{32}$ direction as shown.

COMMENT: Parts (b) and (c) generally well done. Some students missed out the d'Alemberts forces in the free body diagram. Students did not seem to use dimensional analysis for a quick and simple check to their solutions to spot algebraic errors.

Q6

(a) Static balance $\sum_i M_i r_i = 0$

Total balance mass for static balance

$$= \frac{5}{4} \times 40g = 50g$$

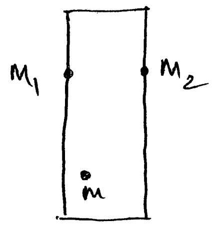
This mass should be placed 180° opposite to the 40g mass on the rim.

Dynamic balance $\sum_i M_i r_i x_i = 0$.

Split 50 g mass m_1 and m_2 as shown

$$m_2 = \frac{1}{4}(50) = 12.5g.$$

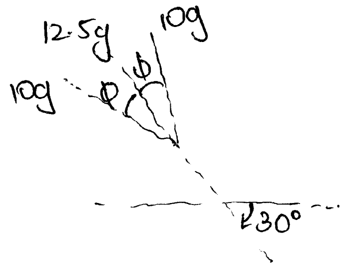
$$M_1 = \frac{3}{4}(50) = 37.5g.$$



Both masses placed at $\theta = 210^\circ$.

COMMENT: Some students were not able to visualize the problem. Several did not know how to formulate the condition for dynamic balance correctly. Some students came up with a solution to use four masses when two would have sufficed.

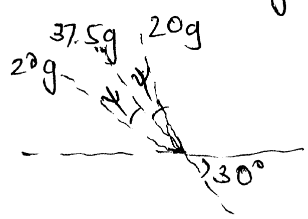
(b) Use two 10 g weights to implement 12.5g.



$$10 \cos\phi = 6.25$$

$$\phi = 51.3^\circ.$$

Use two 20 g weights to implement 37.5g.

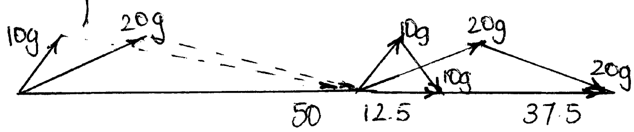


$$20 \cos\psi = 18.75$$

$$\psi = 20.36^\circ.$$

(c) Worst net out-of-balance force if 10g mass remains.

worst remaining out of balance force.



COMMENT: Parts (b) and (c) generally well done.

AAS
JUNE 2004