PART IB

PAPER 2 STRUCTURES, 2004

1)

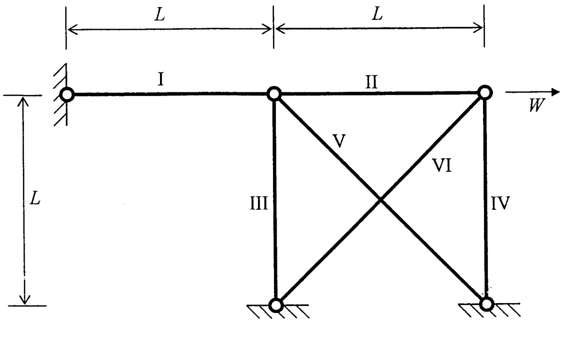


Figure 1

Bar tensions

$$\underset{\sim}{t} = \begin{bmatrix} t_{I} \\ t_{II} \\ t_{III} \\ t_{IV} \\ t_{V} \\ t_{VI} \end{bmatrix}$$

(a) Maxwell's equation $s - m = b + r - dj$

$m = 0$ by inspection $\therefore$ $s = 6 + 2\times3 - 2\times5$

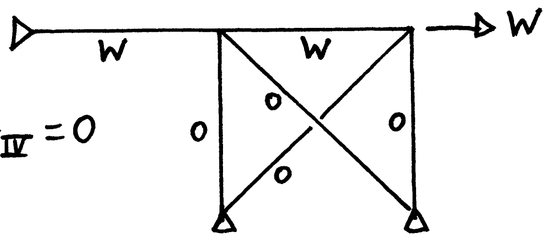
$s = 2$ states of self-stress (redundancies)

Alternatively remove two bars to make statically determinate

(b) Use bars III and IV as redundant bars.

(i) Hence find a particular equilibrium solution:

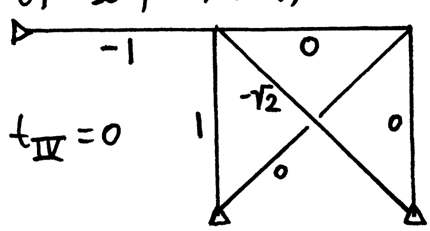
$t_{III} = t_{IV} = 0$



$$\therefore \underset{\sim}{t}_0 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} W$$

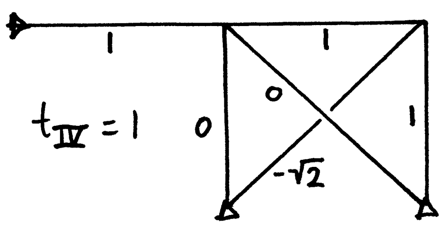
(ii) States of self-stress

$t_{III} = 1 \quad t_{IV} = 0$



$$\therefore \underset{\sim}{s}_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ -\sqrt{2} \\ 0 \end{bmatrix}$$

$t_{III} = 0 \quad t_{IV} = 1$



$$\therefore \underset{\sim}{s}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ -\sqrt{2} \end{bmatrix}$$

PART IB,

PAPER 2, STRUCTURES, 2004

Q1, cont'd.

(b) (iii) General solution $\underset{\sim}{t} = \underset{\sim}{t}_0 + x_1 \underset{\sim}{s}_1 + x_2 \underset{\sim}{s}_2$

Flexibility matrix $\underset{\sim}{F} = \begin{bmatrix} 1 & & & & \cdots & 0 \\ & 1 & & & & \vdots \\ & & 1 & & & \vdots \\ \vdots & & & 1 & & \\ \vdots & & & & \sqrt{2} & \\ 0 & \cdots & & & & \sqrt{2} \end{bmatrix} \dfrac{L}{EA}$

Bar extensions $\underset{\sim}{e} = \underset{\sim}{F}\,\underset{\sim}{t}$

Hence $\underset{\sim}{e} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \dfrac{WL}{EA} + \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ -2 \\ 0 \end{bmatrix} \dfrac{x_1 L}{EA} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ -2 \end{bmatrix} \dfrac{x_2 L}{EA}$

Compatibility: Use virtual work with $\underset{\sim}{s}_1$ and $\underset{\sim}{s}_2$ as sets of bar forces in equilibrium with zero external load to find x_1 & x_2 $\quad \Sigma\, \overbrace{F.d}^{} = \Sigma\, \underbrace{S.e}_{}$

$\therefore\ \underset{\sim}{s}_1 \cdot \underset{\sim}{e} = 0 \Rightarrow -\dfrac{WL}{EA} + (2+2\sqrt{2})\dfrac{x_1 L}{EA} + \dfrac{-x_2 L}{EA} = 0$

$\therefore\ W = (2+2\sqrt{2})x_1 - x_2 \quad ①$

$\underset{\sim}{s}_2 \cdot \underset{\sim}{e} = 0 \Rightarrow \dfrac{2WL}{EA} + \dfrac{-x_1 L}{EA} + (3+2\sqrt{2})\dfrac{x_2 L}{EA} = 0$

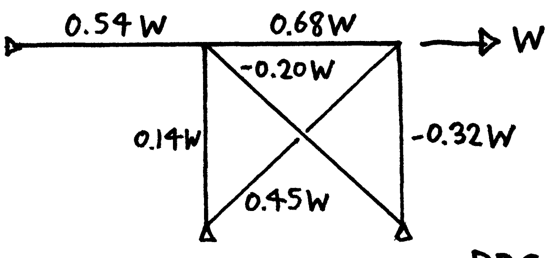
$\therefore\ 2W = x_1 - (3+2\sqrt{2})x_2 \quad ②$

① into ② $\quad 2W = x_1 - (3+2\sqrt{2})\left[(2+2\sqrt{2})x_1 - W\right]$

$\left[2 - (3+2\sqrt{2})\right] W = \left[1 - (3+2\sqrt{2})(2+2\sqrt{2})\right] x_1$

$\therefore\ x_1 = \dfrac{1+2\sqrt{2}}{13+10\sqrt{2}} W = 0.14105\,W \qquad x_2 = \dfrac{-3-4\sqrt{2}}{13+10\sqrt{2}} W = -0.31895\,W$

$\therefore\ \underset{\sim}{t} = \begin{bmatrix} 0.54000 \\ 0.68105 \\ 0.14105 \\ -0.31895 \\ -0.19947 \\ 0.45106 \end{bmatrix} W$



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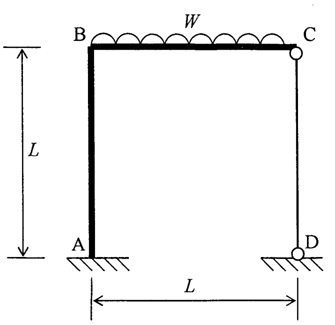
PART IB

PAPER 2, STRUCTURES, 2004

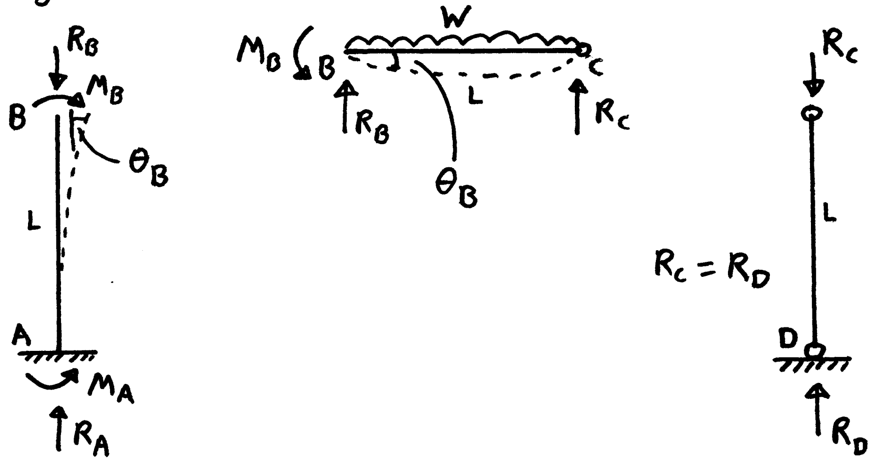
2)



(a) Introducing a hinge at B releases one d.o.f. and makes the frame statically determinate.

Hence there is one redundancy.

(b) (i) Let M_B be the unknown internal moment at B.



$R_A = R_B$

$M_A = M_B$

$R_C = R_D$

AB: $\theta_B = \dfrac{M_B L}{EI}$ BC: $\theta_B = \dfrac{WL^2}{24EI} - \dfrac{M_B L}{3EI}$

Compatibility of rotations: $\dfrac{M_B L}{EI} = \dfrac{WL^2}{24EI} - \dfrac{M_B L}{3EI}$

$$\therefore \quad M_B = \frac{WL}{24} - \frac{M_B}{3} \qquad \therefore \; M_B = \frac{WL}{32}$$

Moments about B for BC: $R_C L = \dfrac{WL}{2} - M_B = \dfrac{WL}{2} - \dfrac{WL}{32} = \dfrac{15WL}{32}$

$$\therefore \; R_D = R_C = \frac{15W}{32}$$

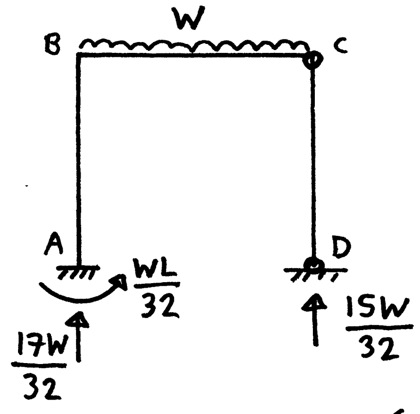
Vertical equil. $R_A = R_B = W - R_D = W - \dfrac{15W}{32} = \dfrac{17W}{32}$

PART IB

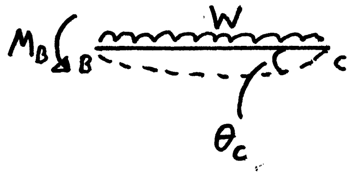
PAPER 2, STRUCTURES, 2004,

Q2 cont'd

Reaction forces and moments



(b) (ii) Rotation of C



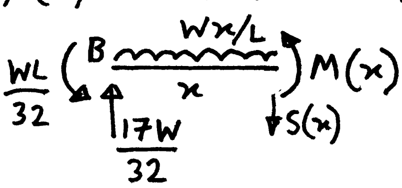
$$\theta_c = \frac{WL^2}{24EI} - \frac{1}{2}\frac{M_B L}{3EI} = \frac{WL^2}{24EI} - \frac{1}{2}\frac{WL}{32}\frac{L}{3EI}$$

$$= \frac{WL^2}{EI}\left(\frac{1}{24} - \frac{1}{192}\right) = \frac{7}{192}\frac{WL^2}{EI}$$

Deflection of C = horizontal displacement of B

$$= \frac{M_B L^2}{2EI} = \frac{WL}{32}\cdot\frac{L^2}{2EI} = \frac{WL^3}{64EI}$$

(b) (iii) Find maximum sagging moment in BC



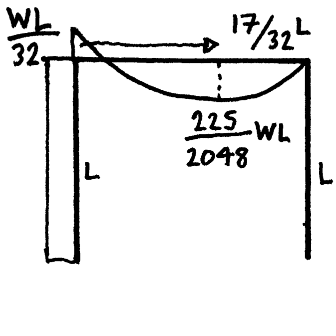
$$S(x) = \frac{17W}{32} - \frac{Wx}{L} = W\left(\frac{17}{32} - \frac{x}{L}\right)$$

$M(x)$ is a maximum when $\frac{dM}{dx} = S = 0$ $\therefore$ @ $x = \frac{17}{32}L$

$$M(x) = \frac{17W}{32}x - \frac{Wx}{L}\cdot\frac{x}{2} - \frac{WL}{32} = W\left(\frac{17}{32}x - \frac{x^2}{2L} - \frac{L}{32}\right)$$

$$\therefore M_{max} = W\left(\frac{17}{32}\cdot\frac{17}{32}L - \left(\frac{17}{32}L\right)^2\Big/2L - \frac{L}{32}\right) = \frac{225}{2048}WL$$

Bending moment diagram



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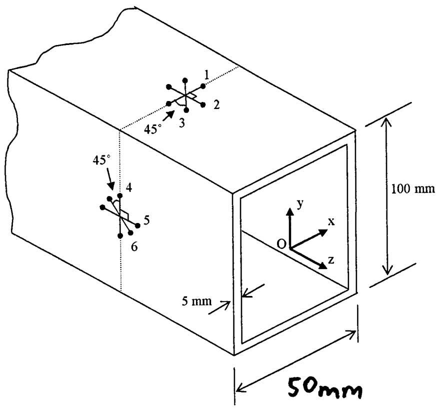
PART IB

PAPER 2, STRUCTURES, 2004

3) (a)



123 : $\varepsilon_{xx} = \varepsilon_{11} = -1.8\times10^{-4}$

$\varepsilon_{zz} = \varepsilon_{22} = 6\times10^{-4}$

$\varepsilon_{45°} = \varepsilon_{33} = 3\times10^{-4}$

456 : $\varepsilon_{yy} = \varepsilon_{44} = 0$

$\varepsilon_{zz} = \varepsilon_{55} = 0$

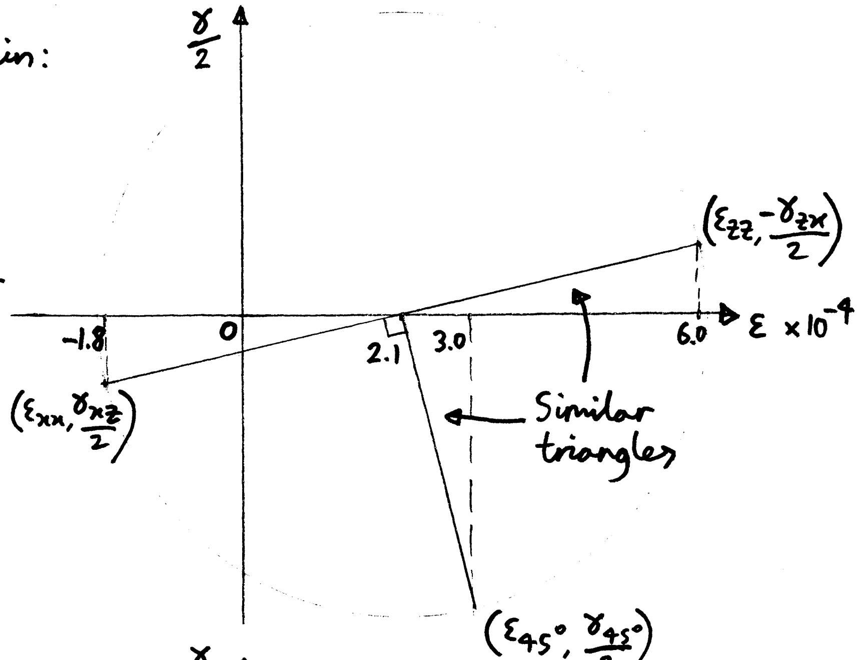
$\varepsilon_{45°} = \varepsilon_{66} = 2\times10^{-4}$

Mohr's circles of strain:

123 : Top flange:

$$\frac{\gamma_{xz}}{2} = -0.9\times10^{-4}$$

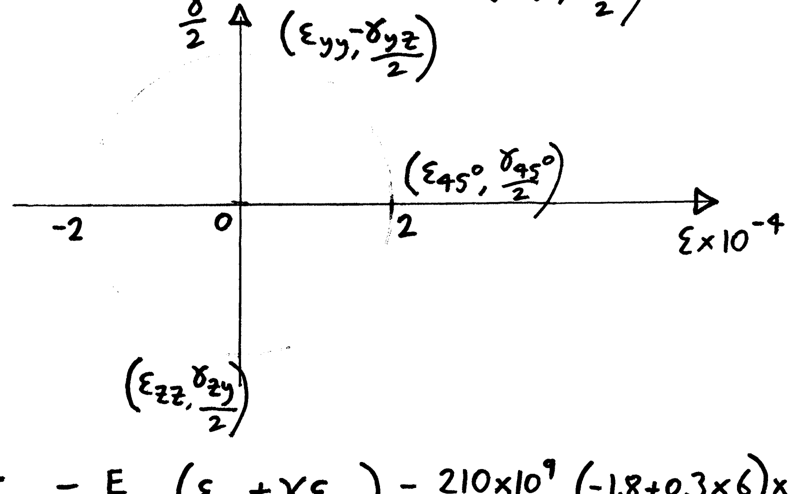
$$\therefore \gamma_{xz} = -1.8\times10^{-4}$$



456 : Web :

$$\frac{\gamma_{zy}}{2} = -2\times10^{-4}$$

$$\therefore \gamma_{zy} = -4\times10^{-4}$$



(b) Top flange: $\sigma_{xx} = \frac{E}{1-\nu^2}(\varepsilon_{xx} + \nu\varepsilon_{zz}) = \frac{210\times10^9}{0.91}(-1.8+0.3\times6)\times10^{-4}$

$= 0$

$\sigma_{zz} = \frac{E}{1-\nu^2}(\varepsilon_{zz} + \nu\varepsilon_{xx}) = \frac{210\times10^9}{0.91}(6+0.3\times-1.8)\times10^{-4}$

$= 126$ MPa

PART IB,
PAPER 2, STRUCTURES
Q3 cont'd

$$\tau_{zx} = G\gamma_{zx} = 81\times10^9 \times -1.8\times10^{-4} = -14.6 \text{ MPa}$$

Web: $\sigma_{yy} = \sigma_{zz} = 0$

$$\tau_{zy} = G\gamma_{zy} = 81\times10^9\times -4\times10^{-4} = -32.4 \text{ MPa}$$

(c) Moment about O_{yy}, axial tension and shear in x direction all zero.

$$I = \frac{1}{12}(50\times100^3 - 40\times90^3) = 1.7367\times10^6 \text{ mm}^4$$

Moment about O_{xx} $= \dfrac{\sigma_{zz\,flange}}{y} I = \dfrac{126\times10^6}{0.05}\times 1.7367\times10^{-6}$

$= 4376$ Nm $= 4.376$ kNm

Because there is no shear force in the direction Ox the shear in the top flange must be the result of a torque about z

Torque about z axis $= -2A_e t\,\tau_{zx\,flange} = 2\times95\times45\times5\times(10^{-3})^3 \times14.6\times10^6$

$= 624.2$ Nm $= 0.624$ kNm

Torque will cause a constant shear flow (and $\therefore$ shear stress) around section. Hence the excess shear stress in the web of $-32.4 - -14.6 = -17.8$ MPa must be due to a shear force F in the direction $-Oy$

Estimate F assuming constant shear stress in webs:

$$F \simeq 17.8\times10^6 \times 2\times100\times10^{-3}\times5\times10^{-3} = 17800\text{ N} = 17.3\text{ kN}$$

Revised estimate using $S = \dfrac{FA\bar{y}}{I}$ where $S = 2\tau t$

$A = 50\times5 + 2\times45\times5 = 700\text{ mm}^2$

$\bar{y} = (50\times5\times47.5 + 2\times45\times5\times22.5)/700$
$= 31.43$ mm

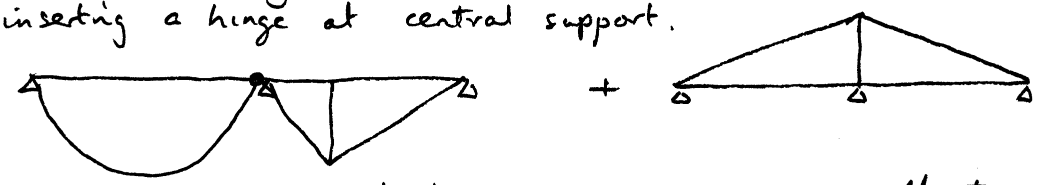
$$F = \frac{SI}{A\bar{y}} = \frac{2\times17.8\times5\times1.7367}{10^{-6}\times700\times31.43} = 14.05\text{ kN}$$

Paper 2 STRUCTURES. 2004.

SECTION B.

Q4.

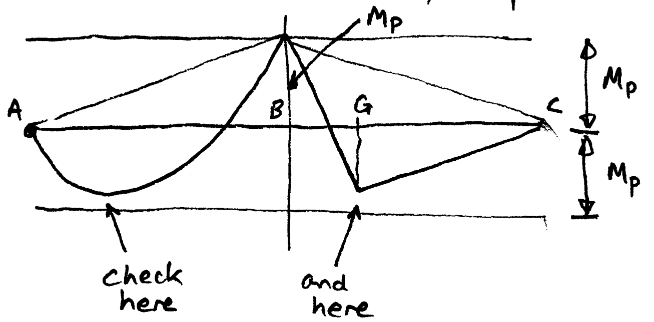
a) Use particular solution + state of self-stress
("free bending moment" + "reactant")
– by inserting a hinge at central support.



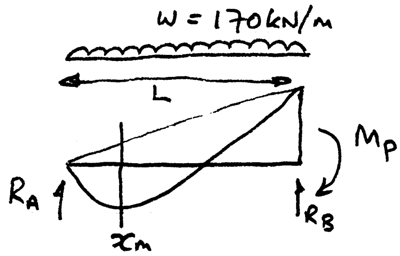
(one redundancy
∴ one state of self-stress).

(**NB** Do **NOT** waste time calculating particular solutions at this stage.).

To obtain maximum benefit from continuity over central support, set moment at B to be full plastic moment of resistance M_p.



Consider span AB.



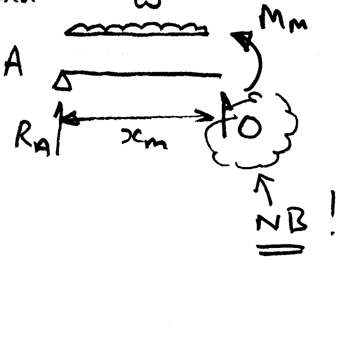
Determine location of maximum by finding position of <u>zero shear</u> by statics.

Moments about B:

$$R_A L + M_p - (wL)\frac{L}{2} = 0$$

$$\therefore \; R_A = \left(\frac{wL^2}{2} - M_p\right)\frac{1}{L}$$

Left hand end



$$w x_m = R_A \qquad \text{(VERT EQUILIB)}$$

$$\therefore \; x_m = \left(\frac{wL^2}{2} - M_p\right)\frac{1}{wL}$$

Paper 2 STRUCTURES. 2004.
SECTION B
Q4 a) Cont'd.

Moments about A :

$$\underline{\underline{\frac{w x_m^2}{2} = M_m}}$$

Insert values.

533 x 210 x 109 UB $\Rightarrow Z_p = 2828 \text{ cm}^3 = 2.828 \times 10^6 \text{ mm}^3$

$$\begin{aligned} M_p &= Z_p \sigma_y \\ &= (2.828 \times 10^6 \text{ mm}^3)(255 \text{ N/mm}^2) \\ &= 721 \times 10^6 \text{ Nmm} = \underline{\underline{721 \text{ kNm}}} \end{aligned}$$

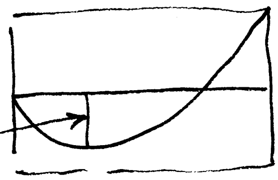
STRENGTH.

$$\begin{aligned} x_m &= \left(\frac{wL^2}{2} - M_p\right)\frac{1}{wL} = \left(\frac{L}{2} - \frac{M_p}{wL}\right) \\ &= \frac{6.86}{2} - \frac{721}{(170)(6.86)} = 3.43 - 0.618 = \underline{\underline{2.812 \text{ m}}} \end{aligned}$$

$$M_m = \frac{w x_m^2}{2} = \frac{(170)(2.812)^2}{2} = \underline{\underline{672 \text{ kNm}}}$$

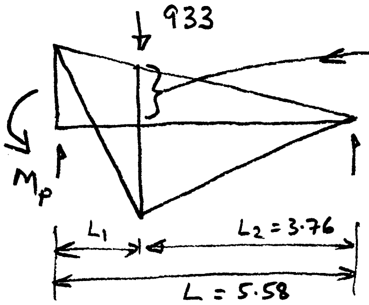
which is less than 721kNm = M_p.

$\therefore$



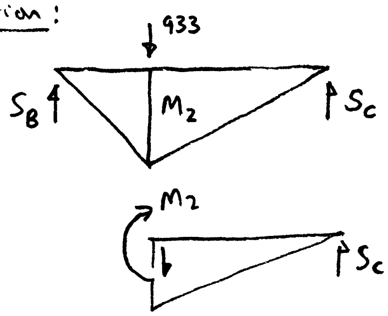
$\therefore$ $\underline{\underline{\text{OK}}}$ ✓

Consider span BC



$$\begin{aligned} M_1 &= M_p\left(\frac{3.76}{5.58}\right) \\ &= 721\left(\frac{3.76}{5.58}\right) \\ &= \underline{\underline{485.8 \text{ kNm}}} \end{aligned}$$

Particular solution:



Moments about B:

$$S_c L = 933 L_1$$

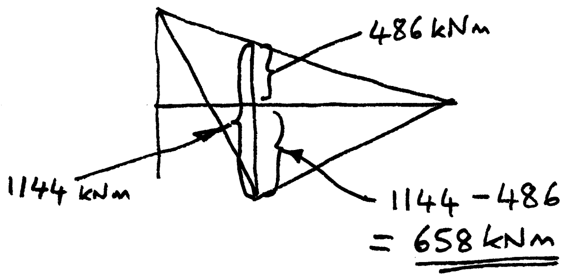
$$\therefore S_c = \frac{933\,(1.82)}{5.58} = 304 \text{ kN}.$$

$$M_2 = S_c L_2 = (304)(3.76) = \underline{\underline{1144 \text{ kNm}}}$$

Paper 2 STRUCTURES 2004

Q4 a) cont'd.

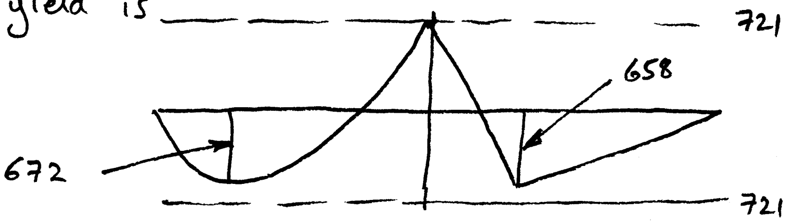
Add particular + self stress



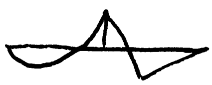
OK ✓ (LESS THAN 721 kNm)

Summary

A solution which satisfies equilibrium and nowhere defies yield is



∴ OK by LOWER BOUND THEOREM.

b) Since  works,

then  works

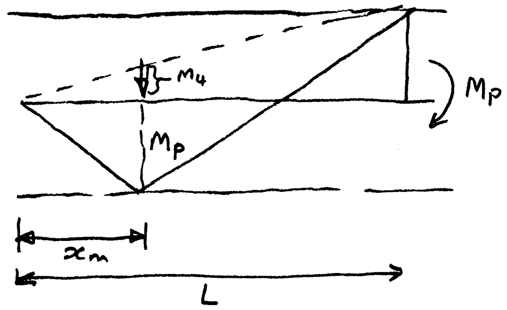
and  works

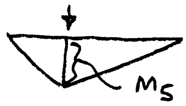
(i.e. can find equilib. systems (trivially) that don't defy yield)

∴ OK if loads applied separately.

c) PTO.

Q4 (c). The worst case will be when the rolling load is on the longer span, AB. The worst case will <u>NOT</u> be when it is at the midspan of AB, but at some position slightly to the left of the midspan, with a BMD

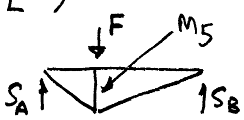


This BMD =  + 

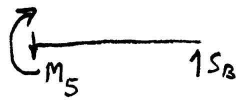
and $M_5 = M_4 + M_p$

with $M_4 = \frac{x_m}{L} M_p$ from geometry of top BMD.

$$\therefore \quad M_5 = \left(1 + \frac{x_m}{L}\right) M_p \qquad (1)$$

Consider particular solution 

Moments about A: $S_B L = F x_m \qquad (2)$

Right hand portion 

Moments about LH end $M_5 = S_B(L - x_m) = S_B L\left(1 - \frac{x_m}{L}\right)$

$= F x_m \left(1 - \frac{x_m}{L}\right)$ from (2)

$$\therefore \quad FL\frac{x_m}{L}\left(1 - \frac{x_m}{L}\right) = \left(1 + \frac{x_m}{L}\right) M_p \qquad \text{from (1)}$$

Define $\beta \equiv \frac{x_m}{L}$

$$FL\,\beta(1-\beta) = (1+\beta) M_p$$

or $m \equiv \frac{M_p}{FL} = \frac{\beta(1-\beta)}{1+\beta} = \frac{\beta - \beta^2}{(1+\beta)} \qquad (3)$

$\therefore$ For given F, L look for value of β that requires max M_p.

$$\frac{\partial m}{\partial \beta} = 0 \quad \Rightarrow \quad 0 = \frac{(1-2\beta)(1+\beta) - (\beta - \beta^2)\,1}{\text{denom}}$$

$$\Rightarrow \quad 1 + \cancel{\beta} - 2\beta - 2\beta^2 - \cancel{\beta} + \beta^2 = 0$$

$$1 - 2\beta - \beta^2 = 0 \quad \text{or} \quad \beta^2 + 2\beta - 1 = 0$$

Q4 (c) cont'd.

$$\beta^2 + 2\beta - 1 = 0$$

$$\beta = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

Positive (sensible) solution is $\beta = \sqrt{2} - 1 \approx \underline{\underline{0.414}}$.

$\therefore$ Worst location is $(\sqrt{2}-1)^{th}$ of the way along AB

$$x_m = 0.414\,(6.86) = 2.84 \text{ m from A.}$$

Substitute into ③

$$M_p = FL \frac{\beta(1-\beta)}{1+\beta}$$

$$= (933 \text{ kN})(6.86 \text{ m}) \frac{0.414\,(1-0.414)}{1.414}$$

$$= (933)(6.86)(0.172) = \underline{\underline{1098 \text{ kNm}}} \text{ req'd}$$

$$Z_p \text{ req'd} = \frac{M_p}{\sigma_y} = \frac{1098 \times 10^6 \text{ Nmm}}{255 \text{ N/mm}^2}$$

$$= 4306 \times 10^3 \text{ mm}^3 = \underline{\underline{4306 \text{ cm}^3}}$$

Q4. **Plastic design of a two span beam.** 179 attempts (75%). Average mark 9.7 / 20

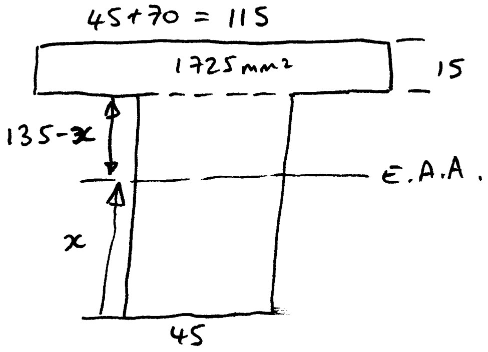
This question was done badly by most students. Almost all had grasped the idea of adding free and reactant bending moment diagrams ("a particular solution plus a state of self-stress"). Quite a few, however, drew states of self-stress that did not satisfy equilibrium (i.e. had moments at the pinned ends). It was remarkable that many students, having the right idea about what was required, could not then perform the simple calculations correctly. A common approach was to insert a pin at B, look at the left hand beam under the UDL and write down an equation for the shear force. This was then integrated to obtain an equation for the moment. This was then differentiated (integrate up then differentiate back down again!) to obtain the location of the maximum, x = 3.43m (i.e. L/2 – the maximum is at the midspan, surprise!!). This was then substituted into the equation for the moment, to obtain $M = wl^2/8$ = 1000kNm. Of course, by this time, most had introduced errors, and had wasted perhaps 10 minutes and two sides of paper. Students did not know that $M = wl^2/8$ (it is not in the Data Book), nor did they recognise that this is actually irrelevant anyway. The maximum sagging moment is required for the combined free and reactant. The students thus needed to repeat this whole process with the linear reactant line included, and few did. No-one determined the location of maximum moment by saying that it occurs at the point of zero shear. Equal numbers of problems then presented themselves on the right hand span, with only a minority being able to compute the correct numerical value of the free moment beneath the point load. Given that very few obtained the correct numerical solutions to part a), it became difficult to make sense of many answers to parts b) and c). In part b), a high proportion erroneously said that applying the loads separately would lead to collapse in one span or the other, variously, for various reasons – often because they had calculated that M = 2000kNm, 4000kNm or 8000kNm in the left hand span. In part c) almost every answer asserted (wrongly) that the worst case for the rolling load was at the centre of the left-hand span. In summary, the students demonstrated that they had grasped some high-level and subtle ideas about lower bounds, indeterminate structures, particular solutions, states of self-stress, etc., (all of which used to be third year material), but were woefully unable to undertake simple first year calculations, such as determining moments in simply supported beams.

(Incidentally, this question can also be answered by upper bound mechanism analysis to obtain exactly the same answers, and a few did this.)

PAPER 2: STRUCTURES

Q5. a)

First determine equal area axis

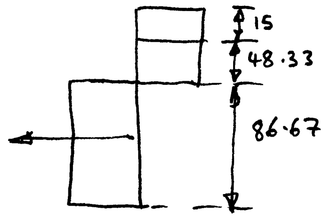


$$(135-x)45 + 1725 = 45x$$

$$(135-x) + \frac{1725}{45} = x$$

$$173.3 = 2x$$

$$\underline{\underline{x = 86.67\text{ mm}}}$$

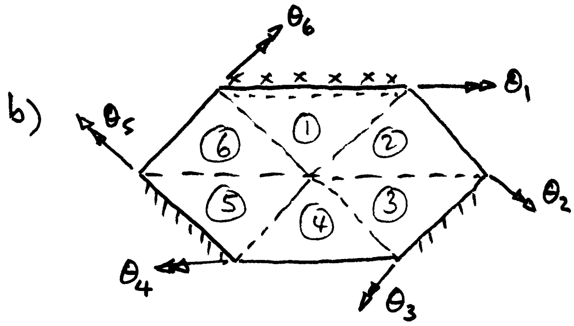


Area	lever arm	Product
1725	55.833	96 312
2175	24.17	52562
3900	43.33	169,000

$\Sigma = \underline{\underline{317\,875\text{ mm}^3}} = \underline{\underline{317.9\text{ cm}^3}}$

5 marks.

b)



Let the central deflection be Δ

External Work Done

$= W \times \text{Area of slab} \times \Delta/3$

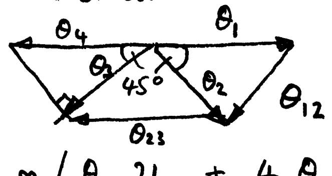
average deflection of pyramid

$= W.\,6L^2.\,\Delta/3 = \underline{\underline{2WL^2\Delta}}$

Internal Work Done at hinges

Using rotation vector method

Vector diag.



$\theta_1 = \frac{\Delta}{L} = \theta_4$

$\theta_2 = \frac{\Delta}{\sqrt{2}L} = \theta_{3,5,6}$

Internal W.D $= m\left(\theta_1 . 2L + 4\,\theta_{12} L_{12} + 2\,\theta_{23} L_{23}\right)$

1 hog   (1,2), (3,4), (4,5), (6,1)

$= m\left(2\theta_1 L + 4.\frac{\theta_1}{\sqrt{2}}.\sqrt{2}L + 2.\theta_1 . 2L\right)$

$= m\theta_1 L(2+4+4) = 10m\theta_1 L = \underline{\underline{10m\Delta}}$

Equate Internal and External:

$2WL^2\Delta = 10m\Delta \Rightarrow \underline{\underline{W = \frac{5m}{L^2}}}$

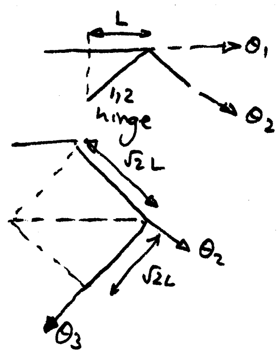
Q5 b)

(Alternative solution using projection method:

① hogging $= m.\theta_1 . 2L$

①,② $= m(L.\theta_1 + 0.\theta_2)$ $= mL\theta_1$

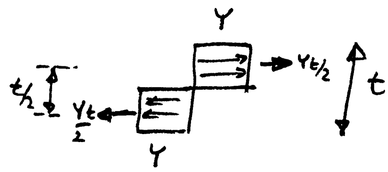
②③ $= m(\sqrt{2}L.\theta_2 + \sqrt{2}L\,\theta_3)$ $= 2\sqrt{2}\,mL\theta_2$ $= 2\sqrt{2}mL\dfrac{\theta_1}{\sqrt{2}}$ $= 2mL\theta_1$



Total $=$ ① hog $+ 4\times$ ①,② $+ 2\times$ ②③

$= m\theta_1 2L + 4.mL\theta_1 + 2(2mL\theta_1) = 10m\theta_1 L$ again)

Plastic Moment of resistance per unit length $m = \dfrac{Yt^2}{4}$



(Moment $= \dfrac{Yt}{2} \times \dfrac{t}{2} = \dfrac{Yt^2}{4}$)

From earlier, $W = \dfrac{5m}{L^2}$

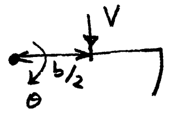
$\therefore W = \dfrac{5Yt^2}{4L^2}$

Q5. **Plastic section modulus/yield lines.** 211 attempts (88%). Average mark 13.3 /20
In part a) a sizeable proportion of students evaluated the plastic section modulus around the centroidal rather than the equal-area axis. In part b), common errors were geometrical: quite a few took the hexagon to be regular. The simple method of considering mechanisms via orthogonal slices is rather tricky here, but none approached it that way as that method is not taught. Approximately equal numbers approached it via the vector rotation method and the projection method (which was introduced to the teaching this year, possibly prompted by the comments of last year's examiner). For this problem, both methods are equally straightforward, and many correct answers or reasonable attempts were obtained by each route. Finally though, very few students knew, or knew how to work out, the plastic moment of resistance per unit length of a plate.

PAPER 2 : STRUCTURES 2004

Q6

a) Let the slip circle rotate by a small angle θ 

External Work Done =  $= V \, b/2 \, \theta$

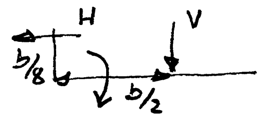
Internal Work Done $= k \times \underbrace{\pi b}_{\text{length of failure surface}} \times \underbrace{b\theta}_{\text{distance slipped}} = k\pi b^2 \theta$

Equating the two
$$V \frac{b}{2} \theta = k\pi b^2 \theta$$
$$V = 2\pi k b \qquad 2\pi = 6.28$$

So if $V > 2\pi kb$ it must have collapsed by this or some othe mechanism.

so if $V > 6.3\,kb$ ditto.

b) Slip circle mechanism, when $H \neq 0$

External Work Done =  $= \left(V\frac{b}{2} - H\frac{b}{8}\right)\theta$

Internal Work Done
$$= k\pi b^2 \theta \quad \text{again}$$

Equating :
$$\left(V\frac{b}{2} - H\frac{b}{8}\right)\cancel{\theta} = k\pi b^2 \cancel{\theta}$$

$$4\frac{V}{kb} - \frac{H}{kb} = 8\pi$$

when $V = 0$ $\quad \frac{H}{kb} = -8\pi$

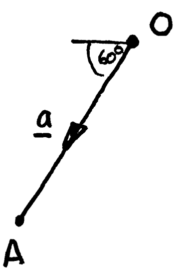
when $H = 0$ $\quad \frac{V}{kb} = +2\pi$

6 b) cont'd.

Sliding block mechanism.

Draw displacement diagram.

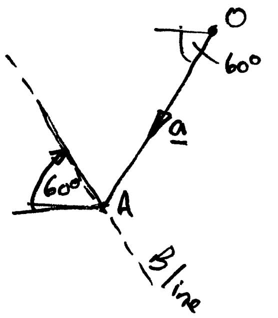
Let block A move down and to left with displacement $\underline{a}$



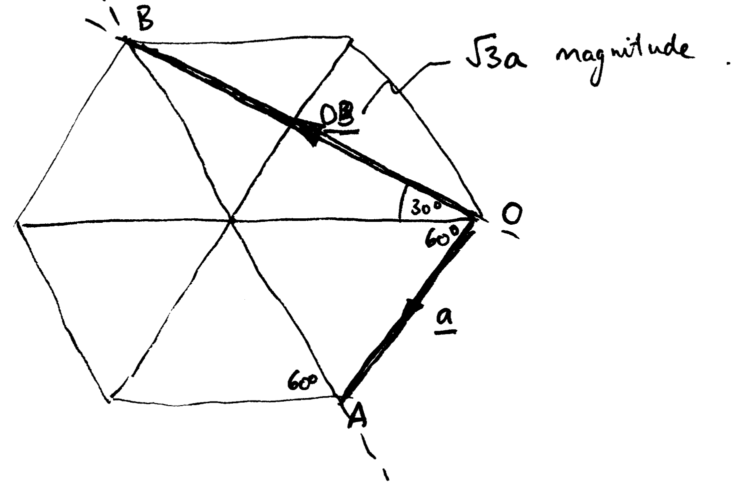
B must move up at 30°

Relative velocity of A and B must be directed along their common interface

i.e.

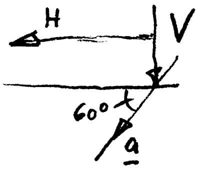


Total diagram:



6 b) contd 2.

External W.D.



$$= V a \cos 30° + H a \sin 60°$$

$$= V a \frac{\sqrt{3}}{2} + \frac{Ha}{2}$$

Internal W.D. $=$ " AO + BO + AB "

$$= \underset{\text{length}}{b} . \underset{\text{displ}}{a} . k + \underset{\text{length}}{\sqrt{3} b} . \underset{\text{displ}}{\sqrt{3} a} . k + \underset{\text{length}}{b} . \underset{\text{rel. displ.}}{2a} . k$$

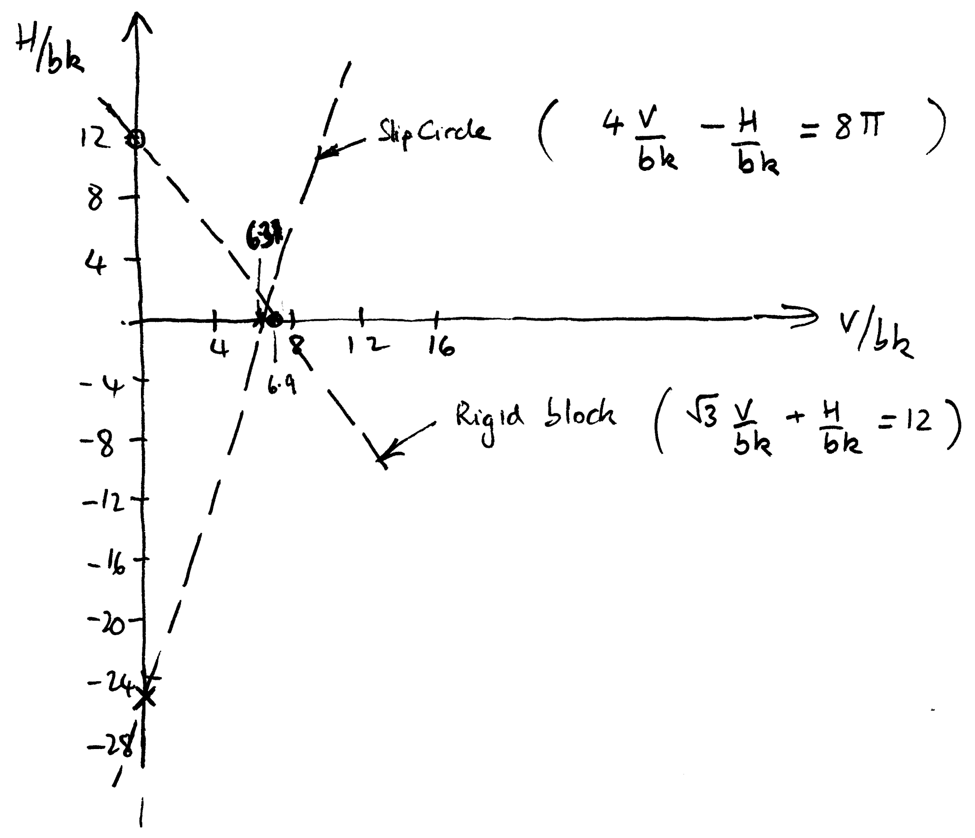
$$= bak\,(1 + 3 + 2) = 6\, b.a.k.$$

Equating $\quad V \cancel{a} \frac{\sqrt{3}}{2} + \frac{H\cancel{a}}{2} = 6\, b\cancel{a}\, k$

$$\underline{\underline{\sqrt{3}V + H = 12bk.}}$$

$V=0 \quad \frac{H}{bk} = 12$

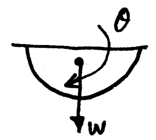
$H=0 \quad \frac{V}{bk} = \frac{12}{\sqrt{3}} = 6.9$



6 (c).

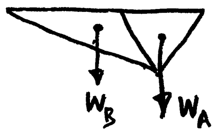
There will be no effect whatsoever. It is a general result for indentation problems into rigid plastic half-spaces with a horizontal surface. It can be seen to be true in the individual cases by consideration of the external W.D. by the weights of each block considered as acting through the centres of mass of the individual blocks.

Slip circle:



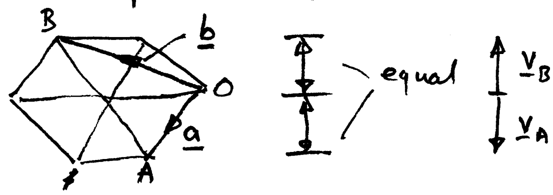
Vertical displacement of C. of M = 0 (to first order in θ).

Rigid sliding blocks



$W_A = W_B$

Downwards displacement of block A = upwards displacement of block B



$$\therefore \quad \underline{W}_A \cdot \underline{v}_A + \underline{W}_B \cdot \underline{v}_B = 0$$

(A number of students thought the self-weight applied to the footing, but the question clearly states that it applies to the continuum).

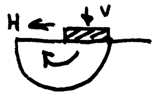
Q6. **Slip circle/plastic failure of footings**.133 attempts (55%). Average mark 12.2 / 20
Many good attempts. Almost all students correctly solved part a). Errors came in part b), and were mostly algebraic. As pointed out by the External Examiner before the exam, there are a number of subtleties associated with the fact that the horizontal force tends to cause the slip circle to rotate in the opposite direction to the vertical force. These will be covered in detail in the crib. However no student found any confusion in this. The most common substantive error in part b) was the assumption that wedge A moves vertically downward, in defiance of compatibility. In part c), many students stated erroneously that including the self-weight of the continuum would "help V" and so affect the interaction diagram.

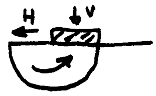
PAPER 2, STRUCTURES 2004

Q6 Additional comments (not required in order to obtain full marks on the examination).

The slip circle mechanism is a little unusual in that the vertical force V tends to cause rotation in one direction and the horizontal force H is trying to cause it to rotate in the opposite direction.

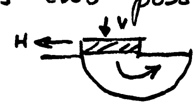
For this mechanism there are thus two possible motions, clockwise and anticlockwise.

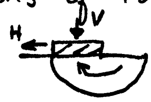
(1) 

and 

(2)

There is also another mechanism (which students were not asked to consider) which is the mirror image of this. This also has two possible directions of rotation

(3) 

and 

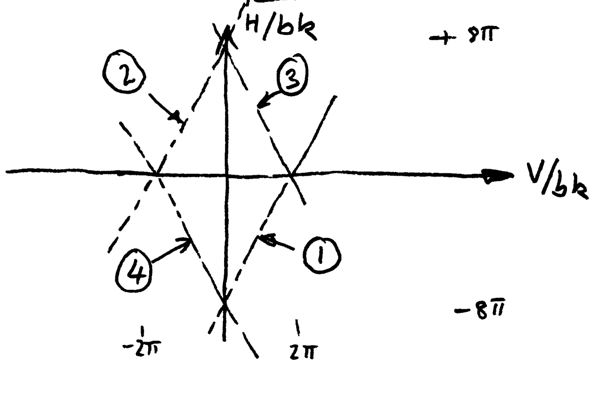
(4)

For a small rotation of magnitude $|\theta|$, in all four cases, the internal work dissipated at the slip boundary is $k\pi b^2|\theta|$, a positive quantity. The external work done by H and V is however

(1) $+\frac{Vb}{2} - \frac{Hb}{8}$ $\qquad$ $-\frac{Vb}{2} + \frac{Hb}{8}$ (2)

(3) $+\frac{Vb}{2} + \frac{Hb}{8}$ $\qquad$ $-\frac{Vb}{2} - \frac{Hb}{8}$ (4)

for the four cases respectively. Equating each of these to the internal work done leads to four lines on the interaction diagram



The exam question referred to mechanism  in either rotation sense (1) or (2).

Note that there also exist four interaction lines for the rigid block mechanism and its mirror image (with both senses of rotation).