

ENGINEERING TRIPOS PART IB

Tuesday 1 June 2004 9.00 to 11.00

Paper 3

MATERIALS SOLUTIONS - FINAL VERSION

1 (a) The production of plain medium carbon steels with high combined hardness and toughness involves three main processes;

1. Austenising. The steel is heated to above the temperature at which austenite forms (depends on the carbon content of the steel), and is typically around 850°C for medium carbon steels 0.25 wt% < C < 0.6 wt%). The steel is held at this temperature for up to 1 hour to ensure homogenisation of the austenite phase.

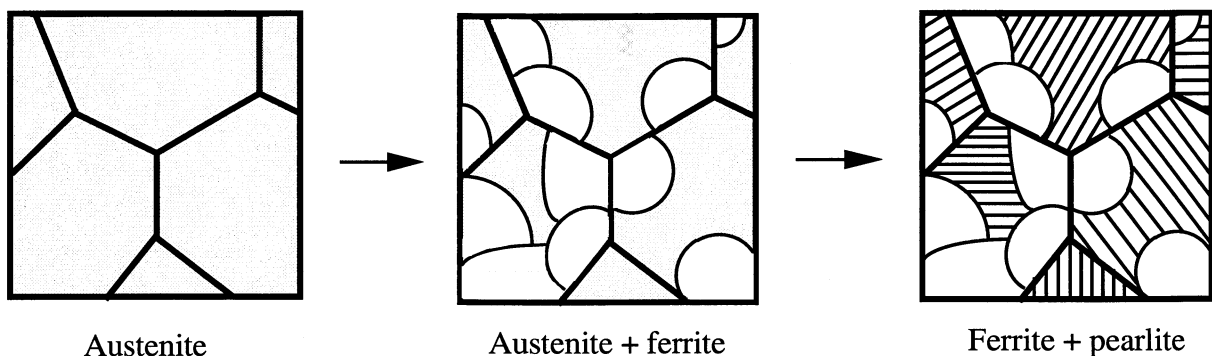
2. Quenching. The fully austenised steel is quenched from the austenising temperature at a rate sufficiently large to beat the nose of the TTT curve. This results in the formation of metastable martensite (α'), which is hard and brittle and of little practical use.

3. Tempering. The martensite is re-heated to temperatures in the range 200-700°C, to precipitate out spheroidal particles of cementite (Fe_3C). The matrix reverts partially to the stable α phase during this process. The uniform distribution of Fe_3C particles increases the yield stress of the steel. Tempering temperatures and times determine the final properties of the steel.

The knee of the TTT diagram moves to shorter times for decreasing carbon content, which decreases its hardenability.

This process is not used for low carbon steels (< 0.25 wt% C) since the critical cooling rate for the formation of martensite is too high (low carbon content). The microstructure of low carbon steels usually consists of ferrite and pearlite.

(b) Hypoeutectoid steel has a carbon content of less than 0.8% (i.e. the eutectoid composition). The α phase nucleates at austenite grain boundaries in the $\alpha + \gamma$ region of the phase diagram. Pearlite forms at the eutectoid temperature to produce a final microstructure consisting of ferrite and pearlite.

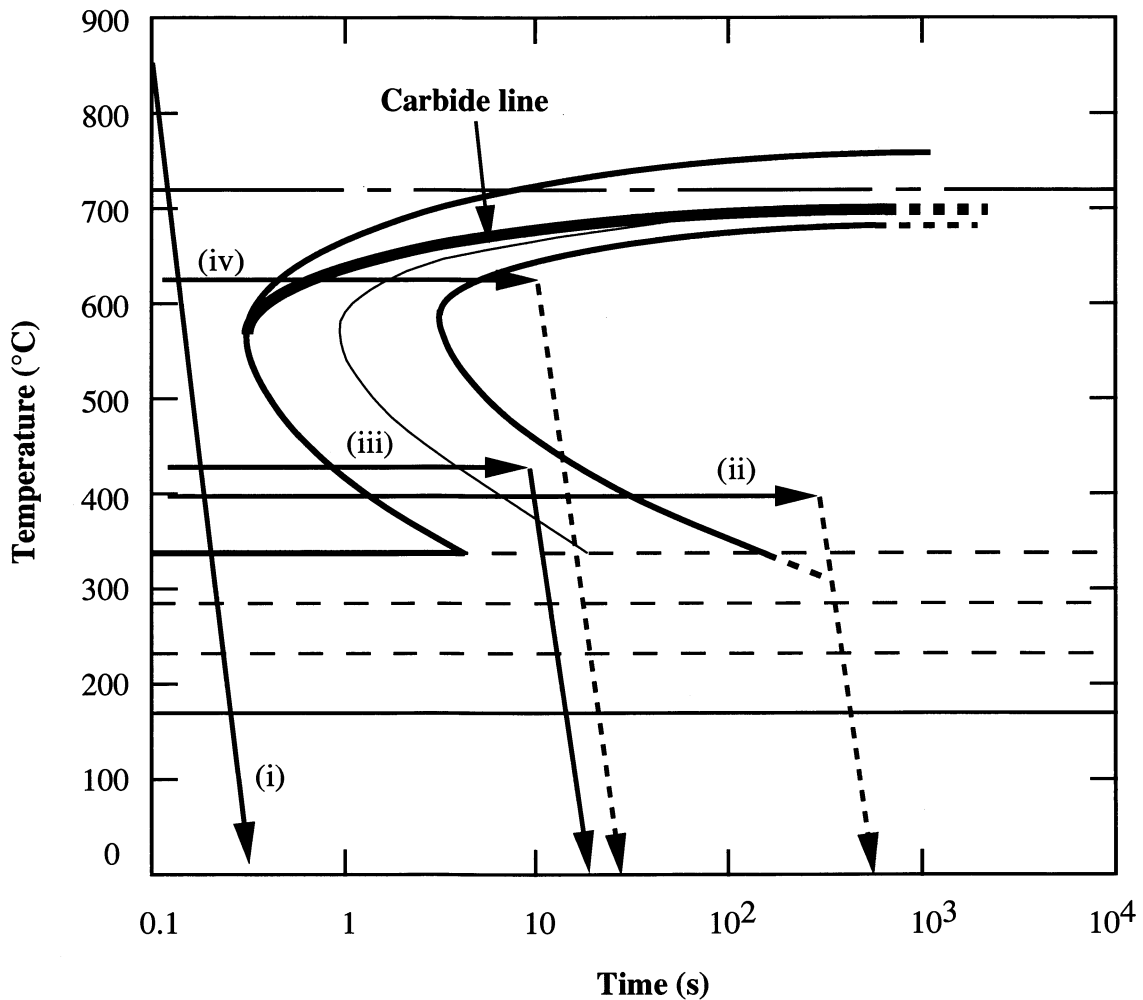


Use of the lever rule to determine the proportion of Fe_3C in 0.45 wt. % carbon steel by measuring relevant lengths on the phase diagram.

$$F_{\text{Fe}_3\text{C}} = \frac{C_{0.45} - C_{\alpha}}{C_{\text{Fe}_3\text{C}} - C_{\alpha}} \approx \frac{0.45}{6.7} \approx 6.7\% \text{ Fe}_3\text{C}$$

- (c) A: Austenite; phase that forms above 800°C for 0.45 wt. % carbon steel.
 B: Ferrite; phase that precipitates out first on cooling between 800 and 723°C.
 C: Pearlite; coarse microstructure of ferrite and cementite plates that forms at temperatures between 723 and 600°C (diffusion limited).
 D: Bainite; fine microstructure of ferrite with needles of cementite that forms at lower temperatures between 600 and 330°C (diffusion limited).
 E: Martensite; phase that forms at temperatures below 330°C for cooling rates of at least 70°C/s

A, B and E are phases.



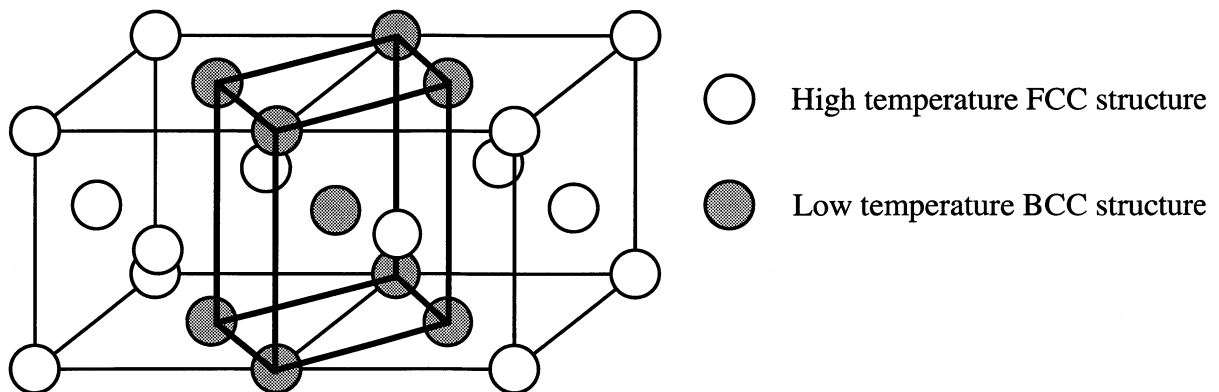
(i) 100% martensite \Rightarrow Cool rapidly to room temperature.

(ii) 100% bainite \Rightarrow Cool rapidly to 400 °C, hold for 500 s, then cool (rate unimportant).

(iii) 75% bainite and 25% martensite \Rightarrow Cool rapidly to 450 °C, hold for 10 s, then quench to room temperature.

(iv) \approx 10% Ferrite (α) and 90% fine pearlite \Rightarrow Cool rapidly to 625 °C, hold for 10 s; (proeutectoid)

(d) The displacive structural transition takes place when martensite forms from austenite in plain carbon steels, and is driven by the difference in free energy of the two states (G for α' is less than γ). This involves the lattice shearing from a FCC to a body centred tetragonal (or BCC) structure, with each Fe atom being displaced by a fraction of an atomic spacing.



Martensite does not appear in the iron-carbon phase diagram since it is a metastable form of ferrite. The phase diagram describes only equilibrium conditions (i.e. phase content for infinite times at temperature).

2. (a) Both power law and diffusional creep are examples of steady-state creep and are described by the following equation;

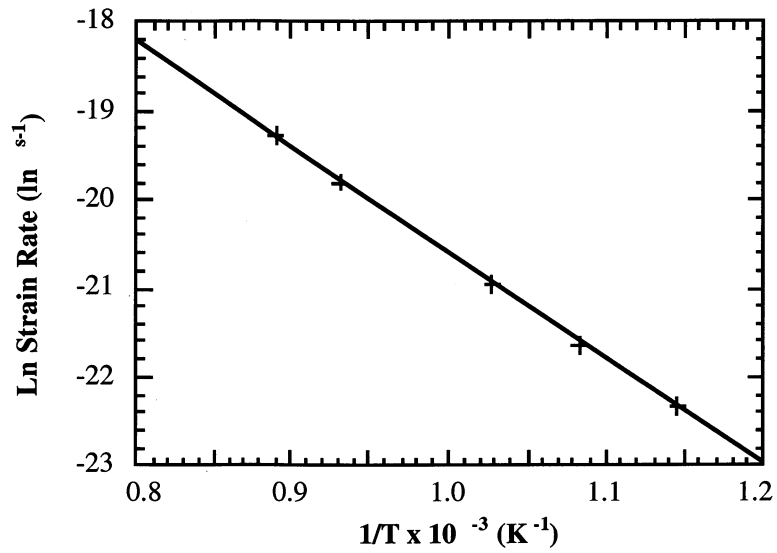
$$\dot{\epsilon} = A \sigma^n \exp\left[-\frac{Q}{RT}\right]$$

Power law creep occurs at $T > 0.4T_m$ at relatively high stress and is very sensitive to σ ($n = 3 - 10$ in the above equation). Strain rates in the power law creep regime occur over a wide range ($10^{-10} < \dot{\epsilon} < 1 \text{ s}^{-1}$). The mechanism of power law creep is (stress-sensitive) core diffusion at lower temperature and bulk diffusion at higher temperatures.

Diffusional creep occurs at $T > 0.4T_m$ and lower stress and is less sensitive to this parameter ($n = 1$ in the above equation). Strain rates in the diffusional creep regime occur over a narrower range ($10^{-10} < \dot{\epsilon} < 10^{-7}$). The mechanism of diffusional creep is grain boundary diffusion at lower temperatures and bulk diffusion at higher temperatures.

(b) $\dot{\epsilon} = A \sigma \exp\left[-\frac{Q}{RT}\right]$ $\ln \dot{\epsilon} = \ln(A \sigma) - \frac{Q}{RT}$

An Arrhenius plot of $\ln \dot{\epsilon}$ against $1/T$ yields a straight line of gradient $-Q/R$. i.e.

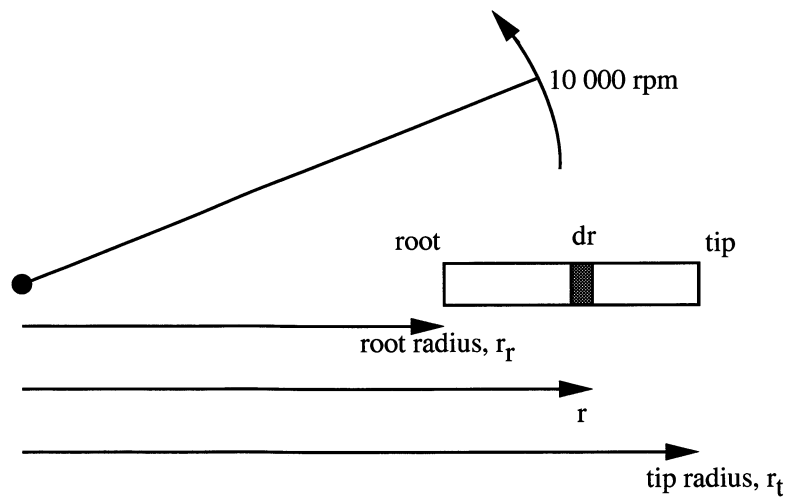


$$-\frac{Q}{R} = \frac{(-18.2) - (-23.0)}{(0.8 - 1.2) \times 10^{-3}} = -12 \times 10^3$$

$$Q = 12 \times 10^3 R = 12 \times 10^3 \times 8.31 \approx \mathbf{100 \text{ kJ/mol}}$$

(Note that failure to convert temperature from °C to K produces an answer of around 50 kJ/mol.)

(c)



The force is greatest at the root of the blade, decreasing to zero at the blade tip (free surface). Centripetal force at a position r is given by $m r \omega^2$. But mass = $\rho \times \text{volume} = \rho \times \text{area} \times dr$. Hence centrifugal force at a position r is given by $\rho \times \text{area} \times r \omega^2 dr$. The total force acting on an element of width dr at a radius r is obtained by integrating over radius between r and r_t ;

$$F = \int_r^{r_t} \rho \times \text{area} \times \omega^2 r dr = \frac{\rho \times \text{area} \times \omega^2}{2} (r_t^2 - r^2)$$

$$\sigma = \frac{F}{\text{area}} = \frac{\rho \times \text{area} \times \omega^2}{2 \times \text{area}} (r_t^2 - r^2) = \frac{\rho \omega^2}{2} (r_t^2 - r^2)$$

Substitute for stress in $\dot{\epsilon} = A \sigma \exp\left[-\frac{Q}{RT}\right]$;

$$\dot{\epsilon} = A \frac{\rho \omega^2}{2} \exp\left[-\frac{Q}{RT}\right] (r_t^2 - r^2)$$

The rate of change of length of an element is given by $\dot{\epsilon} dr$. Rate of change of length of whole blade is given by integrating this over length of blade;

$$\frac{dl}{dt} = \int_{r_r}^{r_t} \dot{\epsilon} dr = A \frac{\rho \omega^2}{2} \exp\left[-\frac{Q}{RT}\right] \int_{r_r}^{r_t} (r_t^2 - r^2) dr$$

$$\frac{dl}{dt} = A \frac{\rho \omega^2}{2} \exp\left[-\frac{Q}{RT}\right] \left[r r_t^2 - \frac{r^3}{3} \right]_{r_r}^{r_t}$$

Determine A from the ordinate intercept of the Arrhenius plot (4.3×10^{-11}) and insert numbers;

$$\begin{aligned} \frac{dl}{dt} = \dot{\epsilon} &= 4.3 \times 10^{-11} \times \frac{8900}{2} \times \left(\frac{2\pi \times 10\,000}{60}\right)^2 \exp\left[-\frac{100\,000}{8.31 \times 873}\right] \times \\ &\quad \left[\left(0.25^3 - \frac{0.25^3}{3}\right) - \left(0.2 \times 0.25^3 - \frac{0.2^3}{3}\right) \right] \\ &= 1.26 \times 10^{-10} \text{ m/s.} \end{aligned}$$

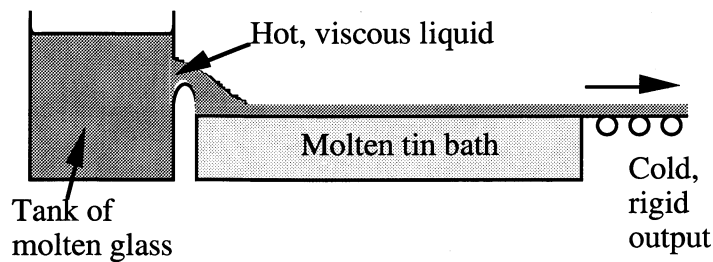
In 100 hours the blade will extend $1.26 \times 10^{-10} \times 100 \times 3600 = 4.5 \times 10^{-5} \text{ m} = 45 \mu\text{m}$.

3 (a) (i) Extrusion of polymers

Used to produce tubes, pipes, sheets and rods of thermoplastics. Thermoplastic granules are used to charge the extruder, which converts them to linear product by melting. The barrel of the extruder consists of a feed section, a compression section, a metering section, a

perforated breaker plate and a die. A screw of increasing cross-sectional area transports the granules through the various sections. The granules melt as they move along the barrel due to external heating and friction with the inner walls of the screw housing. The compression section densifies the melt and the metering section produces a controlled liquid flow to the breaker plate, to produce a linear rather than a rotating output, and finally to the die, which defines the cross sectional area of the product. The product is cooled as it leaves the die. A significant property is the alignment of the polymer molecules as the product leaves the die. This increases the strength and Young's modulus parallel to the axis of extrusion.

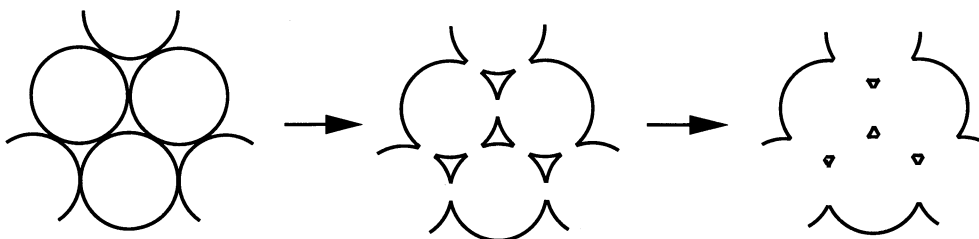
(ii) Float glass technique (Pilkinton)



Molten glass is fed from a tank under gravity onto the surface of a molten tin bath (molten tin has a significantly lower melting temperature and higher density than molten glass). The surface of the molten tin is perfectly flat, which produces a uniform thickness of glass under the combined action of surface tension and gravity. Significantly, the molten tin and glass do not react chemically. The glass solidifies as it cools on contact with the molten tin, with the solid product removed by rollers at constant speed. The thickness of the glass is controlled by the feed rate and the speed of the rollers. A significant property of the float glass process is the uniformity of the thickness of the glass due to a uniform cooling and extraction rate.

(iii) Hot isostatic pressing of ceramics

Ceramics are produced by a powder production route. Fine, calcined (reacted) powder (typical grain size $1\ \mu\text{m}$) of the constituent metal oxides, carbides or nitrides are mixed with a small amount of organic binder (later burned out at around 500°C) and compacted into a pellet in a die usually of circular or square cross-sectional area using a uniaxial press to produce a 'green' body. A rubber mould can be used if complex shapes are required. Hot isostatic pressing involves simultaneously applying a uniform pressure and heat to the green body. The heat and isostatic pressure enables individual atoms to diffuse more rapidly, which speeds-up the process of densification, or sintering (the fusion of individual grains - see below). Sintering typically occurs at temperatures of $2/3$ the melting temperature of the ceramic. A significant property of HIPing is the relatively high density that can be achieved compared to conventional sintering.



(b) The strength of polymers can be increased by aligning the polymer chains along the direction of load. This is achieved in extrusion, blow moulding and injection moulding. In

addition, the application of pressure to molten thermoplastics on solidification reduces porosity and increases strength. Finally, the degree of crystallinity influences strength.

The strength of glass is controlled by surface defects (microscopic flaws or cracks). Strength is increased by inhibiting the growth of these cracks and by preventing the formation of new flaws. This can be achieved by coating the surface of the glass with a polymer immediately after drawing from the melt. Alternatively a compressive stress can be introduced to the surface of the glass by thermal treatment to suppress crack growth.

The strength of ceramics is determined by residual pores due to incomplete sintering and microcracks due to thermal stress on cooling. These defects are similar in dimensions to the grain size. Hence their effect can be reduced by reducing the grain size of the ceramic by either reducing processing times (by HIPing) or by using a finer precursor powder. Introducing second phase particles to the ceramic can also blunt cracks as they propagate and increase the fracture toughness of the ceramic. Finally, transformation toughening can be used, which involves the structural transformation of a small region of secondary, stabilising phase material within the ceramic body.

(c) It is assumed that the dimensions of the bar do not change during heating (the effect of Poisson's ratio is negligible since the diameter is small compared to its length). There is no thermal stress parallel to the diameter of the bar at its centre since there is no constraint. The thermal stress at the ends of the bar parallel to the diameter is small compared to the thermal stress parallel to the length of the bar (therefore d does not appear in the analysis). It is assumed further that the mounting sites at the end of the bar are maintained at elevated, constant temperature as the bar cools. On cooling to $T - \Delta T$ the bar will be placed in tension. If the ends of the bar were not fixed, this would cause the bar to contract by $L_0 \alpha \Delta T$. As a result the bar experiences a strain of;

$$\epsilon = \frac{\text{extension}}{\text{original length}} = \frac{L_0 \alpha \Delta T}{L_0} = \alpha \Delta T$$

The stress due to this strain must necessarily be less than the fracture strength of the glass, σ_f .

i.e. $\sigma_f > E \alpha \Delta T$ Maximum allowable temperature change = $\Delta T = \frac{\sigma_f}{E \alpha}$

Glass	$\alpha \times 10^{-6}$ K ⁻¹	E GPa	σ_f MPa	ΔT K
Soda glass	9	70	33	52
Silica	0.5	71	80	2254
Borosilicate	4	63	27	107

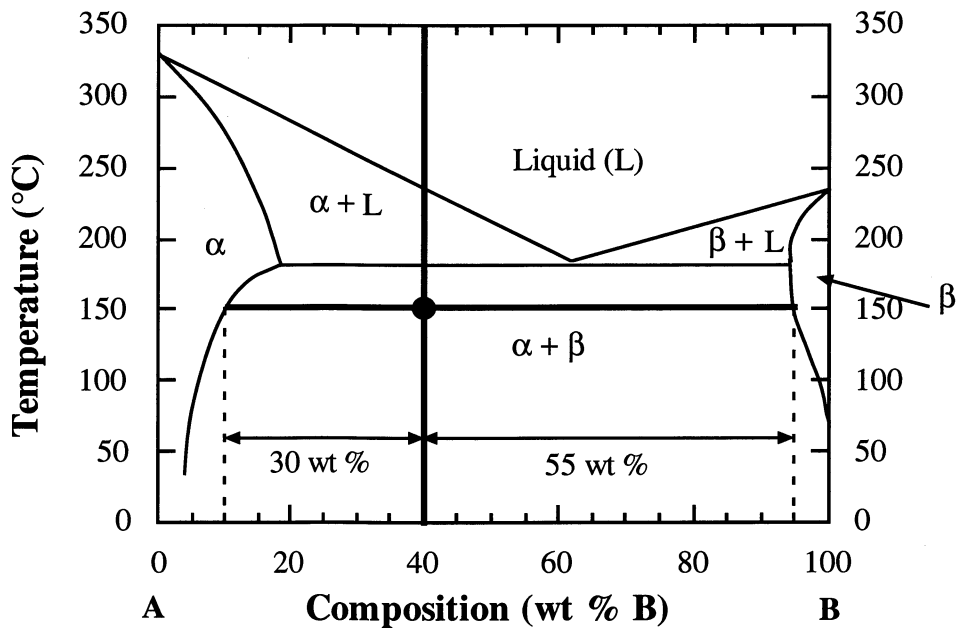
Silica glass is the most suitable for a temperature change of 400°C.

If the temperature were to increase, rather than decrease, the bar would be placed in compression, rather than tension if the same assumptions are valid. Given that the bar is relatively thin and is significantly stronger in compression than in tension, it would fail by buckling. In this case the calculation would become limited by the elastic modulus of the bar, rather than by its strength.

4 (a) A phase diagram plots the boundaries between different phases of an alloy on a graph with axes of temperature (ordinate) and concentration (abscissa). The regions between the lines are labeled with the phases present at equilibrium (i.e. lowest free energy). It is an equilibrium diagram obtained by cooling slowly.

The lever rule is used to estimate the proportions of the phases present in a two phase region for a given starting composition and temperature. A vertical line is drawn at the concentration of the alloy and a horizontal line at the temperature of interest. The distance from where the lines meet to the phase boundary can be regarded as a lever arm, and the weight of the phase at the boundary balances the weight of the phase at the other boundary.

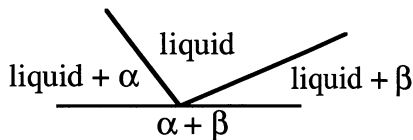
For example, the phase content of alloy AB at 40 wt. % B and 150°C;



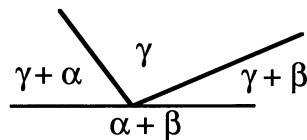
$$\text{Proportion of phase } \alpha = \frac{55}{85} = 0.65$$

$$\text{Proportion of phase } \beta = \frac{30}{85} = 0.35$$

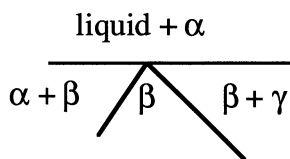
(i) a eutectic;



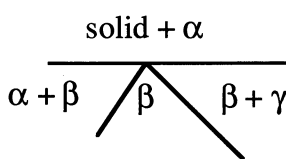
(ii) a eutectoid;



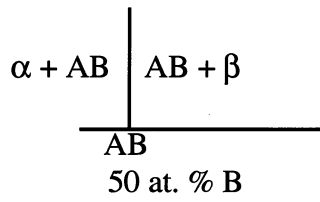
(iii) a peritectic;



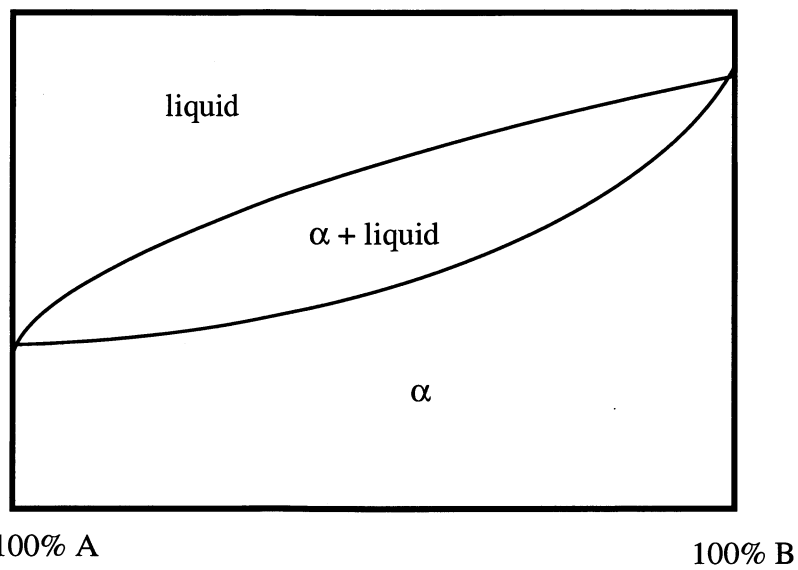
(iv) a peritectoid;



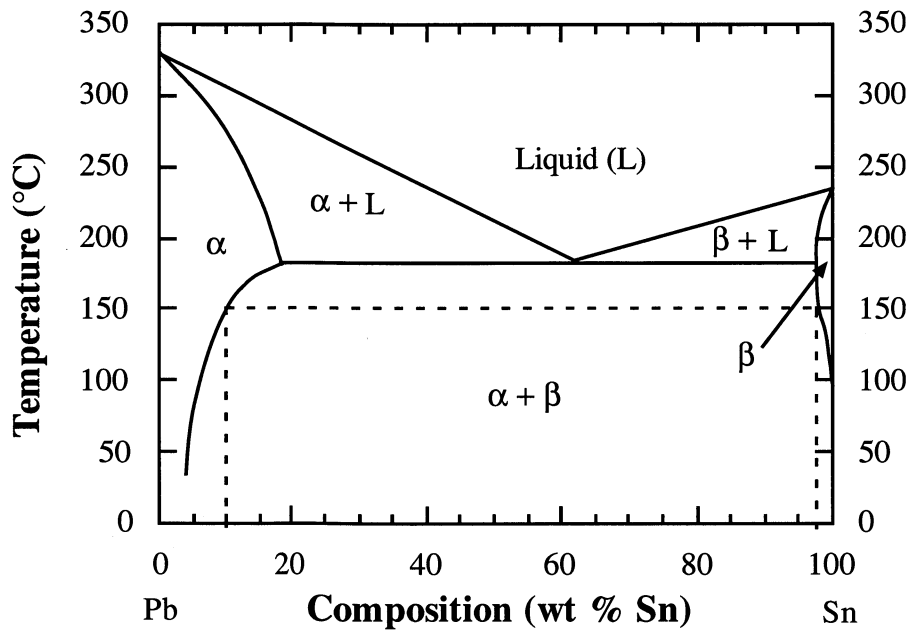
(v) an intermetallic compound, AB;



Phase diagram for two materials which are mutually soluble in each other at all concentrations;



(b) Pb-Sn phase diagram;

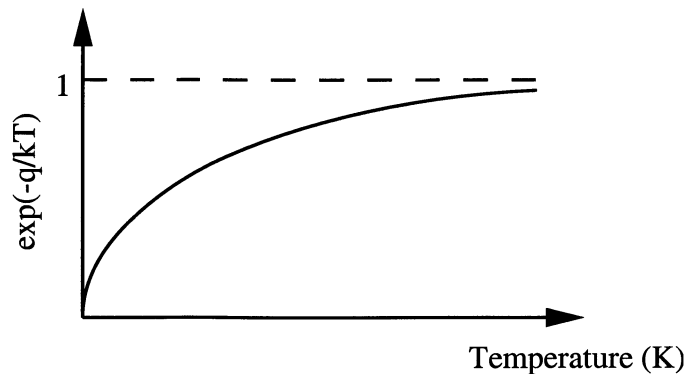


The precipitates in the white grains can only be tin in lead, since lead is virtually insoluble in Sn at the eutectic temperature (183°C) It is around half eutectic and half α , so the composition is approximately 40 wt.% lead.

The white (lead) grains (α) form on cooling from the melt at around 230°C, indicating a non-eutectic composition. The liquid becomes tin-rich as lead precipitates out until it reaches the eutectic composition at the eutectic temperature. At the eutectic temperature, the liquid forms a fine array of tin and lead-rich plates (or lamellae). Some Sn precipitates out of the lead grains on further cooling. This only happens for slow cooling of the initial composition. Sketches should include the microstructure above (precipitates of lead in a Sn-Pb liquid mixture) and below (precipitates of lead and lamellae of Sn metal and Pb metal) the eutectic temperature.

5 (a) The Arrhenius temperature dependence of diffusion is based on statistical considerations. Atoms at finite temperature vibrate with an average energy per atom of $3kT$. Statistically, some atoms have an energy greater than this. The probability, p , of an atom having greater energy than q (J per atom) is given by a Boltzman distribution;

$$p = \exp\left(-\frac{q}{kT}\right)$$



There is an effective energy barrier of magnitude between adjacent atoms. Atoms can move over this barrier (i.e. diffuse) as they vibrate with a probability that is strongly temperature-dependent.

Since there is an equal probability that an atom can move (or jump) either way, a driving force is required to tip the balance one way. This driving force can be a concentration gradient, a stress field or an electric field.

The factors that determine how rapidly one material diffuses into another are the activation energy, the size of the diffusing atom, the temperature and the number of grain boundaries.

(b) Diffusion-controlled processes to optimise the properties of;

(i) heat-treatable aluminium alloys

Diffusion is responsible for ageing effects in heat-treatable aluminium alloys. In this case precipitates form and grow with time via diffusion, which reduces their ability to restrict the motion of dislocations.

(ii) non heat-treatable aluminium alloys

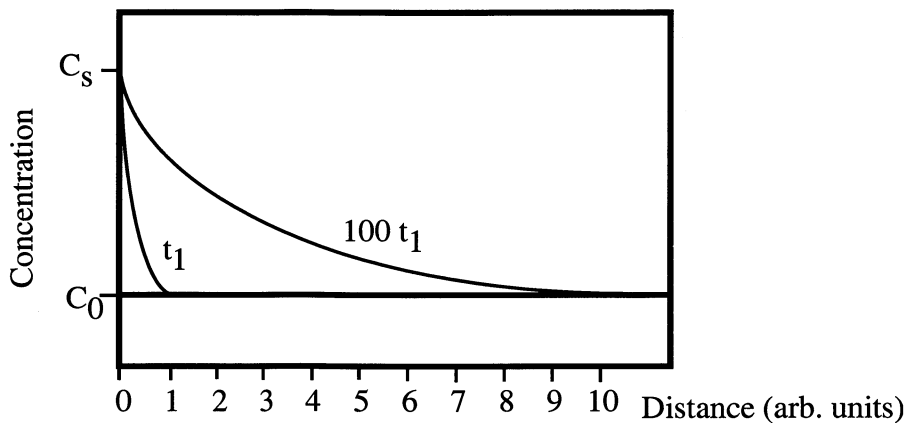
Diffusion is the mechanism by which non heat-treatable (solid solution hardened) aluminium alloys recover their softness and ductility following deformation processing in a process known as recovery and recrystallisation. Dislocations, introduced during deformation, move to low-energy configurations, or annihilate each other at elevated temperature (thermally activated). Dislocations rearrange into sub-grains (small regions with the same crystal orientation).

(iii) semiconductors.

Diffusion is the mechanism responsible for the doping of semiconductors. For example, p-type silicon may be doped with donor atoms to introduce extra electrons to balance the holes that originate from the acceptor atoms. Thus the electrical properties of the semiconductor may be engineered for devices such as FETs and diodes.

(c) Characteristic diffusion length in time t ; $x \approx \sqrt{Dt}$

Therefore, for example, the diffusion length is 10 times that at 100 times the carburising time, i.e.;



$$\text{At } 900^\circ\text{C}; 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right) = \frac{0.6 - 0.2}{0.9 - 0.2} = 0.57 \rightarrow \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right) = 0.43 \quad \text{so } \frac{x}{2\sqrt{Dt}} = 0.4$$

$$\text{At } 1000^\circ\text{C}; 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right) = \frac{0.7 - 0.2}{0.9 - 0.2} = 0.71 \rightarrow \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right) = 0.29 \quad \text{so } \frac{x}{2\sqrt{Dt}} = 0.26$$

[Values of erf obtained from the tabulated data on page 8 of the Materials Data Book.]

$$\frac{\sqrt{D_{1000}}}{\sqrt{D_{900}}} = \frac{0.4}{0.26} = 1.54$$

$$\frac{D_{1000}}{D_{900}} = 2.37$$

The ratio of the diffusion coefficients is;

$$\frac{D_{1000}}{D_{900}} = \exp\left(\frac{-Q}{1273 R} + \frac{Q}{1173 R}\right)$$

$$Q = \frac{\ln(2.37)}{\left(\frac{1}{1173 R} - \frac{1}{1273 R}\right)} = 106 \text{ kJ/mol}$$

6 (a) The main process attributes (defined as such in the notes) that define the physical capabilities of a process are;

- (i) materials class
- (ii) mass
- (iii) section thickness
- (iv) tolerance and
- (v) roughness.

(b) Crankshaft process specification;

Number = 10,000
 Length = 30 cm
 Minimum diameter = 6 cm
 Finish = 1 μm
 Tolerance = ± 0.1 mm
 Tolerant to moderately high temperatures
 Resistant to fatigue
 High yield stress
 Not too brittle
 Hard.

The material must be a metal, and since hardening is required, is a medium carbon or high alloy steel. Fabrication route determined by considering the process attribute charts on pages 23 - 25 of the Materials Data Book. Process selection considerations restricted to metals.

1. Mass considerations;

$$\text{Volume of crankshaft} \approx \pi \times (0.03)^2 \times 0.3 = 8.5 \times 10^{-4} \text{ m}^3$$

$$\text{Density of steel} = 7.9 \text{ Mgm}^{-3}$$

$$\text{Mass} = \text{density} \times \text{volume} = 7.9 \times 10^6 \times 8.5 \times 10^{-4} = 6.7 \text{ kg}$$

From figure 4.2 of the Materials Data Book, any of the metal shaping processes are suitable for fabricating the crankshaft.

2. Section thickness considerations

From figure 4.3 of the Materials Data Book, sand casting, rolling, forging, extrusion or machining are suitable processes for a section thickness of 6cm.

3. Surface roughness considerations

From figure 4.4 of the Materials Data Book, die casting (eliminated in 2 above), extrusion or machining are suitable processes for a surface roughness of 1 μm .

4. Dimensional tolerance considerations

From figure 4.5 of the Materials Data Book, investment casting (eliminated in 2 above), sheet forming (eliminated in 2 above) or machining are suitable processes for a dimensional tolerance of 0.1 mm.

5. Economic batch size considerations

From figure 4.6 of the Materials Data Book, die casting, rolling, forging, powder methods or machining are suitable processes for producing 10,000 crankshafts.

Choice of process, therefore, is as follows;

Use a casting, forging (followed by quenching and tempering) or powder process to produce a near net-shape crankshaft, followed by machining and case hardening case hardening of the bearing surfaces. The surfaces can be ground to the final finish.

(c) Simple cost model for batch shaping processes;

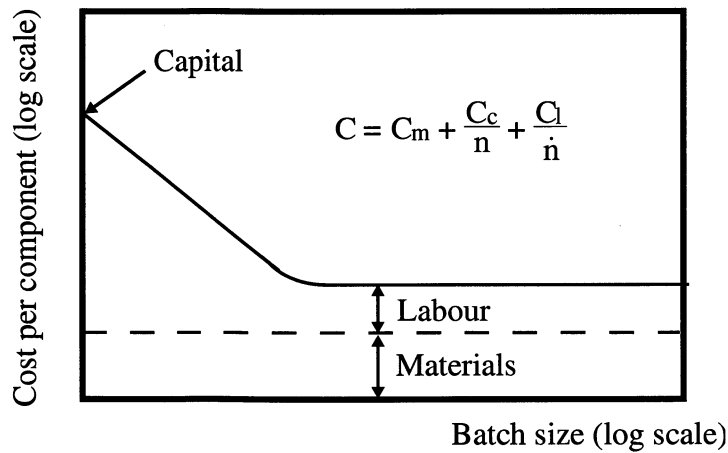
The total cost of the component depends on three contributions;

1. Materials cost (including consumables), C_m
2. Capital cost, C_c
3. Labour cost, C_l

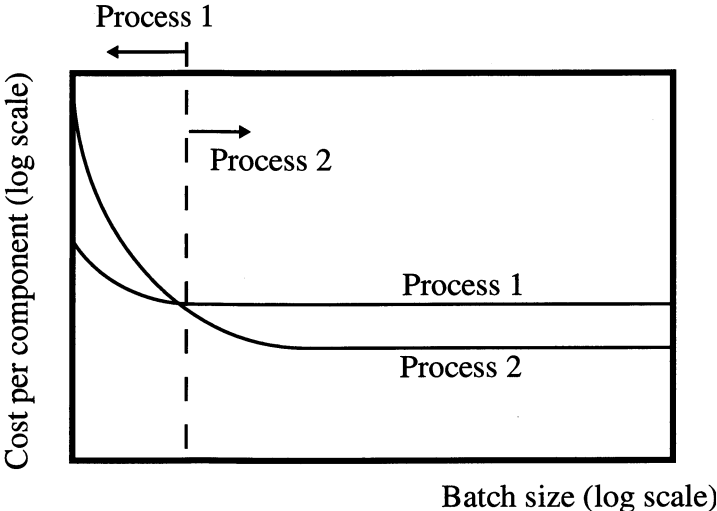
The relative importance of these depends on the rate of production and on the total number of components being made.

Batch size = n
 Production rate = \dot{n}

General cost equation is;
$$C = C_m + \frac{C_c}{n} + \frac{C_l}{\dot{n}}$$



The total cost equation allows competing processes to be ranked approximately in order of increasing cost, with ranking determined by batch size. For example, process that involve relatively high capital and labour costs are only cost effective for relatively high batch sizes (i.e. material cost don't vary significantly between processes that involve similar levels of wastage).



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