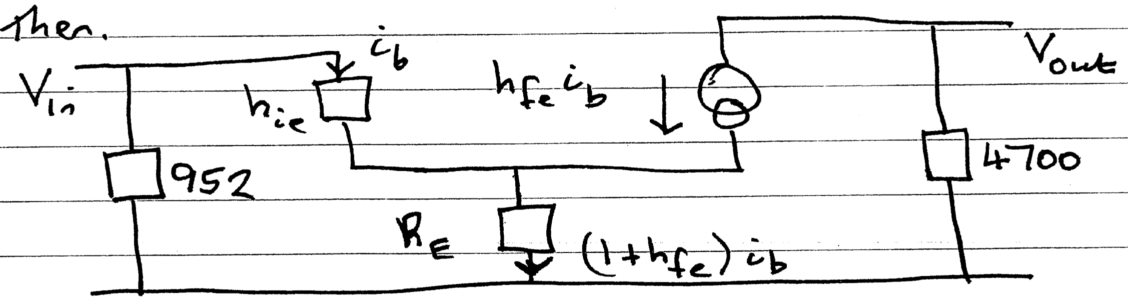
OUTLINE ANSWERS
ENGINEERING TRIPOS
2004 Part Ib Paper 5

2004
Ib Paper 5 Q1 $\quad$ DGm 1.2.2004

a) To construct the small scale equivalent circuit work out the parallel combination of the 20k and 1k resistors.

$$\frac{1}{Z_T} = \frac{1}{20000} + \frac{1}{1000} = 0.00105 \qquad \therefore Z_T = 952\,\Omega$$

Then.



$$\text{Gain} = \frac{V_{out}}{V_{in}} = \frac{-h_{fe}\, i_b\, 4700}{h_{ie}\, i_b + R_E(1+h_{fe}) i_b} = \frac{-4700\, h_{fe}}{h_{ie} + (h_{fe}+1)R_E}$$

In this case $h_{fe} = h_{FE}$ which is effectively infinite

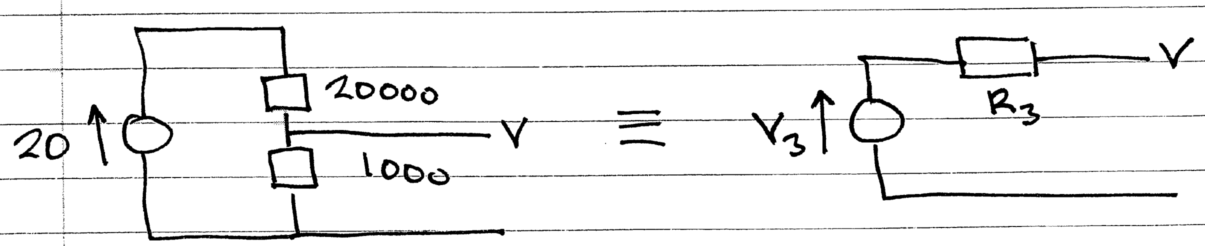
$$\therefore \frac{V_{out}}{V_{in}} = \frac{-4700\, h_{fe}}{h_{fe} R_E} = \frac{-4700}{R_E}$$

For gain $= -10$ require $10 = \frac{4700}{R_E}$ $\quad \therefore \boxed{R_E = 470\,\Omega}$

b) Now $h_{fe} = 200$, $\quad$ gain $= -10 = \frac{-200 \times 4700}{1000 + 201 R_E}$

$$\therefore 1000 + 201 R_E = 94000 \qquad \therefore \boxed{R_E = 462.7}$$

Transform the input part of the circuit in Figure 1



2004 Ib 5Q1 (continued) ✳

Dfm 12.2004

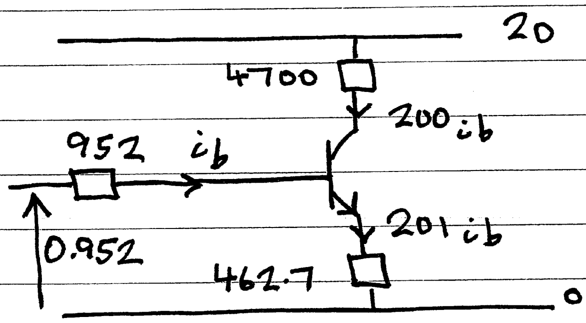
To find R_3 and V_3

Short the output: $I = \frac{20}{20000} = \frac{V_3}{R_3}$ $\therefore V_3 = \frac{R_3}{1000}$

open circuit output: $V = \frac{1000 \times 20}{1000 + 20000} = V_3$ $\therefore V_3 = 0.952$

Hence $R_3 = 952$

Transformed version of Figure 1.



$V_{BE} = 0.7$ volt

Find i_b Input voltage $= 0.952 = 952\,i_b + 0.7 + 201 \times 462.7$

$\therefore 0.252 = 93955\,i_b$

$i_b = 2.68 \times 10^{-6}$ amp.

Collector voltage $V_c = 20 - 4700 \times 200\,i_b$

$= 20 - 940000 \times 2.68 \times 10^{-6}$

$V_c = 17.48$ volt

Emitter voltage $V_E = 201 \times 462.7 \times 2.68 \times 10^{-6}$

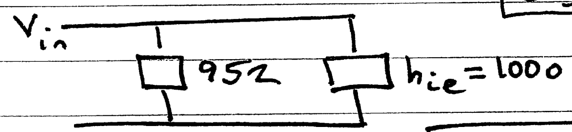
$= 0.25$ volt

Hence $V_{CE} = 17.48 - 0.25 = \boxed{17.23 \text{ volt}}$

Maximum output swing limited by supply to ± 2.77 volt

Hence maximum input swing $\boxed{\pm 0.277 \text{ volt}}$ $\boxed{\text{Peak to peak } 0.55 \text{ volt}}$

c) Input equivalent circuit for $R_E = 0$

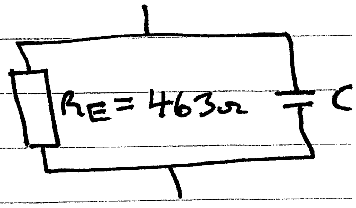


$\frac{1}{R_{in}} = \frac{1}{952} + \frac{1}{1000} = 0.00205$ $\therefore \boxed{R_{in} = 488\,\Omega}$

d) Adding the capacitance C_E effectively shorts R_E and results in increased circuit gain at mid band frequencies.

Require $\frac{1}{2\pi R_E C_E} = 1000$ Hz

$\therefore C_E = \frac{1}{2\pi \times 463 \times 1000} = \boxed{0.34 \times 10^{-6} \text{ F}}$



2004

Ib Paper 5 Q2 RJM 1.2.2004

a) A differential amplifier is chosen because it has a higher gain for the 1 kHz differential signal than for the common mode 50 Hz which is unwanted.

$$CMRR = \frac{A_{diff.}}{A_{common.}}$$

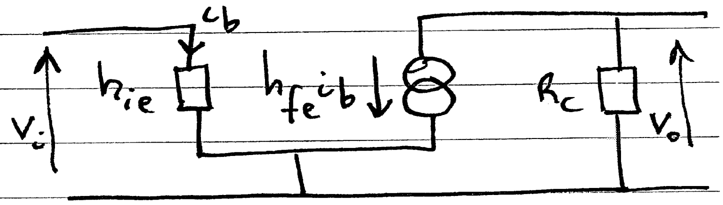
40 dB corresponds to 100 times

$$\frac{V_{out}(diff)}{V_{out}(common)} = \frac{A_{diff} \times 0.002}{A_{common.} \times 0.050} = \frac{A_{diff}}{25\,A_{common}}$$

$$\text{Require } 100 = \frac{1}{25}\frac{A_{diff}}{A_{com.}} = \frac{CMRR}{25}$$

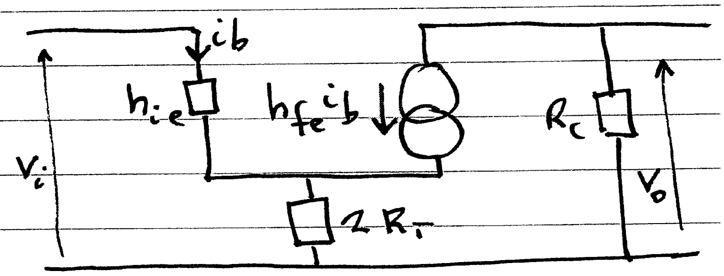
Common mode rejection ratio 2500 or $\boxed{68\text{ dB}}$

b) From Figure 2 to analyse the differential halfcircuit ignore R_T



$$A_{diff} = \frac{V_o}{V_i} = \frac{-h_{fe}\,i_b\,R_c}{h_{ie}\,i_b} = \frac{-h_{fe}\,R_c}{h_{ie}}$$

For the common mode half circuit take half the conductance of R_T



$$A_{common} = \frac{V_o}{V_i} = \frac{-h_{fe}\,i_b R_c}{h_{ie}\,i_b + 2R_T(1+h_{fe})\,i_b} = \frac{-h_{fe}\,R_c}{h_{ie} + 2R_T(1+h_{fe})}$$

$$CMRR = \frac{A_{diff}}{A_{comm}} = \frac{h_{ie} + 2R_T(1+h_{fe})}{h_{fe}R_c} \cdot \frac{h_{fe}R_c}{h_{ie}}$$

2004 Ib Paper 5 Q2 (continued)

Dfm 1.2.2004

Hence $CMRR = \dfrac{h_{ie} + 2R_T(1+h_{fe})}{h_{ie}}$

$= 1 + \dfrac{2R_T(1+hfe)}{h_{ie}} = 1 + \dfrac{2R_T\,160}{1000}$

$CMRR = 1 + 0.302\ R_T$

In this case $50\,dB \equiv 316 = 1 + 0.302\ R_T$

require $R_T = \dfrac{315}{0.302} = \boxed{1043\ \Omega}$

In this case $A_{diff} = 177.8 = \dfrac{150}{1000} R_C$ $\quad\therefore R_C = \dfrac{177.8}{.15} = \boxed{1185\ \Omega}$

c) For 0.01 amp the potential difference across R_C is $0.01 \times 1185 = 11.85V$

For $V_{BE} = 0.7\ V$, $V_{CE} = 10V$ and $V_B = 0$ require $V_C = 9.3V$

Hence $V_{CC} = 11.85 + 9.3 = \boxed{21.15\ \text{volt}}$

Total current through R_T is 0.02 amp (assuming the base currents are negligible)

Hence potential difference across R_T is $0.02 \times 1043 = 20.86\ V$

But for $V_{BE} = 0.7$ and $V_B = 0$ require $V_E = -0.7\ V$

Hence $V_{EE} = -0.7 - 20.86 = \boxed{-21.56\ \text{volt}}$

d) The common mode rejection ratio could be increased by placing a Zener diode or a transistor in place of R_T

This will reduce the required power supply voltage.

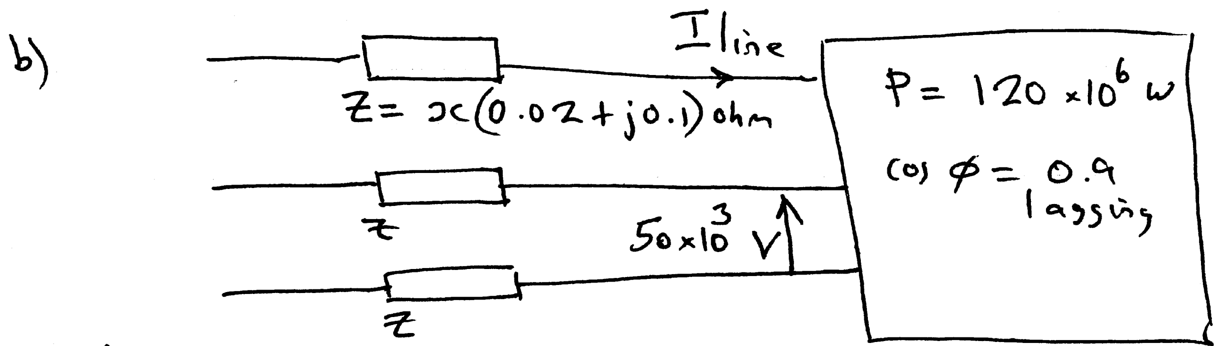
2004
Ib Paper 5 Q3 Dfm 1.2.2004

a) Large generators are used where possible because they are more efficient than small generators. Hence power is transmitted to many customers some a long distance away.

Reliability of supply is improved by inter connecting many large generators on a grid. If one generator were to fail then supply can usually be maintained. Hence long distance transmission which can be very efficient when low currents (and high voltages) are used.

In the UK burning fossil fuels such as oil coal and gas to generate steam to drive turbine generators is the main source of power. Nuclear power is significant but becoming less important. Hydro electric power is very cost effective in a few locations but only contributes a few % of the total power as do wind, tidal and solar sources.

b)



The line current per phase $I_{line} = \frac{1}{\sqrt{3}} \frac{P}{\cos\phi \, V}$

$$I_{line} = \frac{120 \times 10^6}{1.732 \times 0.9 \times 50 \times 10^3} = \boxed{1540 \text{ Amp}}$$

c) Consider the case with line length x and losses of 5%

Total Real Power $P_{in} = 120 \times 10^6 + 6 \times 10^6 = 126 \times 10^6$ Watt

Real Loss in the line $3 \, (I_{line})^2 \, \text{Real}(Z)$

$\therefore \; 6 \times 10^6 = 3\,(1540)^2 \; x \; 0.02$

$$x = \frac{6 \times 10^6}{0.06\,(1540)^2} = 42.19 \text{ km}$$

$\boxed{\text{length } 42.2 \text{ km}}$

2004 Ib 5 Q3 (continued)

DfM 2.2.2004

For one line $Z_{total} = (0.02 + j0.1)\, 42.19$

$$= 0.8438 + j\,4.219$$

Total Reactive Power $Q_{in} = 3\,(I_{line})^2\, I_{mag}(Z) + P\tan(\cos^{-1}0.9)$

$$= 3\,(1540)^2\, 4.216 + 120\times10^6 \tan(\cos^{-1}0.9)$$
$$= 30 \times 10^6 + 58.12 \times 10^6$$
$$= 88.12 \times 10^6$$

Evaluate $S_S = \sqrt{(Q_{in})^2 + (P_{in})^2}$

$$= \sqrt{(88.12)^2 + (126)^2} \times 10^6$$

$$= 153.8 \times 10^6 \text{ VA}$$

The voltage at the sending end is V_S

where $S_S = \sqrt{3}\, I_{line}\, V_S$

$$\therefore V_S = \frac{S_S}{\sqrt{3}\, I_{line}} = \frac{153.8\times10^6}{1.732 \times 1540}$$
$$= \boxed{57.7\times10^3 \text{ Volt}}$$

As expected the sending end voltage (57.7 kV) is greater than the voltage needed at the city (50 kV) because of the line loss.

d) By national standards 42 km is a short length of feeder. Increasing the line voltage (to 300 kV for example) is the most effective way to increase the maximum feasible length ie reduce the line current.

Using a thicker cable to reduce the ohmic loss per km is probably less cost effective, and with a power factor 0.9 increasing that the unity would make a relatively small improvement in the maximum feasible length of feeder.

2004

Ib Paper 5 Qu.4 DB 2.2.2004

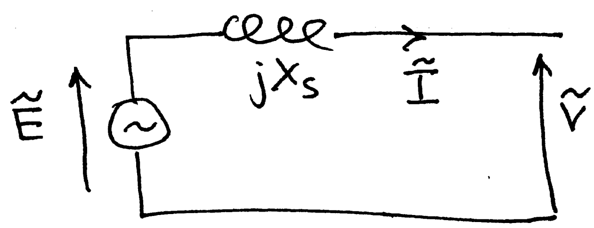
a) The national grid system is so large that no individual generator can influence the magnitude or the frequency of its terminal voltage. To convert power the machine must rotate at its synchronous speed which is determined by the number of poles on the machine and the frequency of the grid. The load angle determines whether the machine is acting as a generator or motor.

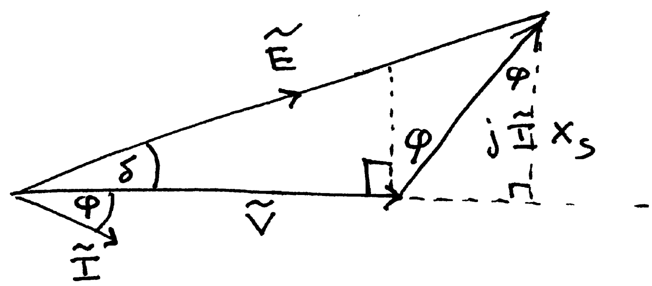
50Hz i) For 2 pole 3000 rpm or 314 radian/sec are needed

ii) For 20 pole 300 rpm or 31 " " "

To generate hydro electric power, and also in diesel electric generators, a relatively small rotation speed is desired hence synchronous machines have a large number of poles in these applications.

b)





c) Power delivered $= \sqrt{3}\ V_L I_L \cos\varphi$

Hence $300\times10^6 = 1.732\ 50\times10^3\ I_L\ 0.7$

$$I_L = \frac{300\times10^6}{1.732\times50\times10^3\times0.7} = \boxed{4949\ \text{Amp}}$$

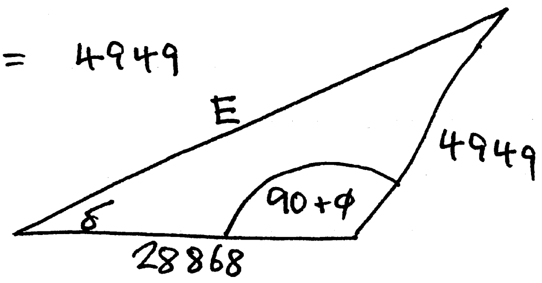
2004 IB5 Qu.4 (continued)

Dfm 1.2.2004

Voltage $V = \dfrac{V_{line}}{\sqrt{3}} = 28868$ Volt

$\bar{I} X_s = 4949 \times 1 = 4949$

Apply the cosine rule in the triangle to determine E



$$E^2 = V^2 + (\bar{I} X_s)^2 - 2|V||\bar{I} X_s| \cos(90+\varphi)$$

But $\cos(90+\varphi) = -\sin\varphi$ $\quad$ $\sin\varphi = 0.714$

$$\therefore \quad E^2 = (28868)^2 + (4949)^2 + 2 \; 28868 \; 4949 \; \sin\varphi$$

$$= 833.4 \times 10^6 + 24.5 \times 10^6 + 204 \times 10^6$$

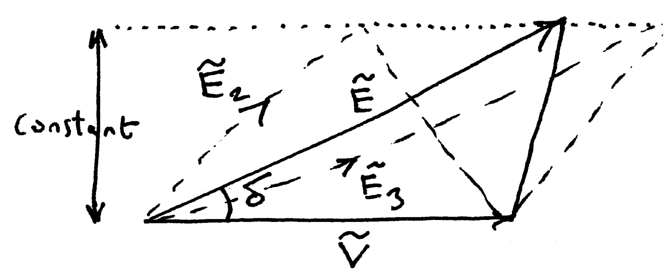
$$E = 32.59 \times 10^3 \text{ Volt}$$

The generator phase voltage is $\boxed{32.6 \text{ kV}}$

d) If the excitation is increased but the prime-mover power is held constant then the amount of reactive power delivered to the load changes.

E_3 is over excited example

E_2 is under excited.



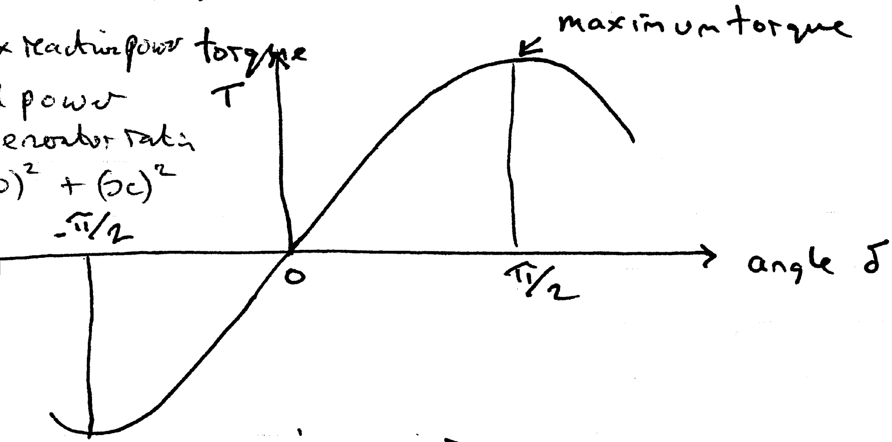
Let x be max reactive power torque

300 MW is real power

600 MVA is generator rating

$(600)^2 = (300)^2 + (x)^2$

$\boxed{\therefore x = 520 \text{ MVA}}$



acting as a motor | generating electric power

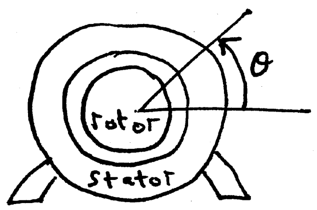
The point $\delta = 0$ is special because no mechanical power is transfe

2004

Ib Paper 5 Q 5

DFm 1.2.2004

a)



A rotating magnetic field is established in the air gap by the balanced three phase coils wound on the stator

The rotor also has a balanced three phase winding.
The stator and rotor-driven magnetic fields have the same number of poles and rotate at the same speed producing a steady torque (except at the synchronous speed when the torque is zero)

b) For an eight-pole motor $p = 4$

The slip is defined by $S = \dfrac{\omega_s - \omega_r}{\omega_s}$

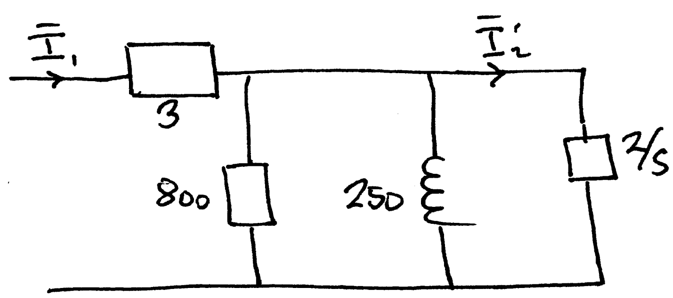
$= 1 - \dfrac{\omega_r}{\omega_s}$

In this case $\omega_r = 738$ rpm

Hence $S = 1 - \dfrac{738}{\frac{50}{4} \times 60}$

$= 1 - \dfrac{738}{750} = \boxed{0.016 \quad \text{Slip}}$

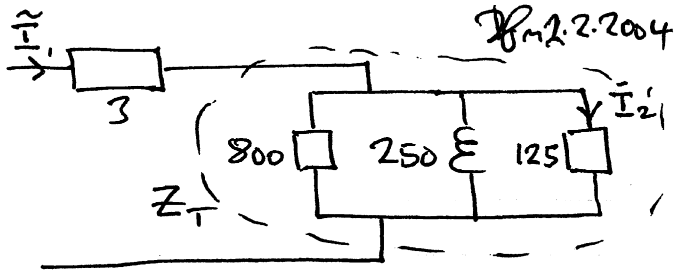
c) The equivalent circuit simplifies from that in the data book to



In this case $\dfrac{2}{s} = \dfrac{2}{0.016} = 125$ ohm

1/4

To determine the complex input impedance per phase the three parallel resistors must first be combined.



$$\frac{1}{Z_T} = \frac{1}{800} + \frac{1}{j250} + \frac{1}{125} = 0.00925 + j0.004$$

$$\therefore Z_T = \frac{0.00925 - j0.004}{(0.00925)^2 + (0.004)^2} = 91.08 + j39.4$$

Hence total complex input impedance is $\boxed{94.1 + j39.4 \text{ ohm}}$

Driving the motor with 600 volts gives a stator line current $= \frac{\sqrt{3}\, V}{Z}$

$$= \frac{1.732 \times 600}{|94.1 + j39.4|} = \frac{1.732 \times 600}{102}$$

$\boxed{\text{Line current} = 10.2 \text{ Amp}}$

Using the data book formula electromagnetic torque developed is

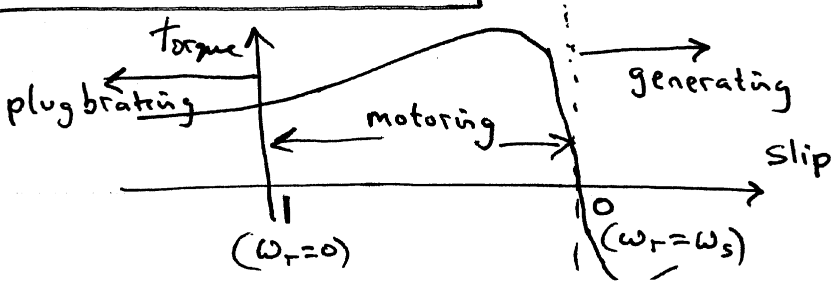
$$T = \frac{3 (I_2)^2}{\omega_s} \frac{R_2'}{S}$$

Find an approximate value for I_2 by neglecting the stator winding resistance.

$$I_2 = \frac{\sqrt{3}\, 600}{125} = 8.3 \text{ amp.}$$

$$T = \frac{3 \left(\frac{8.3}{\sqrt{3}}\right)^2 \times 125}{2\pi \times 50/4} = \frac{3 \times 23 \times 500}{100\pi}$$

$\boxed{T = 110 \text{ Nm torque}}$



2004

Ib Paper 5 Q6 JBm 1.1.2004

a) The characteristic impedance in the ratio between the voltage and the current of a unidirectional wave on a transmission line at any point. It is always positive.

$$Z_o = \frac{V_F}{I_F} = \sqrt{\frac{L}{C}}$$

The wave velocity is $\frac{1}{\sqrt{LC}}$ and for an air cored transmission line is equal to the velocity of light $3\times10^8\ ms^{-1}$

b) Power transmission lines

Wave velocity $\frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.17\times10^{-6} \times 3.5\times10^{-12}}} = 3\times10^8\ ms^{-1}$ as expected for free space.

Wavelength $\lambda = \frac{v}{f} = \frac{3\times10^8}{50} = 6\times10^6$ metres.

Impedance $Z_o = \sqrt{\frac{L}{C}} = \sqrt{\frac{3.17\times10^{-6}}{3.5\times10^{-12}}} = 952$ ohm.

Transmission lines are always much less than the 6000 km wavelength so a simple lumped element equivalent circuit is appropriate @ 50 Hz

c) Ethernet cable

Wave velocity $\frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.2\times10^{-6} \times 90\times10^{-12}}} = 2.36\ \cancel{10}^8\ ms^{-1}$

Wavelength $\lambda = \frac{v}{f} = \frac{2.36\times10^8}{10\times10^6} = 23.6$ m

Impedance $Z_o = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.2\times10^{-6}}{90\times10^{-12}}} = 47\ \Omega$

The wave velocity is less than the free space velocity of light because a dielectric medium is used.

In this case the wavelength is small compared with a typical system size and transmission line effects are important.

2004 Ib Paper 5 Q6 (continued)

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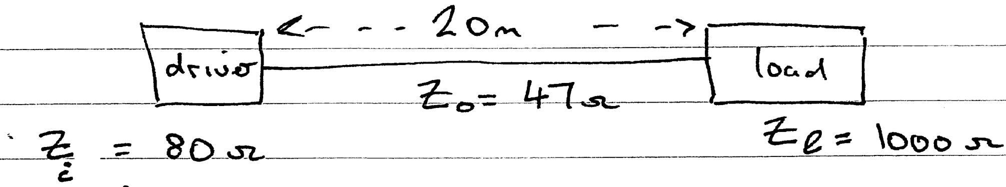

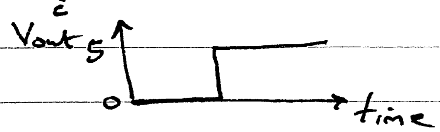

The driver switches at time $t=0$

A step voltage pulse travels along the line at $2.36 \times 10^8\ ms^{-1}$ arriving at the load after a delay of $\frac{20}{2.36 \times 10^8} = 8.5 \times 10^{-8}$ sec

Magnitude of first step $= V_{out} \times \frac{Z_0}{Z_0 + Z_i} = 5 \times \frac{47}{47+80}$

$\boxed{= 1.85 \text{ volt}}$

At the load the reflection coefficient is $\frac{Z_\ell - Z_0}{Z_\ell + Z_0}$

$= \frac{1000 - 47}{1000 + 47} = \boxed{0.91}$

Most of the wave is reflected ie amplitude $0.91 \times 1.85 = 1.68$ volt

The potential of the line then rises to $1.85 + 1.68 = 3.5$ volt

After a further 8.5×10^{-8} sec this returning pulse is partly reflected at the source.

But at the centre of the line after 2×10^{-7} seconds the potential is still $\boxed{3.5V}$

The final voltage at the load is essentially 5 V after the succession of reflected waves has died down (because $Z_\ell \gg Z_i$)

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2004
Ib Paper 5 Q7. DJm 1.2.2004

a) For an electromagnetic wave the electric and magnetic field intensities are linked by the intrinsic impedance

$$\eta = \frac{\bar{E}_x}{\bar{H}_y} = \sqrt{\frac{\mu_r \mu_0}{\varepsilon_r \varepsilon_0}}$$

by analogy with a transmission line where the inductance and capacitance give the intrinsic impedance $Z_0 = \sqrt{\frac{L}{C}}$

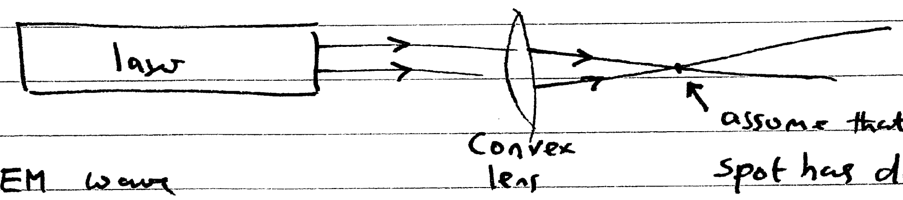
The dielectric strength of a dielectric medium is the largest electrical field that it can support without dielectric breakdown (ionization in the case of air molecules)

For air $\mu_r = 1$ and to a good approximation $\varepsilon_r \simeq 1$

Hence $$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$= \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}} = \boxed{377\,\Omega}$$

b)



assume that the spot has diameter d and uniform light intensity across it.

for an EM wave

$$\text{power density} = \frac{1}{2} \bar{E}\bar{H}^* \qquad \left(\text{but } H_0 = \frac{E_0}{\eta}\right)$$

$$= \frac{1}{2} \frac{E_0^2}{\eta}$$

$\therefore$ Maximum power density to avoid dielectric breakdown $= \frac{1}{2} \frac{(2 \times 10^6)^2}{377}$

$$= 53 \times 10^8 \quad \mathrm{W\,m^{-2}}$$

For focus spot size d assuming uniform power density gives total power $\frac{\pi d^2}{4} \times 53 \times 10^8$ watt

2004 Paper 5 Q7 (continued)
Ib

DPM 1.2.2004

For 1 watt laser

$$1 = \frac{\pi d^2}{4} \times 53 \times 10^8$$

minimum spot diameter $d = \sqrt{\frac{4}{\pi \; 53 \times 10^8}} = \boxed{15.5\,\mu m}$

Magnetic field $H_0 = \frac{E_0}{\eta} = \frac{2 \times 10^6}{377} = 5305 \; Am^{-1}$

Flux density $B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 5305$
$= \boxed{0.0067 \text{ Tesla}}$

c) For a laser output power of 1 milliwatt the reduction is by a factor of 1000

hence at first sight the reduction in the feasible spot size would be a factor of $\sqrt{1000}$ to $0.5\,\mu m$

But diffraction limits make the minimum spot size about $\lambda = 1\,\mu m$. corresponding to the wavelength
ie Power density $\frac{1}{4}$ of maximum
Corresponding electric field $\frac{1}{2} \times$ breakdown field

Field = $\boxed{1 \; MVm^{-1}}$

In this case dielectric breakdown is very unlikely.