UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IB

Monday 31 May 2004 9 to 11

Paper 1

MECHANICS

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments to this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

1 A pencil is modelled as a uniform rod AB of mass m and length $2a$. The pencil is standing upright at the edge of a table O as shown in Fig. 1. At time $t = 0$ a horizontal impulse I is delivered at its base A causing it to leave the table. Coordinates x and y describe the motion of the centre of mass G as it follows the curved path shown and θ is the pencil's rotation.

Initially $x = 0$, $y = a$ and $\theta = 0$.

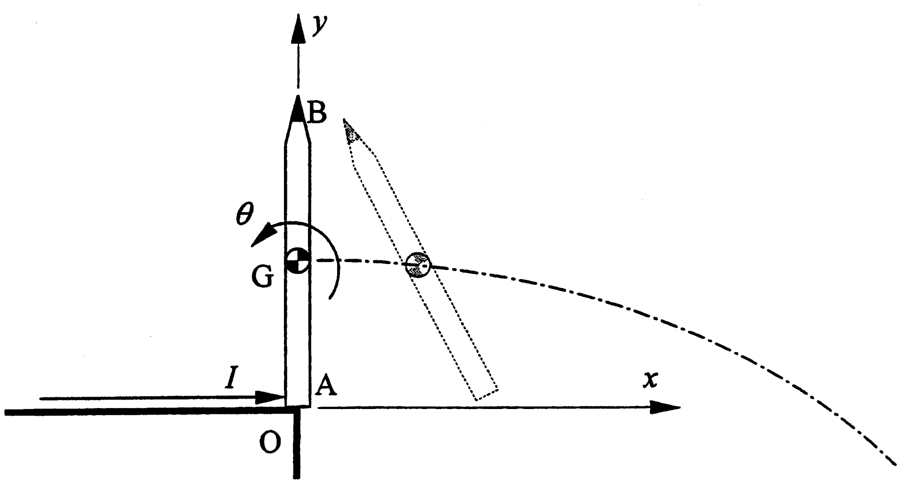

Fig. 1

(a) Find both the speed $\dot{x}$ and angular velocity $\dot{\theta}$ of the pencil immediately after the impulse. [6]

(b) Find the variation with time of x, y and θ and show that $x = a\theta/3$ throughout the motion. [4]

(c) For a certain value of the impulse I_0 the tip of the pencil B just hits the table at O. Using your answers in (b) show that:

(i) the angle $\theta \approx 130.6°$ when the pencil hits the table at O; [4]

(ii) impact occurs when $t \approx 0.836\sqrt{a/g}$; [2]

(iii) the value of the impulse required is $I_0 \approx 0.908m\sqrt{ga}$. [4]

2 In the mechanism shown in Fig. 2 the light link AB and the slider at C constrain the thin uniform flat triangular plate BCD to move in its own plane. A is a fixed pivot and all dimensions are shown in the figure. The plate has mass m and its centre of mass is at G. The centroidal moment of inertia of the plate is $mL^2/9$.

At the instant shown $\angle CAB = 45°$, distance AC = $2L$, the angular velocity of crank AB is ω, the value of the angular acceleration $\alpha = \omega^2$ and the driving torque at A is T.

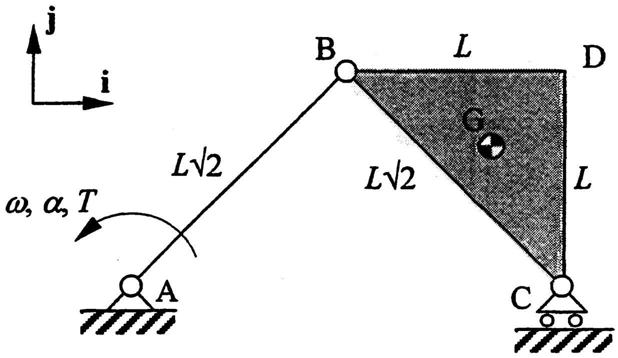

Fig. 2

(a) Find the angular velocity of the plate and show that the velocity of its centre of mass G is

$$-\frac{4}{3}L\omega\,\mathbf{i} + \frac{1}{3}L\omega\,\mathbf{j}. \qquad [4]$$

(b) Find the angular acceleration of the plate and show that the acceleration of its centre of mass G is

$$-3L\omega^2\,\mathbf{i} - \frac{1}{3}L\omega^2\,\mathbf{j}. \qquad [8]$$

(c) Find the torque T driving the mechanism at this instant. [8]

3 A uniform solid spherical ball is "biased" with a small lead pellet P embedded just beneath its surface. The bias causes the ball to roll on a circular path. The ball and small pellet have combined mass M and the ball radius is a. The small pellet has mass m and is at a distance a from the centre of the ball as shown in Fig. 3. The ball is rolling without slip on a horizontal surface. The centre of the ball G is moving with speed V and the path of G is circular with centre at O and radius $R >> a$. At all times points G, P and O lie on a horizontal straight line and the moment of inertia of the ball about the axis PG may be taken as $\frac{2}{5}Ma^2$.

(a) On a free-body diagram of the ball show all forces acting including weight, reaction and d'Alembert forces. Hence or otherwise show that the couple acting on the ball is

$$mga - \frac{MV^2 a}{R} . \qquad [6]$$

(b) Give expressions for:

(i) the angular velocity ω of the ball in terms of V and a;

(ii) the precession rate Ω of the ball in terms of V and R.

Vectors representing these two angular velocities are shown in the figure. [6]

(c) Use your understanding of the gyroscopic effect to find an expression for the path radius R of the ball when it is moving at forward speed V. [8]

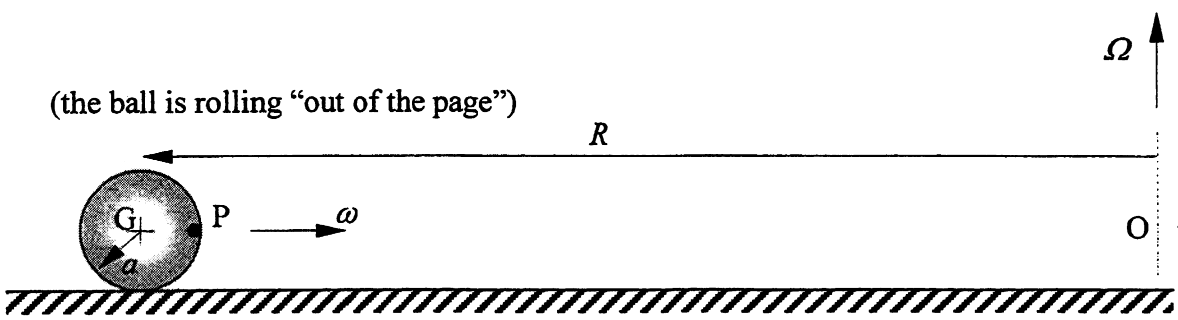

Fig. 3

SECTION B

4 A circular cylinder of radius r is cut in half along a diametral plane to form the rocker shown in Fig. 4. The rocker has mass m and its centre of mass G is at a distance a from the cut surface. The rocker rolls without slip on a smooth horizontal table and its motion is described by the angle θ between the cut surface and the horizontal.

(a) Use the Mechanics Data Book to find the dimension a and by making use of the parallel-axis theorem or otherwise show that the moment of inertia of the rocker about G is $0.320mr^2$. [6]

(b) Find expressions for the potential energy and kinetic energy of the rocker as functions of θ. [8]

(c) Determine the natural frequency of small oscillations. [6]

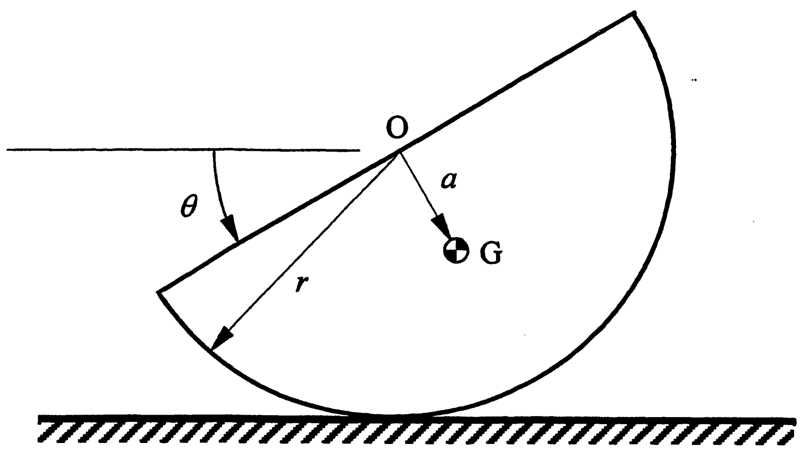

Fig. 4

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5 A thin, uniform rod OA of mass m and length l swings as a pendulum in a vertical plane as shown in Fig. 5. The angle between OA and the vertical is θ as shown.

The rod is held close to vertical ($\theta \approx 0$) and is released from rest.

(a) Find expressions for $\dot{\theta}$ and $\ddot{\theta}$ in terms of θ. [8]

(b) Compute the horizontal and vertical components of the force acting on the bearing O when $\theta = \pi/2$. [6]

(c) Find the bending moment at the mid-point of the rod when $\theta = \pi/2$. [6]

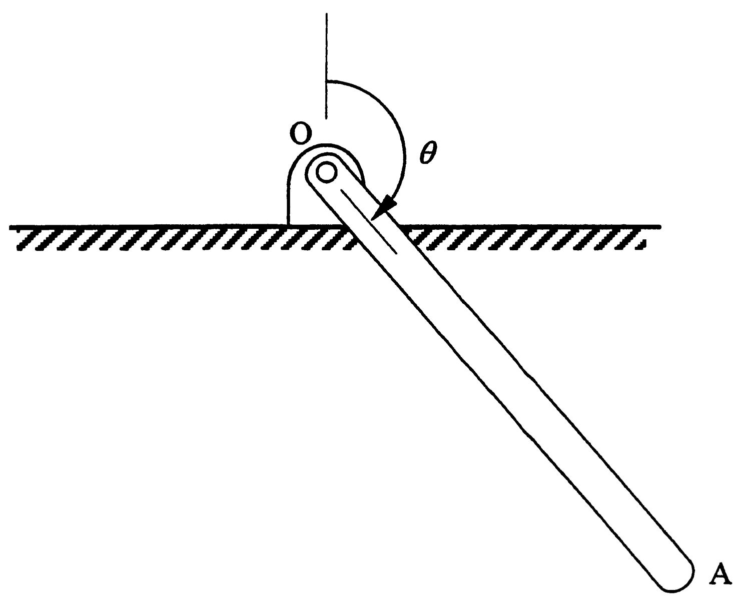

Fig. 5

6 A car wheel is out of balance. Balance weights are attached at the wheel rims at a radius R as shown in Fig. 6. The rims are separated by a distance w. A computer-controlled balancing machine has found that the out-of-balance is equivalent to a mass of $m = 40$ grams located on the wheel at a radius $r = 5R/4$ and at an angle $\theta = 30°$, on a plane at a distance $x = w/4$ from the centreline of the wheel as shown in the figure.

Static and dynamic balance is achieved by adding two balance weights m_1 and m_2, one to each rim of the tyre.

(a) Find the mass and position of the two balancing weights. [6]

(b) In practice weights are only available in steps of 10 grams. Based on your answer to (a) devise a scheme to achieve static and dynamic balance using one pair of 10-gram weights on one rim and one pair of 20-gram weights on the other rim. [8]

(c) Three of the weights from part (b) fall off. Which one remaining will give the worst net out-of-balance force? Sketch the resulting out-of-balance vectorially. [6]

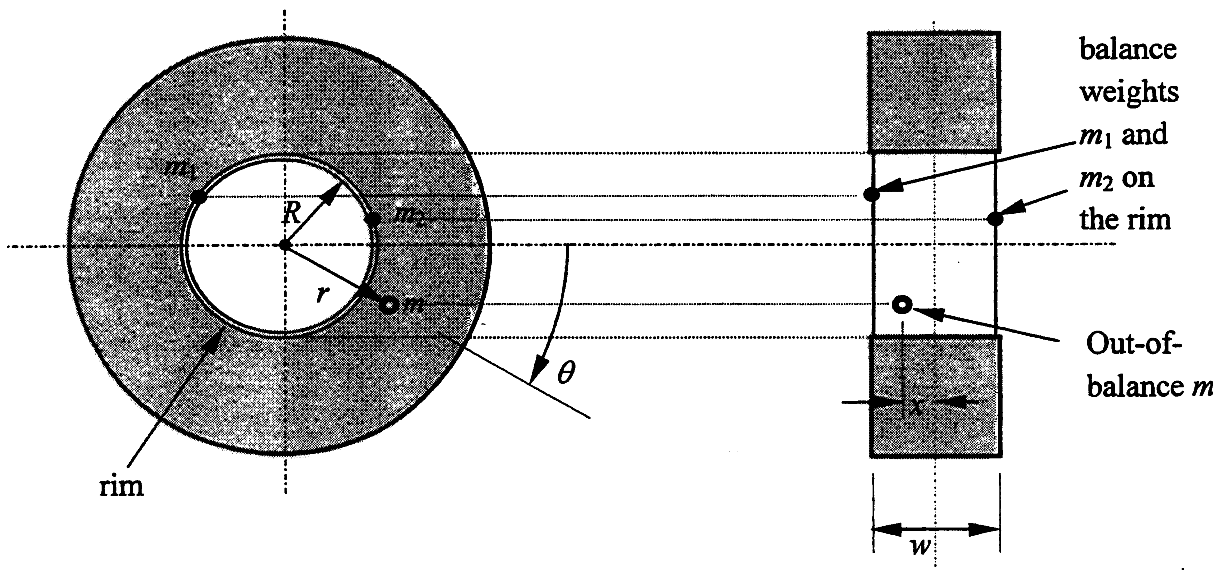

Fig. 6
(note that m_1 and m_2 are shown at arbitrary positions)

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