UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IB

Monday 31 May 2004 2 to 4

Paper 2

STRUCTURES

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments to this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

1 A planar pin-jointed structure has six members, I to VI, as illustrated in Fig. 1. All behaviour may be assumed to be elastic. Each member has axial stiffness EA. The unloaded structure is free of stress and the diagonal bars V and VI are not connected to each other.

(a) Show that the structure has two redundancies. [3]

(b) A horizontal force W is applied to the structure, as shown in Fig. 1. Using III and IV as redundant bars:

(i) find a particular equilibrium solution of bar tensions to the applied load W shown in Fig. 1; [3]

(ii) find two possible states of self stress in the structure; [4]

(iii) find the elastic solution for the bar tensions under the load W. [10]

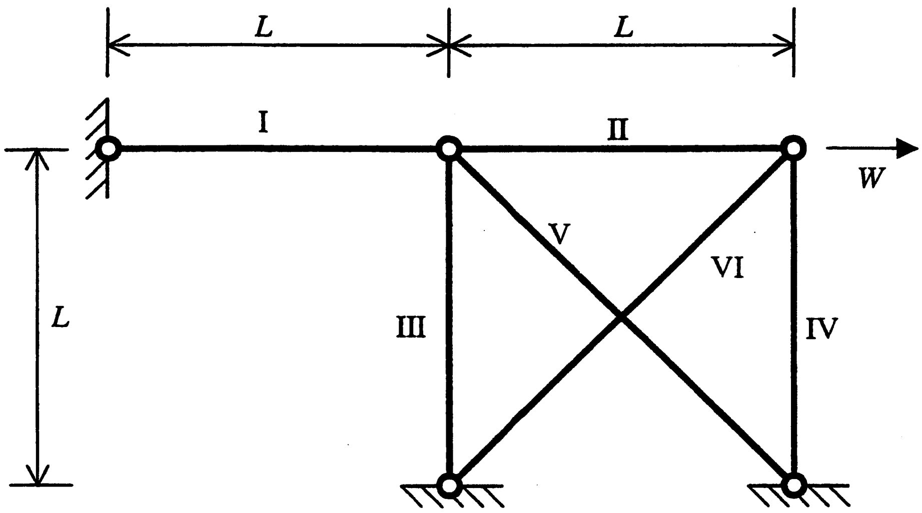

Fig. 1

2 In the light frame shown in Fig. 2 the beam BC is rigidly connected at B to the column AB. Column AB is fully built-in to the ground at A. AB, BC and CD have flexural rigidity EI. Beam BC is propped at C by the pin-ended column CD. The unloaded frame is free of stress and axial strains and buckling are to be ignored. All behaviour may be assumed to be elastic.

(a) Determine the number of redundancies in the structure. [3]

(b) A uniformly distributed vertical load W is applied to beam BC.

(i) Determine the reaction forces and moments at the supports A and D. [4]

(ii) Determine the rotation and displacement of the end C of beam BC. [6]

(iii) Determine the location and magnitude of the maximum bending moment in the frame. Hence sketch a bending moment diagram for the entire structure, indicating important values. [7]

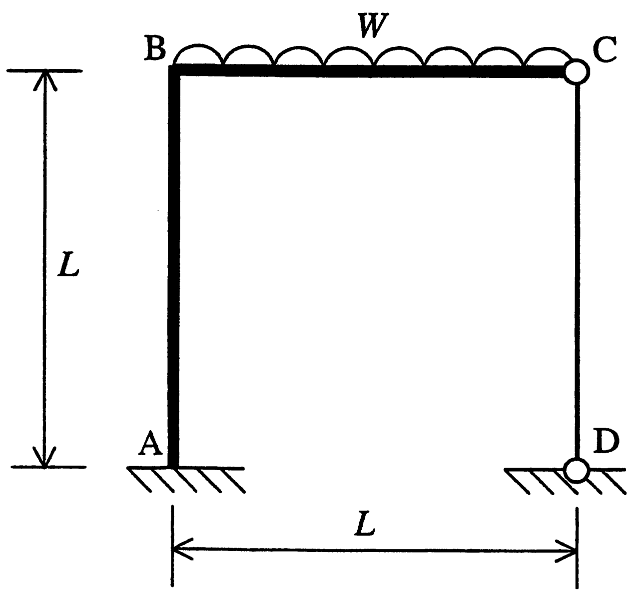

Fig. 2

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3 Two 45° strain gauge rosettes are attached to the surface of a rectangular hollow steel beam, as shown in Fig. 3. One rosette (gauges 1 to 3) is attached to the centre of the top flange while the second rosette (gauges 4 to 6) is attached to one web at mid-depth. The rosettes are aligned with the beam coordinate system $x\ y\ z$. The wall thickness is 5 mm everywhere.

The beam is initially unstressed with all strain gauges reading zero. It is then loaded elastically until the strain gauge readings are as given below:

Gauge	Strain measured
1	-1.8×10^{-4}
2	$+6.0\times10^{-4}$
3	$+3.0\times10^{-4}$
4	zero
5	zero
6	$+2.0\times10^{-4}$

(a) Sketch a Mohr's circle of strain for each rosette location. [6]

(b) Show that the significant stresses in the beam at the gauge positions are as follows:

Top flange (1, 2, 3) $\sigma_{xx} = 0$ MPa $\sigma_{zz} = 126$ MPa $\tau_{zx} = -14.6$ MPa

Web (4, 5, 6) $\sigma_{yy} = 0$ MPa $\sigma_{zz} = 0$ MPa $\tau_{zy} = -32.4$ MPa [8]

(c) Calculate the bending moment about axis Oxx, the shear force in the direction Oy and the torsion. You may assume that the section carries no overall axial tension, bending moment about Oyy or shear force in the direction Ox. [6]

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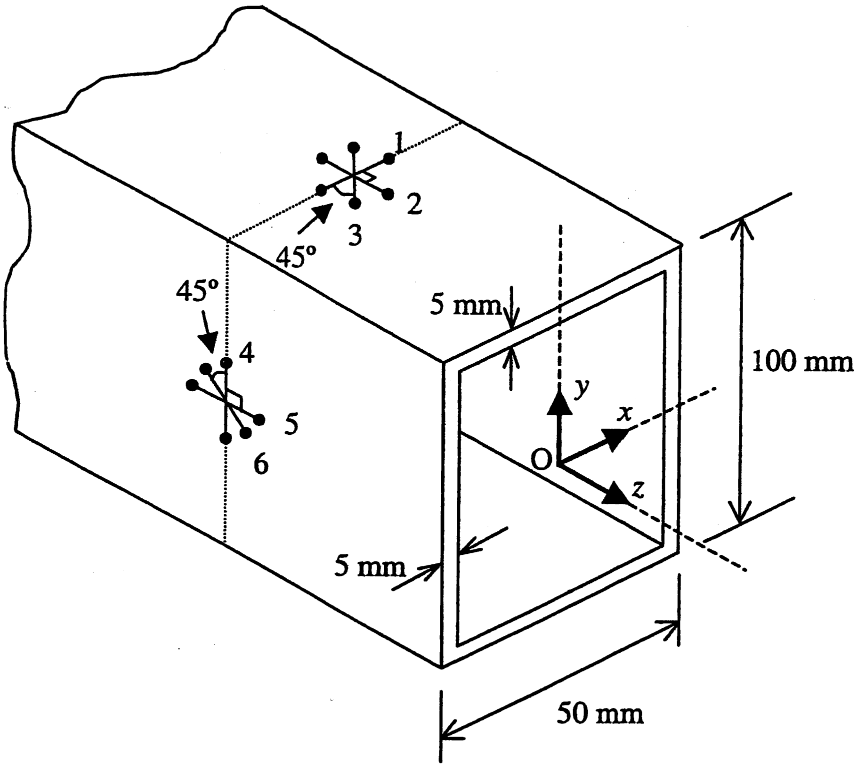

Fig. 3

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SECTION B

4 The steel beam AC is continuous over the intermediate support B, as shown in Fig. 4. It carries a uniformly-distributed live load of 170 kN/m over span AB and a point load of 933 kN at point G on the span BC as shown. The self-weight of the beam is negligible and buckling may be ignored. The yield stress of the steel may be taken as 255 MPa.

a) Determine whether or not a 533 × 210 × 109 Universal Beam bending about its major axis will collapse under the proposed load, and sketch an appropriate bending moment diagram to justify your answer. [10]

b) If the loads in each span were to be applied separately, would the beam collapse, and if so, under which arrangement of loads? Sketch appropriate bending moment diagrams. [4]

c) If the distributed load is removed, and the point load of 933 kN can be applied at any position along the beam AC, determine the minimum value of the uniform plastic section modulus (in cm^3) that that will just carry this "rolling load" without collapse. [6]

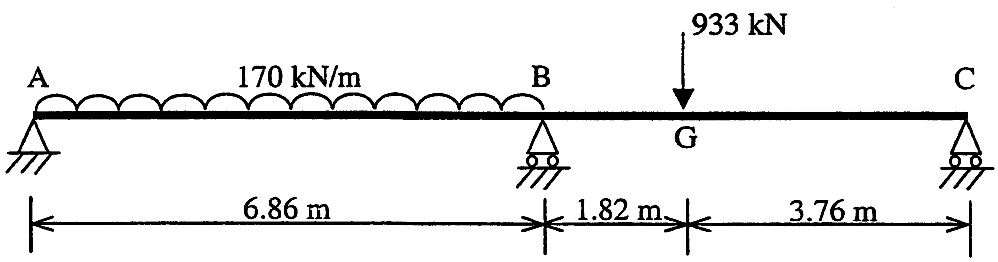

Fig. 4

5 (a) Determine the plastic section modulus for bending about a horizontal axis for the beam cross-section shown in Fig. 5. [5]

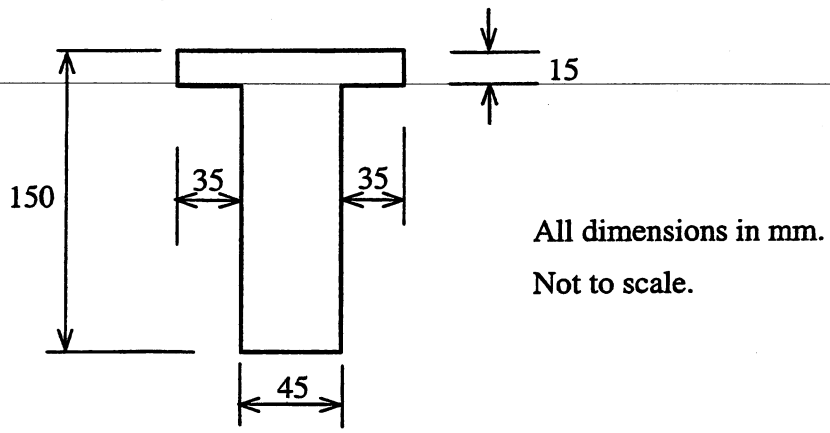

Fig. 5

(b) A hexagonal steel plate of thickness t is shown in plan in Fig. 6. The steel has a uniaxial yield stress in tension of Y. The edge AB of the plate is fully built-in, and the edges CD and EF are simply supported. Edges BC, DE and FA are unsupported. For the collapse mechanism illustrated in Fig. 6 estimate (in terms of Y, t and L) the magnitude of the uniformly distributed load W per unit area needed to cause collapse by this mechanism. [15]

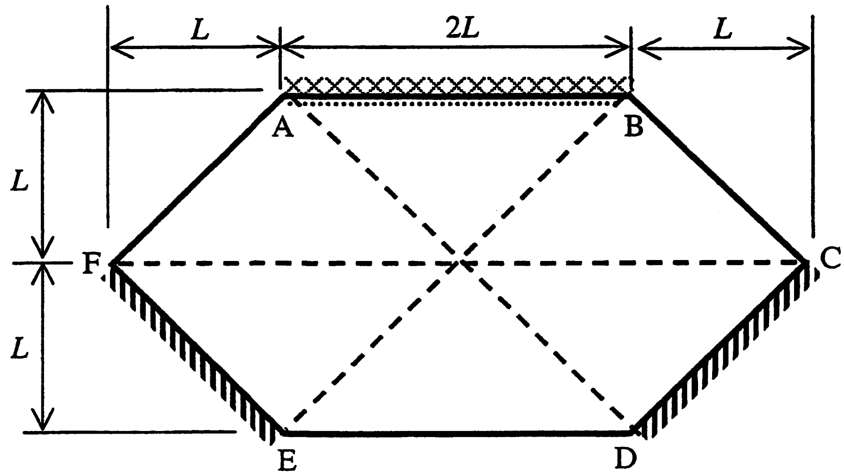

Fig. 6

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6 A long rigid footing P of width b is supported by an ideal rigid-plastic continuum. Uniform horizontal and vertical line loads H and V per unit length are applied to the footing, as shown in Fig. 7. The continuum material is uniform and isotropic, and has a shear strength k. Two rigid-block collapse mechanisms are proposed, as shown in Fig. 7. For the first mechanism, there is a circle of slip failure of radius b. In the second mechanism, two rigid blocks are assumed to move. In both mechanisms there is no slip between the footing and the continuum.

(a) By considering the slip circle mechanism, show that when $H = 0$ the foundation will collapse when $V > 6.3\ bk$. [4]

(b) On an interaction diagram of H/bk against V/bk, construct for each mechanism a line beyond which collapse must have occurred. [12]

(c) State briefly how your calculations would be affected if the self-weight of the continuum were included, and how the interaction diagram in part (b) would change. [4]

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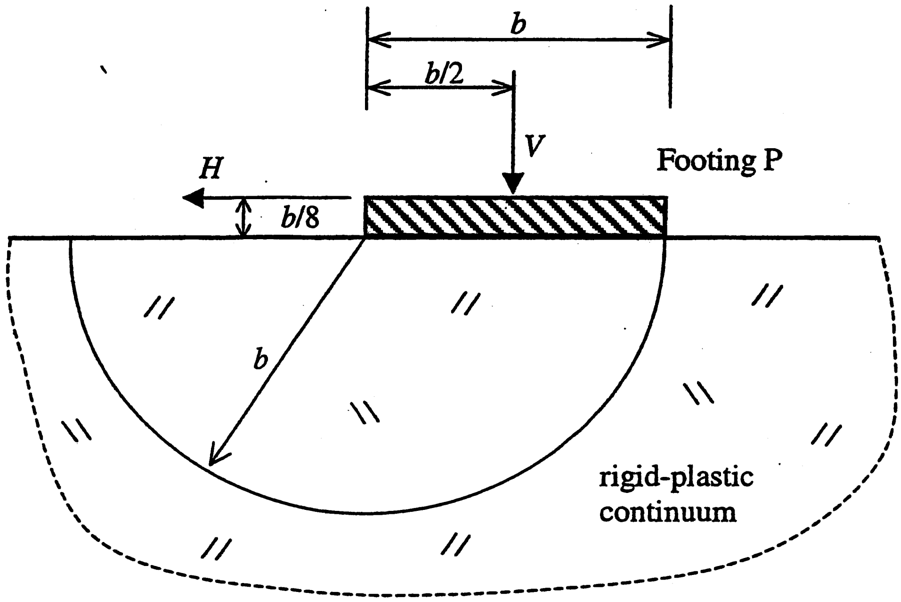
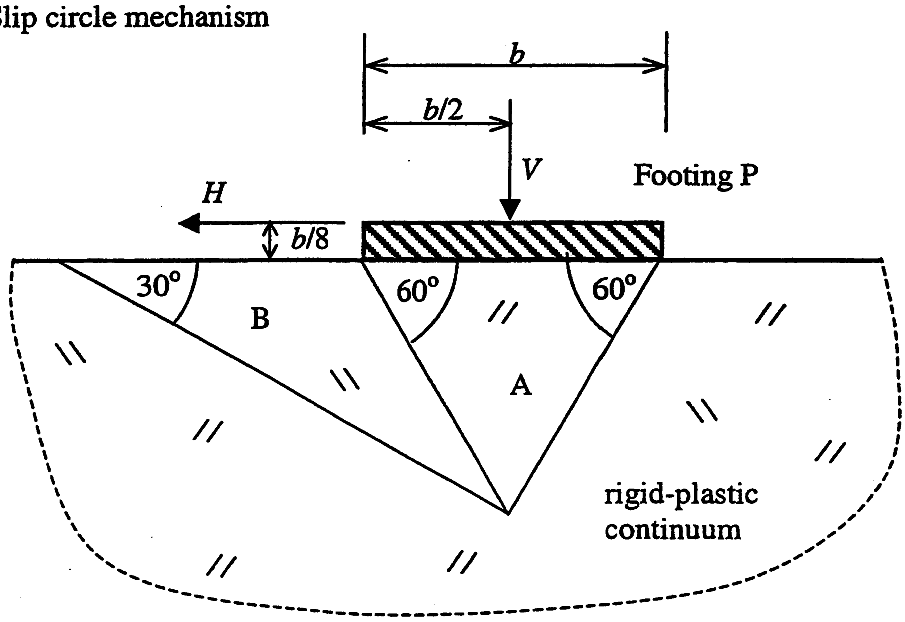


a) Slip circle mechanism



b) Two rigid triangular blocks

Fig. 7

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