UNIVERSITY OF
CAMBRIDGE

ENGINEERING TRIPOS PART IB

Friday 4 June 2004 2 to 4.30
2 to 3.30 Foreign Language Option

Paper 8

SELECTED TOPICS

*Answer **one** question from Section A. In addition:*

> *If you are not taking the Foreign Language option, answer **four** questions, taken from only **two** of Sections B–H. Not more than two questions from each section may be answered.*
> *If you are taking the Foreign Language option, answer **two** questions from **one** of Sections B–H.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Data Sheet for Section B (6 pages)
Data Sheet for Section E (2 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A *Business Economics*

*Answer not more than **one** question from this section*

1 (a) Compare and contrast the assumptions of, and outcomes associated with, the models of perfect competition and monopoly. [10]

(b) In what sense is perfect competition normally deemed to be more desirable than monopoly? [5]

(c) Under what circumstances would monopoly be more desirable than perfect competition? [5]

2 (a) What impact would the following have on the levels of consumption and savings in the macroeconomy:

(i) an increase in the rate of interest; [4]
(ii) a decrease in the rate of income tax; [3]
(iii) an increase in house prices? [3]

(b) What impact would the following have on the level of investment in the macroeconomy:

(i) an increase in the rate of interest; [4]
(ii) an expected increase in national income; [4]
(iii) a rapid technological change? [2]

SECTION B *Civil and Structural Engineering*

Answer not more than **two** *questions from this section.*

You may refer to the data sheet as necessary at the end of the examination paper.

3 (a) Explain why a tunnel constructed directly beneath a masonry building may be less damaging than when it is below and to one side of the building. [4]

(b) Explain how compensation grouting can be used to control damage to buildings in an underground construction project and the role of instrumentation in the process. What should be the principal aim of the process when it is used to protect a masonry building? [4]

(c) Outline two different methods of lining a tunnel, giving advantages and disadvantages of each. [4]

(d) Explain why it is usually safe to construct an open face tunnel in London Clay but not in the marine clays of Singapore, using the concept of stability ratio to illustrate your answer. [4]

(e) Describe the significance of soil permeability in the context of tunnelling below the water table, and give two examples of techniques that may be used to overcome potential problems. [4]

4 A cut-and-cover tunnel is to have a wall with an overall height of 14 m forming one side, as shown in Fig. 1. During construction, excavation in front of the main wall occurs rapidly, with soil and water removed to a depth of 10 m. The wall is temporarily supported by an anchor attached to a 4 m deep wall; this provides support from the passive resistance it can generate. Construction plant, imposing a uniform pressure of 40 kNm^{-2}, is located immediately behind the main wall but does not extend far enough back to influence the passive resistance of the anchor wall. The project is in a land reclamation, in which the ground profile is 4 m of granular fill underlain by 6 m of sand overlying stiff clay containing sand lenses. The water level is at a depth of 4 m.

The properties of the dry granular fill and the saturated sand are the same, with a unit weight of 18 kNm^{-3} and a critical state angle of friction of 35°. The stiff clay has a unit weight of 20 kNm^{-3}, a critical state angle of friction of 25° and an undrained shear strength of 100 kNm^{-2}.

(a) Assuming a translational mode in which both walls move together towards the excavation, derive the values for the limiting active and passive soil pressures acting on *each* wall for the temporary case as soon as the excavation is completed. Sketch their distribution. Assume undrained conditions apply in the clay on both sides of the main wall. [8]

(b) For the temporary case, calculate the factor of safety against failure of the main wall due to translational failure of the anchor wall. [4]

(c) It turns out that the clay just in front of the main wall contains more sand lenses than expected, and these cause drainage to occur very rapidly. Assuming long term conditions are rapidly reached in front of the main wall, with hydrostatic water pressure acting from the excavation level, but that the clay behind the main wall contains only a few sand lenses and hence remains undrained, calculate the following:

(i) the revised factor of safety against failure of the main wall due to translational failure of the anchor wall;

(ii) the factor of safety against rotational failure of the main wall about the anchor (for this calculation ignore the effect of any pressures acting on the main wall in the fill). [8]

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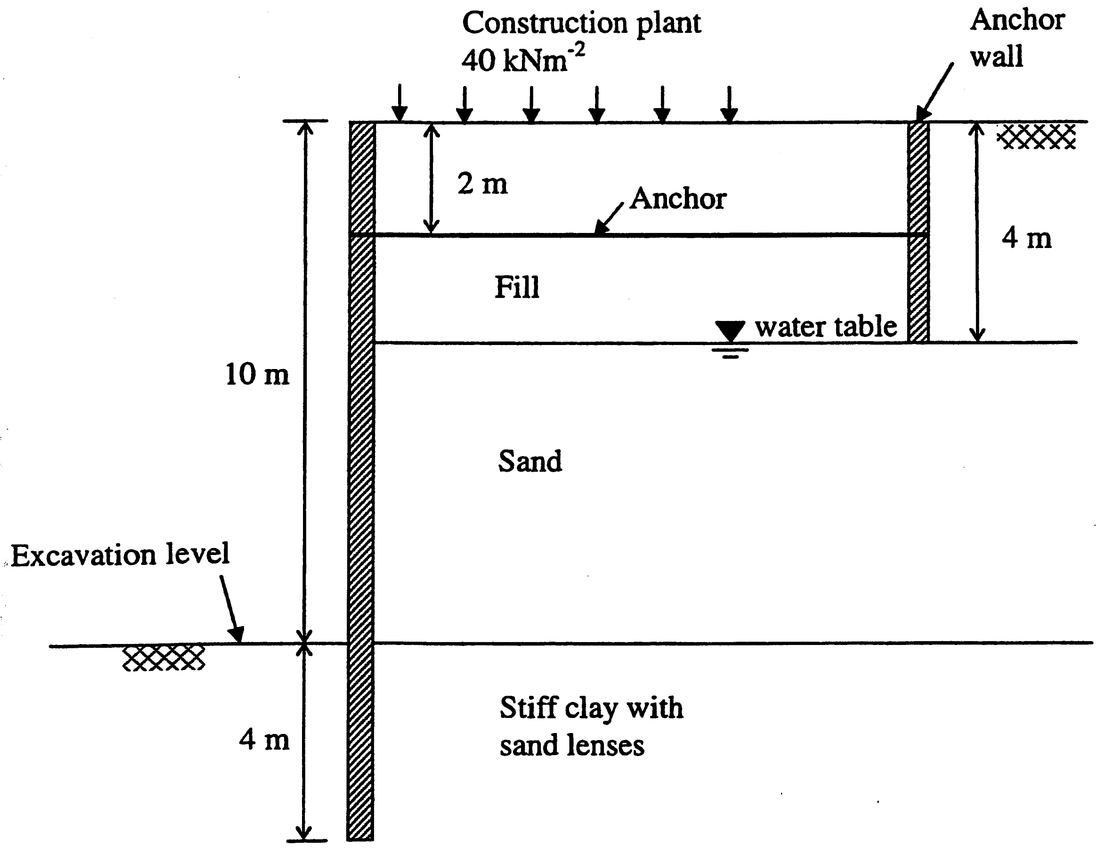
Construction plant
40 kNm^{-2}
Anchor wall
2 m
Anchor
4 m
Fill
water table
10 m
Sand
Excavation level
4 m
Stiff clay with sand lenses

Fig. 1

5 A simply supported reinforced concrete roof beam of an underground facility is to span 7 m between supports, and has 2 m overhangs at each end as shown in Fig. 2. The design live loading consists of six point loads, four of magnitude $P = 500$ kN and two of magnitude $Q = 250$ kN. The loads are positioned as shown in Fig. 2. Partial factors on live loading have already been included in the derivation of P and Q. The reinforcing steel has a yield strength $f_y = 460\ \text{N mm}^{-2}$ and concrete strength $f_{cu} = 30\ \text{N mm}^{-2}$. Consider a uniform beam of rectangular cross-section, with an effective depth to breadth ratio of $d/b = 3/2$.

(a) Draw the live load bending moment and shear force diagrams for the beam, marking on the magnitudes at the position of each point load. [2]

(b) Determine an appropriate value for the effective depth of the beam, d, and hence the overall beam depth, D. Neglect self-weight. [4]

(c) Draw the dead load bending moment and shear force diagrams for the beam chosen in (b) above. Hence include provision for the self-weight of the beam and make any adjustments to your design you deem necessary. [4]

(d) Design the flexural reinforcement. [6]

(e) Check the shear capacity at the critical section and, if required, design the shear steel. [4]

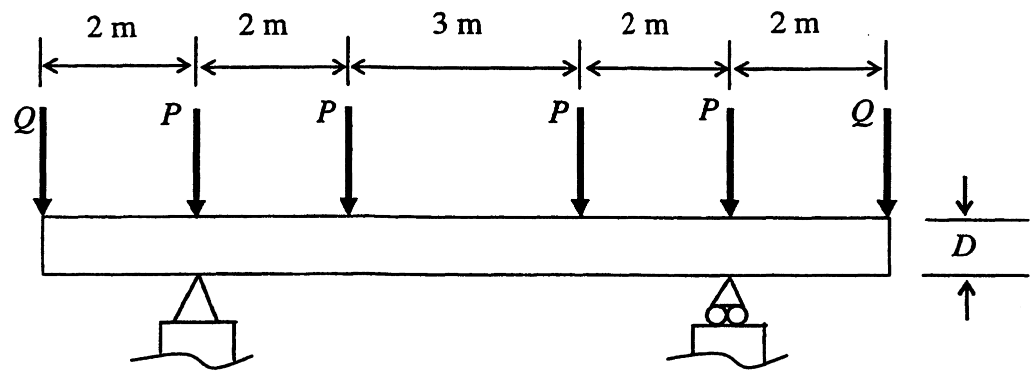

Fig. 2

SECTION C *Mechanical Engineering*

Answer not more than two questions from this section

6 A two dimensional metallic lattice has a periodic microstructure comprising equilateral triangles of side length ℓ and thickness t, as shown in Fig. 3. The walls are made from a rigid, ideally plastic solid of yield strength σ_Y.

(a) Calculate the relative density $\bar{\rho}$ in terms of ℓ and t. [4]

(b) Obtain an expression for the uniaxial tensile strength σ_1 in the x_1 direction, in terms of $\bar{\rho}$. You may assume that the cell walls can be modelled as pin-jointed at the nodes. [4]

(c) Use an upper bound approach to obtain an expression for the uniaxial tensile strength σ_2 in the x_2 direction, in terms of $\bar{\rho}$, by suitable placement of plastic hinges. [8]

(d) Sketch and explain the expected post-yield tensile stress versus strain curve for uniaxial loading in the x_1 direction, and for uniaxial loading in the x_2 direction. [4]

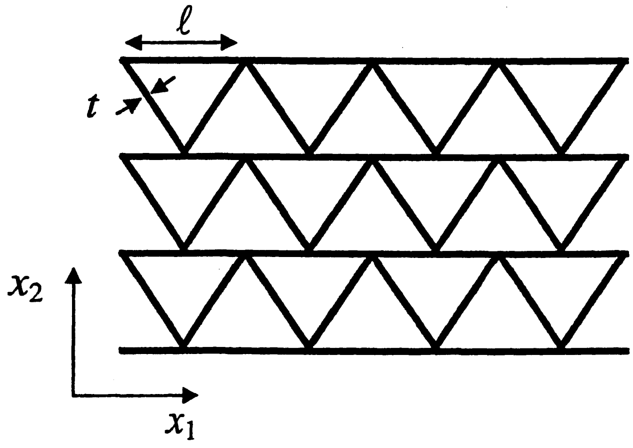

Fig. 3

7 (a) Outline the contributions to the unit cost of a component, and explain the sensitivity of unit cost to batch size. Distinguish between cost, price and value of a product, and explain their relationship for a successful product. [6]

(b) Why are lattice materials superior in mechanical performance to the current generation of metallic foams? [4]

(c) Explain why metallic foams are potentially useful in energy absorbing applications. [4]

(d) Explain how an aluminium alloy foam can be manufactured by a melt route to produce an open cell foam. [6]

8 (a) Explain the advantages of sandwich beam construction over monolithic construction for lightweight structures, and demonstrate this in terms of simple beam theory. [5]

(b) A sandwich beam of length 2ℓ and width b comprises a steel foam core of thickness c and relative density $\bar{\rho}$, with steel faces each of thickness t. The beam is clamped at both ends and loaded by a transverse force P at mid-span.

(i) Obtain an expression for the collapse load assuming the core shears plastically with a shear strength τ_C. Treat the faces as rigid, ideally plastic solids of yield strength σ_Y. [5]

(ii) Obtain an expression for the collapse load for face yield. You may neglect the contribution to bending strength from the core. [5]

(iii) What alternative failure modes might be active? [5]

SECTION D *Aerothermal Engineering*

Answer not more than two questions from this section

9 A simple turbojet operates with an inlet stagnation temperature of 300 K and an inlet stagnation pressure of 1 bar. The design stagnation pressure ratio is 27, and both the compressor and turbine have isentropic efficiencies of 90%. Assume that the working fluid has the properties of air throughout.

(a) At design, the shaft speed is 6500 rpm and the compressor mass flow rate is 50 kg/s. Determine the torque required to drive the compressor. [5]

(b) The turbine has a constant mean radius of 0.5 m, and the stage loading coefficient, $\Delta h_0 / U^2$, is not to exceed 2. Determine the minimum number of turbine stages and calculate the actual stage loading. Given that the design flow coefficient is $V_x / U = 0.55$, calculate also the axial velocity in the turbine, assuming this to remain constant. [3]

(c) The turbine is designed with repeating stages, each having axial flow at inlet. Calculate the absolute flow angle at exit from the stators and the relative flow angles at inlet to and exit from the rotors. Sketch the velocity triangles and blades for a typical stage.

You may use without proof Euler's work equation, $h_o - UV_\theta = \text{constant}$, where the symbols have their usual meanings. [10]

(d) For a new design of this engine it is proposed to reduce the number of turbine stages by one. Discuss briefly the implications of this change. [2]

10 (a) Figure 4 shows a schematic diagram of a twin-spool bypass engine. Sketch separate *T-s* diagrams for the core and bypass flows of such an engine. Annotate your diagrams as fully as possible, labelling the locations in accordance with the figure. [4]

(b) An aircraft cruises at a velocity, *V*, of 250 ms^{-1} at an altitude where the ambient pressure is 0.287 bar. The engines are of the type shown in Fig. 4. Both turbines have an isentropic efficiency of 90%, and both nozzles may be assumed to be loss-free. At cruise, the core flow conditions are as shown in Table 1. Assuming perfect gas relations with the properties of air, determine the stagnation pressure and stagnation temperature downstream of the HP turbine (HPT). [4]

(c) Calculate the core jet velocity, V_{jc}, given that the work extracted from the LP turbine (LPT) is 357 kJ per kg of core flow. [5]

(d) Assuming the bypass and core jet velocities are equal, determine the bypass ratio. Calculate also the net thrust for unit mass flow rate in the core, and the propulsive efficiency. You may assume that the fan efficiency η_{FAN} is given by

$$\eta_{\mathrm{FAN}} = \frac{V_{jb}^{2} - V^{2}}{2c_p \Delta T_{ob}^{\mathrm{FAN}}} = 0.9$$

where c_p is the specific heat capacity of air at constant pressure, and $\Delta T_{ob}^{\mathrm{FAN}}$ is the stagnation temperature rise across the bypass section of the fan. [7]

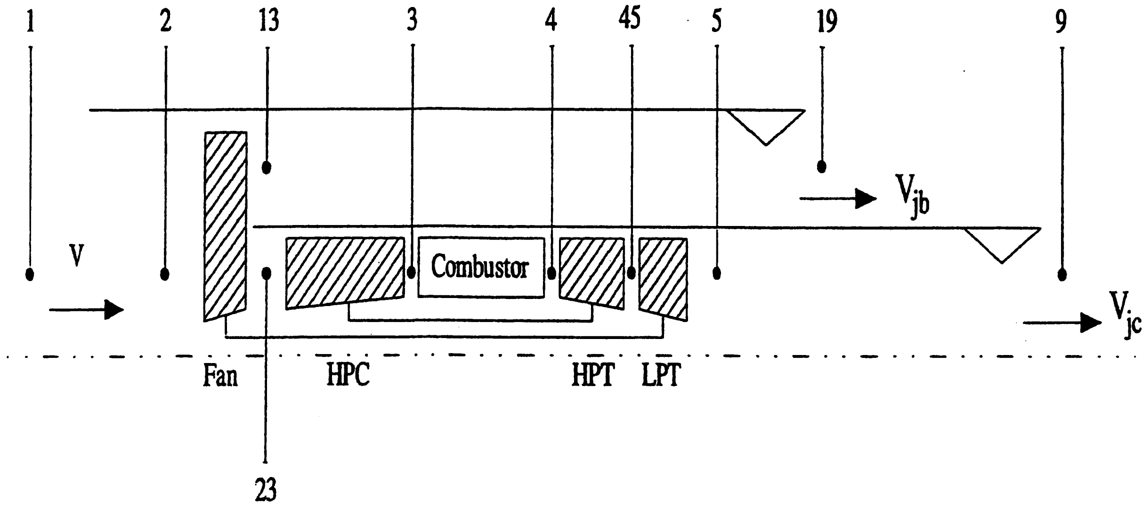

Fig. 4

(cont.

Location	**2**	**23**	**3**	**4**
Stagnation pressure (bar)	0.450	0.720	14.40	14.40
Stagnation temperature (K)	257.8	299.0	748.7	1400

Table 1: Flow conditions for core engine

11 (a) Making appropriate estimates for the relevant quantities, determine the approximate wing area required for an aircraft whose take-off mass is 500 tonnes. Assume all lift comes from the wings. Justify your values for the estimated quantities. [4]

(b) Draw a diagram indicating the principal forces acting on an aircraft climbing steadily at an angle θ to the horizontal. Write down two expressions for the relationships between these forces. [4]

(c) The aircraft of part (a) climbs at a steady rate of 1.5 $\mathrm{m\,s^{-1}}$ to its cruising altitude where the ambient conditions are given in Table 2. During take-off and climb, 20 tonnes of fuel are burnt. The Mach number at the top of climb is 0.85 and the lift to drag ratio is then 20. Calculate the thrust required by the engines at the top of climb. [4]

(d) The thrust is provided by four high bypass engines, each having a nozzle exit area of 2.5 m^2. At the top of climb condition, the total air mass flow rate passed by each engine is 450 kg/s. Estimate the net thrust at take-off, assuming the engines operate at the same non-dimensional condition as for top of climb. Assume the exit nozzles are choked in both cases, and use data from Table 2 where appropriate. [8]

Note that the mass flow rate, $\dot{m}$, of air through a choked nozzle of area A is given in non-dimensional form by

$$\frac{\dot{m}\sqrt{c_p T_o}}{A P_o} = 1.281$$

where T_o is the stagnation temperature, P_o is the stagnation pressure at entry to the nozzle, and c_p is the specific heat capacity at constant pressure.

(cont.

	Top of Climb	Take-Off
Static pressure (bar)	0.287	1.013
Static temperature (K)	226.7	288.2
Stagnation pressure (bar) relative to aircraft	0.460	1.063
Stagnation temperature (K) relative to aircraft	259.5	292.2

Table 2: Conditions at top of climb and take-off

SECTION E *Electrical Engineering*

Answer not more than two questions from this section.
You may refer to the data sheet as necessary at the end of the examination paper.

12 (a) Pattern transfer from a mask to a wafer can be accomplished using various lithographic methods.

(i) Compare and contrast the performance of the proximity and projection optical exposure methods currently utilised by industry. [4]

(ii) Briefly describe two next-generation lithographic methods which may enable the fabrication of nanometre-scale integrated circuits. Specifically mention resolution, registration and throughput in your answer. [4]

(b) Processing at the nanometre scale requires laminar-flow clean rooms. Figure 5 shows the number of particles per cubic foot for various classes of clean room. Using this graph (and assuming that the sticking coefficient of the particles is 5%), estimate how many dust particles of size $\geq 0.5\ \mu\text{m}$ will land on a 300 mm diameter wafer, when the wafer is exposed for two minutes to an airflow of 30 m per minute in a class 10 clean room. State all assumptions made. [5]

(c) As device dimensions continue to shrink, multi-level interconnections are required in order to reduce time delay. The speed gained by shrinking the device dimensions is offset by the propagation delay associated with metal interconnects.

(i) Estimate the time delay associated with two parallel aluminium wires of length 2 mm with square cross-section of side $0.4\ \mu\text{m}$, separated by a $0.5\ \mu\text{m}$ thick silicon dioxide layer. State any assumptions made. [5]

(ii) Apart from modifying the dimensions of the structure, suggest two possible methods of reducing this delay. [2]

Assume the resistivity of aluminium is $2.7\ \mu\Omega\ \text{cm}$ and the dielectric constant for silicon dioxide is 3.9.

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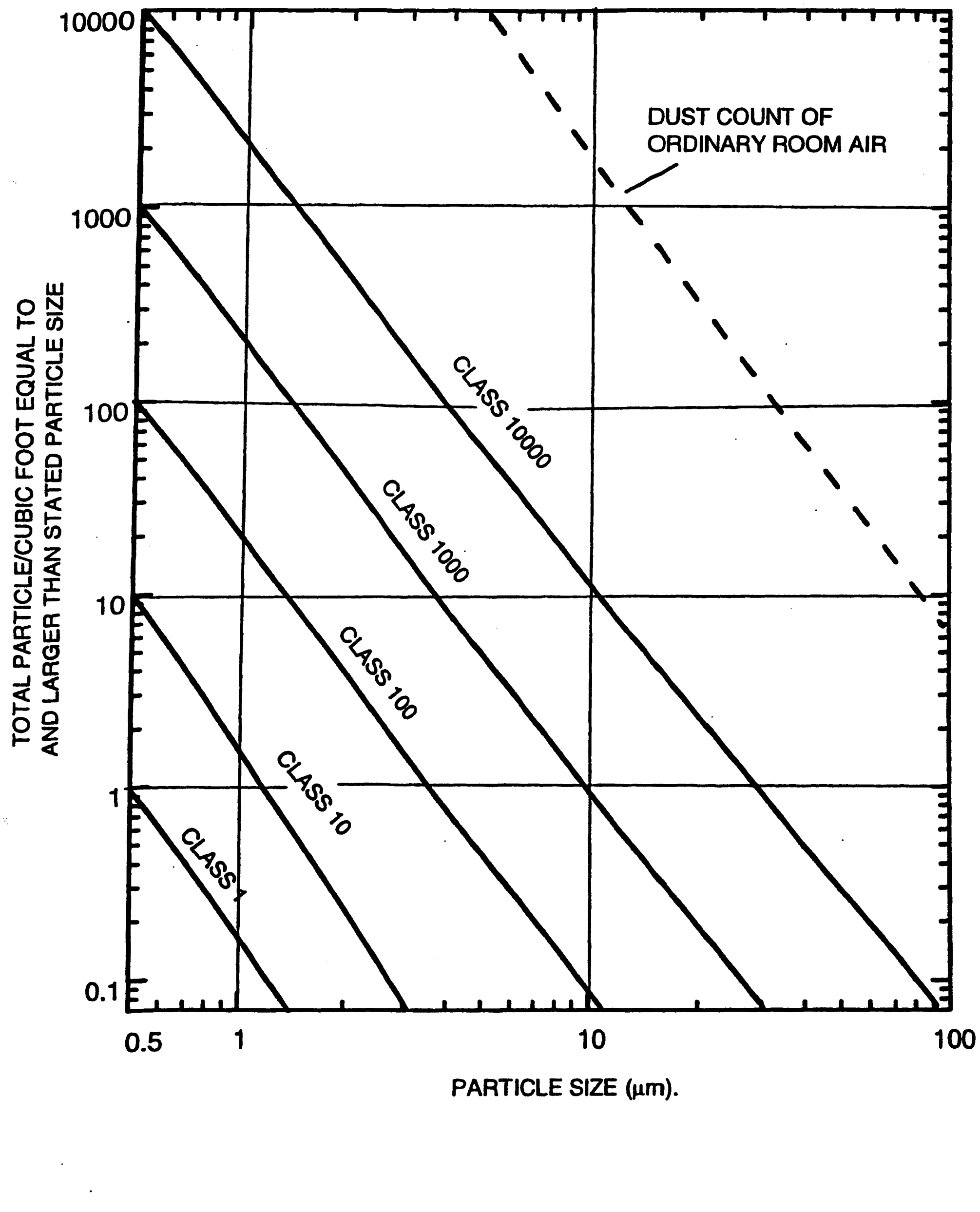

Fig. 5

(TURN OVER

13 (a) Derive the relationship between the conductivity and the mobility of a conductor. [4]

(b) Copper has a face centred cubic lattice with 4 atoms per unit cell, and a lattice constant of 3.61 Angstrom. It has one valence electron per atom, and the electronic charge is 1.6×10^{-19}C. What is the density of valence electrons? Copper has a conductivity of $6 \times 10^{7}\ \Omega^{-1}\mathrm{m}^{-1}$. What is the mobility of its electrons? [3]

(c) What would the conductivity of silicon be if it had the same mobility as copper, and was n-doped to a concentration of $1.2 \times 10^{20}\mathrm{m}^{-3}$, with all the impurities ionised? [2]

(d) Sketch the velocity-field diagram for silicon. Explain the meaning of scattering-limited velocity. [2]

(e) The electron mobility of a semiconductor such as silicon is $0.25\mathrm{m}^{2}\mathrm{V}^{-1}\mathrm{s}^{-1}$ and its scattering limited velocity is $1.25 \times 10^{5}\mathrm{ms}^{-1}$. This material is doped at a donor density of $4 \times 10^{21}\mathrm{m}^{-3}$ and is used to make a depletion mode FET, operating at a supply voltage of 2 V, with a channel of width/length ratio of 50. At what source-drain length does conduction become scattering limited? What is the corresponding transit time? [7]

(f) Can an FET operate at an even higher frequency? [2]

14 (a) Explain how conduction occurs in an n-type depletion mode MOSFET, and how the gate voltage turns off the source-drain current. [3]

(b) Derive the relationship for the gate turn-off voltage in terms of the dopant concentration, stating the assumptions used. [6]

(c) You wish to design a depletion mode MOSFET with a transit time of 2 ps using a $0.2\,\mu\text{m}$ thick layer of n-type semiconductor of conductivity $4\,\Omega^{-1}\text{m}^{-1}$ on an insulating substrate. The supply voltage is 2 V. The semiconductor mobility is $1\,\text{m}^2\text{V}^{-1}\text{s}^{-1}$, its dielectric constant is $1\times10^{-10}\,\text{Fm}^{-1}$, and the electronic charge is $1.6\times10^{-19}\,\text{C}$.

(i) What is the donor density? [3]

(ii) What is the maximum source-drain distance to satisfy the required transit time? [3]

(iii) What gate voltage is required to turn off the MOSFET? [3]

(iv) What is the maximum gate field and how does it compare to the breakdown field of $2\times10^{7}\,\text{Vm}^{-1}$? [2]

SECTION F *Information Engineering*

Answer not more than two questions from this section

15 A standard system requirement is to interface computers to peripheral I/O devices.

(a) Consider the techniques of buffering, spooling, caching and DMA.

(i) Define and explain each of these techniques. For each, point out the problem it solves and provide an example of an I/O scenario in which it should be used.

(ii) Is there any practical difference between buffering and spooling? [8]

(b) Consider blocking and non-blocking I/O.

(i) Define and explain the operation of each of these techniques and describe situations where one of the techniques should be used in preference to the other.

(ii) Distinguish between a "non-blocking read" (from a network, for example) and an "asynchronous read". [8]

(c) Briefly explain the use of polling and interrupts. When is polling more advantageous than using interrupts? [4]

16 Your start-up company has developed a new fingerprint recognition chip and wants to market it as a convenient authentication device (pin/password replacement) for a Personal Digital Assistant (PDA). The PDA add-on product will be a miniature fingerprint reader integrated into a Compact Flash card and supplied with software for all common PDA operating systems. You are in charge of the security design of this product.

(a) Consider the architecture of a typical PDA with respect to PIN/password protection.

(i) What is the threat model?

(ii) When will the PDA ask for a password? What will it do if you type the wrong one?

(iii) How does the PDA check the validity of the password? Why is it done that way?

(iv) How can one define a new password? [8]

(b) Consider the architecture of the system comprising the PDA plus fingerprint reader and any supplied software.

(i) When will the PDA ask for a fingerprint? What components of the system will be involved? Describe the procedure and compare it with the conventional one you discussed in (a).

(ii) Does the fingerprint reader card store any secrets? If so, how are they entered? If not, where else in the system are you storing any secrets?

(iii) Explain how the system is taught the fingerprint of the legitimate owner and how this can be changed later if required.

(iv) Explain what makes your system resistant to substitution of the fingerprint-reading card with a fake one that always says "yes". [12]

17 You are required to design a system for locating personnel and equipment within buildings.

(a) Describe the technology and architecture of a system based on the use of active tags that provides room-scale location information and services in a way that can scale to the largest buildings. [10]

(b) Outline the methods used to maximise battery life of active tags. [6]

(c) List four applications which use in-building location information. [4]

SECTION G *Biological and Medical Engineering*

Answer not more than two questions from this section

18 (a) Sketch the time-course of saccades made to a target suddenly moving to the right through an amplitude of 5°, 10° and 20°. [3]

(b) If a number of such records were made under identical conditions, what aspects of the time-course would be likely to vary? [3]

(c) Sketch the normal relationship between (i) peak saccadic velocity and amplitude; (ii) saccadic duration and amplitude. [4]

(d) How can these relationships be explained in terms of the mechanical properties of the eye, and the commands sent to it by oculomotor nerve fibres to execute a saccade? [4]

(e) What aspects of the oculomotor commands must be regulated to ensure that the eye settles as quickly as possible to its new position after a saccade? Sketch examples of abnormal saccades for which this matching of the command pattern to the plant has not been achieved. [3]

(f) What clinical observations suggest that this regulation is mediated by visual feedback? [3]

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19 (a) The weight mg of a hovering animal is supported by a pair of approximately rectangular wings, each of length R with a constant chord c equal to $0.3R$. The wings flap through an angle Φ in the horizontal plane, and the flapping angle ϕ varies with time t as

$$\phi = \bar{\phi} + \frac{1}{2}\Phi \cos(2\pi f t),$$

where $\bar{\phi}$ is the mean flapping angle and f is the wing beat frequency. The downwash, or induced velocity, is negligible compared with the flapping velocity.

(i) Assume that the lift per unit span, L', is given by the usual blade-element relation

$$L' = \frac{1}{2}\rho c U_r^2 C_L,$$

where ρ is the density of air (1.2 kg m^{-3}), U_r is the relative velocity and C_L is the lift coefficient. Taking C_L to be constant, and the mean of $\sin^2(2\pi f t)$ to be 0.5, show that the vertical force balance requires

$$mg \approx 0.05 \rho \pi^2 \Phi^2 f^2 C_L R^4 .$$ [7]

(ii) A 75 kg student has constructed wings with $R = 2.0$ m that are flapped through an angle of 2 rad. If the maximum lift coefficient is 1.8, what frequency is needed for weight support? [2]

(iii) The wings of flying animals tend to scale isometrically, $R \propto m^{1/3}$. Given that the masses of hovering animals range over six orders of magnitude, how would you expect frequency to vary with body mass? [3]

(b) A 6 cm long *Giant Danio* is swimming along and suddenly makes a 90° c-turn manoeuvre leaving a single vortex ring in its wake. A two-dimensional horizontal cut of the vortex ring reveals two counter rotating vortices forming a jet behind the fish, see Fig. 6. The two vortices have equal magnitude but opposite sign, and each has a circulation of strength $\Gamma = 19.5\,\text{cm}^2\text{s}^{-1}$. Their centre-centre separation is $D_0 = 1.5$ cm. Determine the maximum fluid velocity $V_{\max}$ at the centre of the jet. [8]

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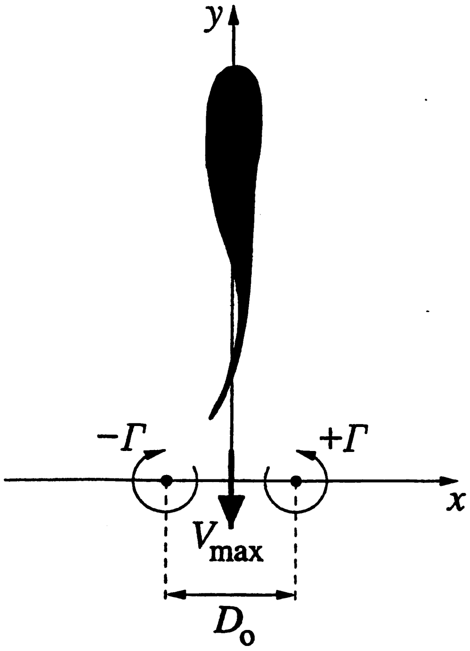

Fig. 6

20 Write brief notes on each of the following. Note that you have a choice of topics in part (e).

(a) The significance of the ‘smooth-swimming’ and ‘tumbling’ modes for the navigation strategy of bacteria such as *E. coli.* [4]

(b) The process of assembly of a tubular structure from identical building blocks, each in an identical environment with respect to its neighbours, inevitably produces a *straight* rod. The bacterial flagellar protein manages to circumvent this constraint, and instead builds *helical* flagellar filaments. [4]

(c) The mode of operation of a virus when it attacks a cell such as a bacterium. [4]

(d) The use of DNA genomic sequences in mapping the evolutionary ‘philogenetic tree’ for different species. [4]

(e) **Either**

(i) The role of ‘receptors’ for specific chemical species, embedded in the cell wall of bacteria, in the phenomenon of chemotaxis. [4]

or

(ii) Problems in the ‘re-engineering’ of bacterial receptors, so that they will respond to molecules such as TNT. [4]

SECTION H *Manufacturing, Management and Design*

Answer not more than two questions from this section

21 Company A has been established for less than two years. It has developed and is now commercialising its first product, based on innovative technology. The sales are building very rapidly.

Company B also develops and sells a wide range of similar products. It has been established for more than ten years and introduces additional or updated products at the rate of one or more per year.

(a) Give an example of an established and well-known technology-based company, and briefly describe how it matches the above description of Company B. [4]

(b) At an earlier stage of its life, Company B had been like Company A. Compare and contrast the management processes, information systems, staff job specifications and range of personalities you would expect to find involved in product development and marketing at these two stages (A and B) in its life. [8]

(c) Discuss the difficulties the younger company may face in developing a sustainable flow of new products similar to that of the older company. [8]

22 (a) Explain what is meant by 'intellectual property' (IP), and list five different forms of IP which are recognised in the UK. [6]

(b) A company is developing a new design of child-proof electrical connector which it proposes to market under the name 'Safetiplug'. The connector is produced by conventional manufacturing processes. Advertisements for the product will include the slogan 'Protect your family with Safetiplug', accompanied by a photograph of the product taken by a professional photographer. Write a memorandum for the Managing Director, discussing the types of IP which the company should consider in planning the development, production and marketing of this product. For each type of IP, outline the steps needed to protect the IP under UK law. Comment on any specific points which the company should consider before the market launch at a trade exhibition. [14]

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23 A start-up company has developed a new range of heaters for the biotechnology market. The range comprises 5 expensive versions and 5 cheap versions. The company is considering two manufacturing options:

Option A is largely manual and can produce all ten versions of the heater. It requires six operators using tools of negligible cost.

Option B is semi-automated and can produce only the five expensive versions. It uses a robotic CNC machine with a capital cost of £100,000, and requires only two operators.

(a) Making use of the data provided below, perform an economic value analysis (EVA) for the two options for the first year of operation. Ignore depreciation. [8]

(b) Discuss any factors not included in the EVA which should be considered in choosing between the two manufacturing methods. [7]

(c) Make a recommendation on which option to adopt, and explain your reasons for choosing it. [5]

REQUIRED DATA
The company is exempt from taxation for the first three years.

Forecast costs per year: sales and marketing costs £15,000; rental of premises £25,000; general administration £25,000; cost of capital 10%; labour £25,000 per operator.

Working capital required for inventory: £50,000. Materials cost per heater: £1,000.

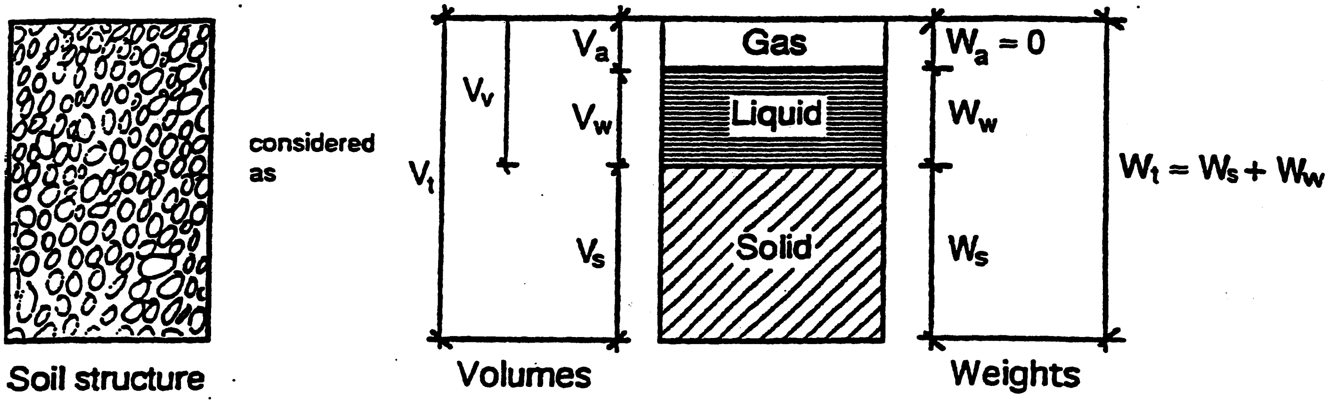
Forecast sales information: the selling price for the expensive version is £6,000 and for the cheaper version is £4,000.

Sales volume per year: ten units of each version, giving a total of 100 heaters for option A or 50 heaters for option B.

END OF PAPER

Data sheet – Soil Mechanics

General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w G_s / e)$
Unit weight of water	$\gamma_w = 9.81$ kN/m^3 (although we assume 10 kN/m^3)
Unit weight of soil	$\gamma = W_t / V_t = \left(\dfrac{G_s + S_r e}{1 + e} \right) \gamma_w$
Buoyant (effective or submerged) unit weight	$\gamma' = \gamma - \gamma_w = \left(\dfrac{G_s - 1}{1 + e} \right) \gamma_w$ (soil saturated)
Unit weight of dry soil	$\gamma_d = W_s / V_t = \left(\dfrac{G_s}{1 + e} \right) \gamma_w$
Relative density	$I_d = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})}$

where e_{max} is the maximum voids ratio achievable in the quick tilt test (for sands), and
e_{min} is the minimum voids ratio achievable by vibratory compaction (for sands).

Classification of particle sizes

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D	equivalent diameter of soil particle
D_{10}, D_{60} etc	particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

Stress components

Principle of effective stress (saturated soil):

$$\text{total stress} \quad \sigma = \text{effective stress } \sigma' + \text{pore water pressure } u$$

$$\tau = \tau' + 0$$

and

σ_v = vertical stress

σ_h = horizontal stresss

τ = shear stress

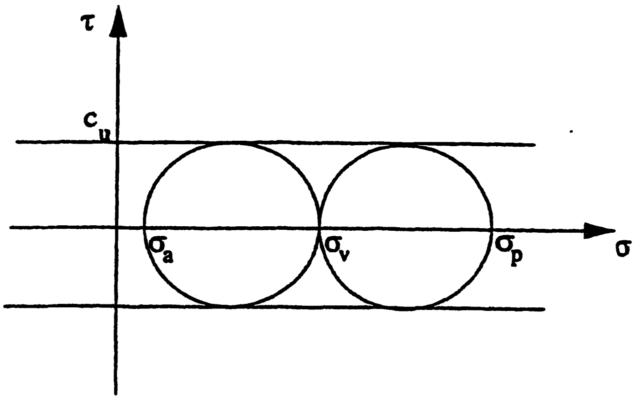
Strength of clays (undrained behaviour only)

Under constant volume (undrained) conditions only, the strength of clays can be characterised by the ***undrained shear strength*** c_u which is mobilized when the shear stress $\tau = c_u$. This conforms to Tresca's criterion, and the active and passive total horizontal stresses, σ_a and σ_p respectively, are given by

$$\sigma_a = \sigma_v - 2c_u$$

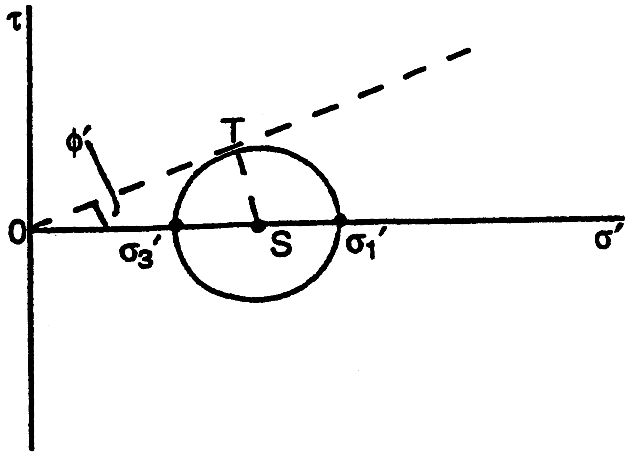
$$\sigma_p = \sigma_v + 2c_u$$

where σ_v is the total vertical stress.



Strength of sands

Mobilised angle of shearing ϕ'
where $\tau = \sigma' \tan \phi'$



$$\sin \phi' = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

$$\therefore \phi' = \sin^{-1}\left[\frac{\left(\sigma_1'/\sigma_3'\right)-1}{\left(\sigma_1'/\sigma_3'\right)+1}\right]$$

Earth pressure coefficient K: $\sigma_h' = K\sigma_v'$

Active pressure: $\sigma_v' > \sigma_h'$ $\therefore$ $\sigma_1' = \sigma_v'$
$\sigma_3' = \sigma_h'$
$K_a = (1 - \sin\phi'/1 + \sin\phi')$

Passive pressure: $\sigma_h' > \sigma_v'$ $\therefore$ $\sigma_1' = \sigma_h'$

[We assume principal stresses $\sigma_3' = \sigma_v'$
are horizontal and vertical] $K_p = (1 + \sin\phi')/(1 - \sin\phi') = \frac{1}{K_a}$

Angle of sharing resistance:
at peak strength ϕ'_{max} at $\left(\sigma_1'/\sigma_3'\right)_{max}$
at critical state ϕ'_{crit} after large strains.

Sand strength data: friction hypothesis

In any shear test on soil, failure occurs when $\phi' = \phi'_{max}$ and

$$\phi'_{max} = \phi'_{crit} + \phi'_{dilatancy}$$

where ϕ'_{crit} is the ultimate angle of shearing resistance of a random aggregate which deforms at constant volume, so the dilatancy, which indicates an increase in volume during shearing, approaches zero $(\phi'_{dilatancy} -> 0)$ as $\phi'_{max} -> \phi'_{crit}$.

ϕ'_{crit} is a function of the mineralogy, size, shape and distribution of particles. For a particular soil it is almost independent of initial conditions. Typical values (±2°):

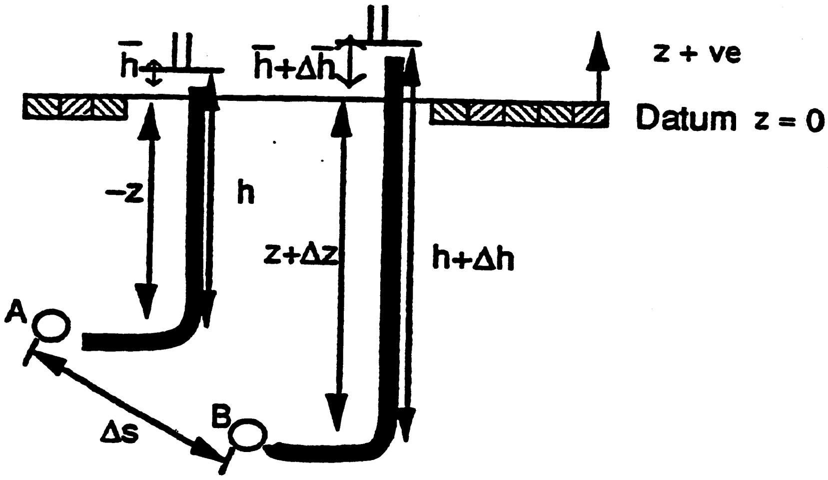
	ϕ'_{crit}	ϕ'_{max}	
feldspar	40°		
quartz	33°	53°	($I_d = 1$, and mean effective stress OS < 150 kPa)
mica	25°		

Seepage

Excess pore water pressure

Head $h = u/\gamma_w$

Potential $\bar{h} = h + z$



Total pore water pressure head at A: $u = \gamma_w h = \gamma_w(\bar{h} - (-z))$

B: $u + \Delta u = \gamma_w(h + \Delta h) = \gamma_w(\bar{h} + z + \Delta\bar{h} + \Delta z)$

[Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w(\bar{h} + \Delta\bar{h})$]

Hydraulic gradient A–B $$i = -\frac{\Delta\bar{h}}{\Delta s} = -\frac{\Delta\bar{u}}{\gamma_w \Delta s}$$

Darcy's law $v = ki$

v = average or superficial seepage velocity

k = coefficient of permeability

Typical permeabilities

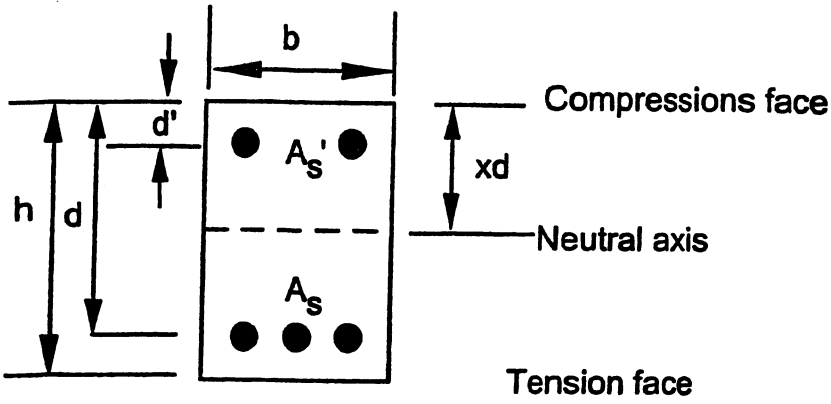
$D_{10} > 10\text{mm}$: non-laminar flow

$10\text{ mm} > D_{10} > 1\mu\text{m}$: $k \cong 0.01(D_{10} \text{in mm})^2$ m/s

clays : $k \cong 10^{-9}$ to 10^{-11} m/s

Design of reinforced concrete

Data sheet for use in Part IB Civil Engineering Elective Course.



Design Stresses

Cube strength for concrete f_{cu}. At failure in bending, stress in concrete $= 0.4f_{cu}$ over whole area of concrete in compression.

Tensile yield stress of steel f_y. At failure in bending, stress in bars in tension $= 0.87f_y$, stress in bars in compression $= 0.75f_y$.

Design Equations

Moment capacity of singly reinforced beam

$$M \leq 0.15 f_{cu}bd^2$$
$$M = 0.87f_yA_sd(1-x/2)$$
$$x = 2.175(f_y/f_{cu}).(A_s/bd) \quad (\leq 0.5 \text{ to avoid over reinforcement})$$

Moment capacity of doubly reinforced beam

$$M = 0.15f_{cu}bd^2 + 0.75f_yA_{s'}(d-d')$$
$$0.87f_yA_s = 0.75f_yA_{s'} + 0.2f_{cu}bd$$

Shear capacity of all beams

Total shear capacity $V = (v_c + v_s)bd$

Where, $v_c = 0.68(100A_s/bd)^{0.33}.(400/d)^{0.25}$ (N/mm^2)

and $v_s = 0.87f_y A_{sq}/(bs)$

in which s = shear link spacing, A_{sq} is total area of all shear bars in a link and A_s is the total area of effective longitudinal *tension* steel at the section.

Standard bar sizes

Diameter (mm)	6	8	10	12	16	20	25	32	40	50
Area (mm^2)	28	50	78	113	201	314	491	804	1256	1963

Available steel types

Deformed high yield steel $f_y = 460$ N/mm^2

Plain mild steel $f_y = 250$ N/mm^2

Lap and anchorage lengths 40 bar diameters

Density of reinforced concrete: $24\ kN/m^3$

Reinforcement areas per metre width

	Spacing of bars (mm)									
	75	100	125	150	175	200	225	250	275	300
Bar Dia. (mm)										
6	377	283	226	189	162	142	126	113	103	94.3
8	671	503	402	335	287	252	224	201	183	168
10	1050	785	628	523	449	393	349	314	285	262
12	1510	1130	905	754	646	566	503	452	411	377
16	2680	2010	1610	1340	1150	1010	894	804	731	670
20	4190	3140	2510	2090	1800	1570	1400	1260	1140	1050
25	6550	4910	3930	3270	2810	2450	2180	1960	1790	1640
32	10700	8040	6430	5360	4600	4020	3570	3220	2920	2680
40	16800	12600	10100	8380	7180	6280	5580	5030	4570	4190
50	26200	19600	15700	13100	11200	9820	8730	7850	7140	6540
Areas calculated to 3 significant figures according to B.S.I recommendations										

April 2004

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Part IB Data Sheet: Fast Transistor Elective

Transistor Design Summary Sheet

Gauss's Theorem

$\varepsilon_0\varepsilon_r E_1 - \varepsilon_0\varepsilon_r E_2$ = charge per unit area enclosed between upper surface 1 and lower surface 2.

FET Design Summary

- $\tau_t \rightarrow$ switching time as 1st approx. (scattering limited transit time).
- $\tau_{eff} = \tau_t + R_{load}C_{eff(output)} \rightarrow$ switching time as 2nd approx.
- $L = v_s \tau_t$ (source-drain spacing).
- $I_{sat} = eNv_s W\ d_s = eN\ WLd_s/\tau_t$.
- Aspect ratio W/L (technology?).

- $(1/2)\, eN (d_s)^2 / \varepsilon_0\varepsilon_r$ = (Max Gate Voltage).
- $E'_{peak} = eN d_s/\ \varepsilon_0\varepsilon_r < E_{breakdown}$.
- Minimum Drain Source Voltage $\sim E_S L$ (E_S is the field required to reach limiting velocities).

Mutual Conductance

$g_{mo} \sim I_{sat}/V_{gate(max)}$.

Mutual conductance reduces with frequency as $g_m(\omega) \approx g_{mo}/(1+j\omega\tau_t)$;

$$v_{out} = g_m(\omega)\, R(1+j\omega RC_{eff(out)}) \approx g_{mo}R/[1+j\omega(\tau_t+RC_{eff(out)})]$$

$$= g_{mo}R/[1+j\omega\tau_{eff}].$$

Capacitances for FET

Parallel Plate Capacitance: $\varepsilon_0\varepsilon_r$ Area/spacing

used for rough estimates of parasitic capacitance.

Effective Capacitances for FET

$C_{eff(out)} \rightarrow C_{gate/drain} \div C_{drain/source} \div C_{load}$;

$C_{eff(in)} \rightarrow MC_{gate/drain} + C_{gate/source\ (proximity)} + C_{gate/source\ (electronic)}$;

$C_{electronic} = g_{mo}\tau_t$; $M = (1 + g_{mo}R_{load})$.

Time Constants for FET

$\mu = e\tau/m^*$ relates mean free time τ and mobility.

Transit time τ_t over distance L and scattering limited velocity v_s are related by $\tau_t = L/v_s$.

$v_{out} \approx g_{mo}R/[1+j\omega\tau_{eff}] = Rg_{mo}/[1+j\omega/(2\pi f_t)]$

$1/(2\pi f_t) = \tau_t \div RC_{eff(out)} = \tau_{eff}$ The transition frequency is f_t.

10% to 90% rise time is $T = 2.2\,\tau_{eff} = (2.2/2\pi)(1/f_t) = 0.35/f_t$.

WIM (Revised MAY 1997)

Fast Transistor Elective:

Tunnel Barrier Design Summary Sheet

Schrodinger's Equation

Complex Wave $\Psi = A \exp(-j\, 2\pi f t + j\, 2\pi x/\lambda) = A \exp(-j\, \omega t) \exp(j\, k x)$;

$<\text{momentum}>\Psi = p\,\Psi = (h/\lambda)\,\Psi = -j\,(h/2\pi)\, \partial\Psi/\partial x$;

$<\text{Total energy}>\Psi = E\Psi = hf\,\Psi = j\,(h/2\pi)\, \partial\Psi/\partial t$

$(h/2\pi) \rightarrow \hbar$; $h = 6.625 \times 10^{-34}$ J/s.

Schrodinger's equation:-

$E\Psi = (1/2m)[-j\, \hbar\partial/\partial x]^2\, \Psi + e\phi\, \Psi$.

Tunnelling (Rectangular barrier $e\phi$)

Propagating waves outside barrier with incident kinetic energy $U_{\text{incident}} = (\hbar k_i)^2/2m^*$

Evanescent waves inside barrier: $-(\hbar \kappa)^2/2m^* = (\hbar k_i)^2/2m^* - e\phi$:

Technology Design Summary

Diffusion

Constant Surface Concentration:

$$C(x,t) = C_s \operatorname{erfc}\left[\frac{x}{2(Dt)^{1/2}}\right]$$

Constant Total Dopant:

$$C(x,t) = \frac{S}{(\pi Dt)^{1/2}} \exp\left[\frac{-x^2}{4\,Dt}\right]$$