DR S D GUEST

ENGINEERING TRIPOS PART IB 2005

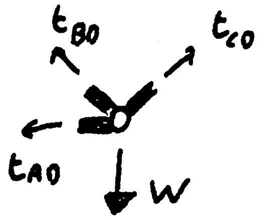
PAPER 2 : STRUCTURES SECTION A

CRIB

1. 3 bars, 2 nodal freedoms ⇒ generically, one state of self-stress

(a) Take AD as redundant bar (this is clearly not a singular arrangement, hence any bar can be chosen)

(i) Take a free body of joint D



Choose $t_{AD} = 0$

hence equilibrium gives $t_{BD} = t_{CD} = \frac{W}{\sqrt{2}}$

Write $\underset{\sim}{t}_0 = \begin{bmatrix} t_{AD} \\ t_{BD} \\ t_{CD} \end{bmatrix} = W \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$ as a particular equilibrium solution.

(ii) For state of self-stress, remove load, set $t_{AD} = 1$

Equilibrium gives $t_{BD} = -\frac{1}{\sqrt{2}}$, $t_{CD} = \frac{1}{\sqrt{2}}$

Write as $\underset{\sim}{s} = \begin{bmatrix} 1 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

examiners note:
256/273 attpts.
av. 13·4/20

1. cont.

(a)(iii) We need <u>compatibility</u> of extensions to find bar forces

General solution for equilibrium

$$\underset{\sim}{t} = \underset{\sim}{t}_0 + x\,\underset{\sim}{s}$$

Extensions are given by

$$\underset{\sim}{e} = \underset{\sim}{F}\,\underset{\sim}{t} \qquad \text{where } \underset{\sim}{F} = \frac{L}{AE}\begin{bmatrix} 1 & & \\ & \sqrt{2} & \\ & & \sqrt{2} \end{bmatrix}$$

Hence

$$\underset{\sim}{e} = \frac{LW}{AE}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + \frac{Lx}{AE}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

By virtual work $\underset{\sim}{s}.\underset{\sim}{e} = 0$ ($\underset{\sim}{s}$ is in equilibrium with zero external loads)

$$\frac{L}{AE}\left(W.0 + x(1+\sqrt{2})\right) = 0$$

$$\Rightarrow x = 0$$

Thus the elastic solution is equal to our choice of particular solution (in this case)

$$\underset{\sim}{t}_e = W\begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

1. cont.

(b)(i) Following heating of bar AD, the general equilibrium expression is unchanged, but the extensions now include an additional term

$$\underset{\sim}{e} = \underset{\sim}{F}\underset{\sim}{t} + \begin{bmatrix} \alpha LT \\ 0 \\ 0 \end{bmatrix}$$

examiners note: students were often careless with dimensions here, using αT rather than αLT, and giving final answers without AE term for tensions

The compatibility expression, $s.e = 0$ now becomes:

$$\frac{L}{AE}\left(x(1+\sqrt{2})\right) + \alpha LT = 0$$

hence

$$x = -\frac{AE\alpha T}{1+\sqrt{2}}$$

thus

$$\underset{\sim}{t} = W\begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} - \frac{AE\alpha T}{1+\sqrt{2}}\begin{bmatrix} 1 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

(ii) Total extensions are given by

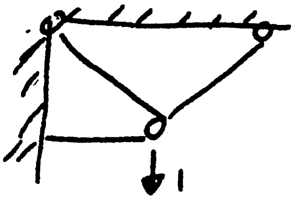
$$\underset{\sim}{e} = \frac{LW}{AE}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \frac{\alpha TL}{1+\sqrt{2}}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + \alpha TL\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

examiners note: many students forgot to include this term

Find deflections using virtual work

1(b)(ii) cont.

Vertical deflection, use a virtual load of 1, down

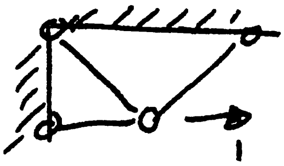


$$t_v^* = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\end{bmatrix}$$

n.b. this is not a unique choice, but any equilibrium system will give the same answer.

$$\delta_v . 1 = \underset{\sim}{t_v^*} . \underset{\sim}{e} = \underline{\sqrt{2}\frac{WL}{AE}}$$

Horizontal deflection, use a virtual load of 1, right



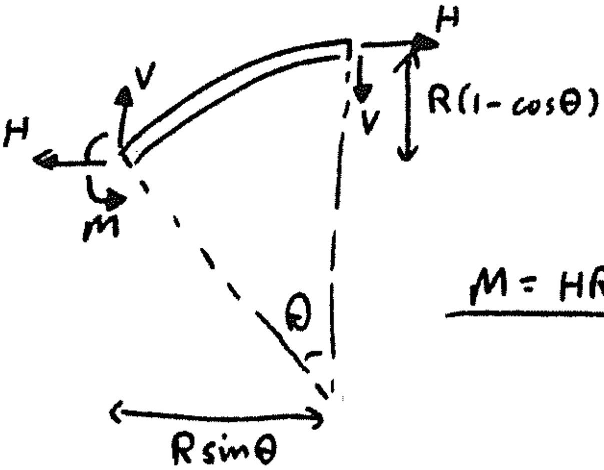
$$t_h^* = \begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$\delta_h . 1 = t_h^* . e = \alpha T L \left(\frac{-1 + (1+\sqrt{2})}{1+\sqrt{2}} \right)$$

$$= \underline{\alpha T L \left(\frac{\sqrt{2}}{1+\sqrt{2}} \right)}$$

examiners note:
95/273 attpts.
av. 13·7/20
27/95 answers received full marks

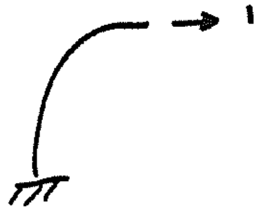
2 (a) Take a free body



$$\underline{M = HR(1-\cos\theta) + VR\sin\theta}$$

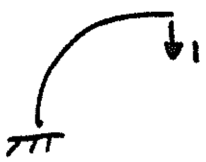
(b) Consider virtual forces

Horizontal



$$M_h^* = R(1-\cos\theta)$$

Vertical



$$M_v^* = R\sin\theta$$

Actual curvatures are given by

$$\kappa = \frac{1}{EI} \cdot M = \frac{HR}{EI}(1-\cos\theta) + \frac{VR}{EI}\sin\theta$$

Find deflections by virtual work

$$1 . \delta_h = \int_0^{\pi/2} M_h^* . \kappa . \underbrace{R\,d\theta}_{ds}$$

$$= \frac{HR^3}{EI}\int_0^{\pi/2}(1-\cos\theta)^2 d\theta + \frac{VR^3}{EI}\int \sin\theta.(1-\cos\theta)\,d\theta$$

2 (b) cont.

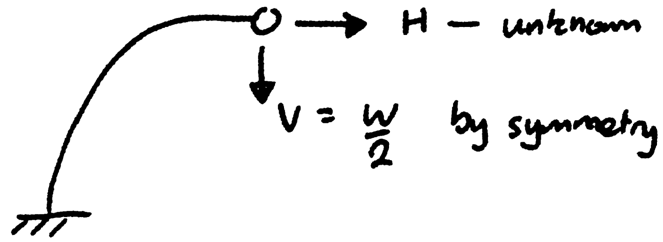
$$\underline{\delta_h = \frac{HR^3}{EI}\left(\frac{3\pi}{4} - 2\right) + \frac{VR^3}{EI} \cdot \frac{1}{2}} \qquad (1)$$

$$1 . \delta_v = \int_0^{\pi/2} M_v^* . k . R d\theta$$

$$= \frac{HR^3}{EI} \int_0^{\pi/2} (1-\cos\theta) . \sin\theta \, d\theta + \frac{VR^3}{EI} \int_0^{\pi/2} \sin^2\theta \, d\theta$$

$$\underline{\delta_v = \frac{HR^3}{EI} . \frac{1}{2} + \frac{VR^3}{EI} . \frac{\pi}{4}} \qquad (2)$$

(c) Consider half of the arch



By symmetry, horizontal deflection will be zero, $\delta_h = 0$

from (1)

$$0 = \frac{HR^3}{EI}\left(\frac{3\pi}{4} - 2\right) + \frac{W}{2} . \frac{R^3}{EI} . \frac{1}{2}$$

$$\Rightarrow H = -\frac{W}{3\pi - 8}$$

sub in (2)

$$\delta_v = -\frac{W}{3\pi-8} . \frac{R^3}{EI} . \frac{1}{2} + \frac{W}{2} . \frac{R^3}{EI} . \frac{\pi}{4}$$

$$= \frac{WR^3}{EI}\left(\frac{\pi}{8} - \frac{1}{6\pi - 16}\right) = \underline{0.042 \frac{WR^3}{EI}}$$

examiners note.
139/273 att pts
av. 10.2/20

3.(a) Total material area in flange & stiffeners

$$= 1800\,mm \times 10\,mm + 8 \times 50\,mm \times 10\,mm$$
$$= 22000\,mm^2$$

Equivalent thickness t

$$1800\,mm \times t = 22000\,mm^2$$
$$t = 12.2\,mm.$$

The flange is constructed with stiffeners to

(i) prevent local buckling

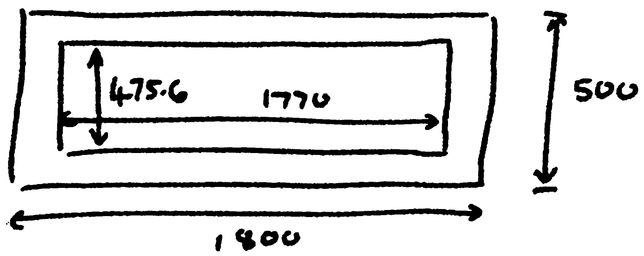
(ii) carry locally concentrated loads

examiners note.

This was answered very poorly. Most answers discussed reasons for adding cross-sectional area, while the question asks about redistributing the flange area. Only 2 candidates got 4 marks for (a)

(b) Require I of section.

Assume smeared thickness.



all in mm

$$I = \frac{1800 \times 500^3}{12} - \frac{1770 \times 475.6^3}{12} = 2.9 \times 10^9\,mm^4$$

examiners note

Any reasonable assumption about dimensions was perfectly acceptable (e.g. 500 mm internal depth, 524.4 external depth

3(b) cont.
$$|\sigma|_{max} = \frac{M\,|y|_{max}}{I}, \quad |y|_{max} = 250\text{ mm}$$

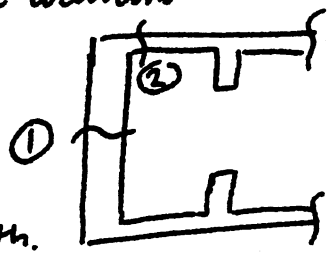
$$\therefore\ |\sigma|_{max} = \frac{1\times10^9\text{ Nmm} \times 250\text{ mm}}{2.9\times10^9\text{ mm}^4}$$

$$= 86\text{ N/mm}^2$$ at outside of top/bottom flange

<u>Shear stress</u>

There are two possible locations

① gives peak shear flow, but ② has a reduced thickness — need to check both.



For ②

$$(A\bar{y})_f = 1800 \times 12.2 \times 243.9 = 5.4\times10^6\text{ mm}^3$$

$$q_2 = \frac{SA\bar{y}}{I} = \frac{400\times10^3\text{ N} \times 5.4\times10^6\text{ mm}^3}{2.9\times10^9\text{ mm}^4}$$

$$= 745\text{ N/mm}$$

Need to take two cuts through unstiffened flange to find stress

$$\tau_2 = \frac{q_2}{2\times10\text{ mm}} = \underline{37\text{ N/mm}^2}$$

For ①

$$(A\bar{y})_1 = (A\bar{y})_f + (240\times15\times120)\times2 = 6.2\times10^6\text{ mm}^3$$

$$q_1 = \frac{SA\bar{y}}{I} = \frac{400\times10^3\times6.2\times10^6}{2.9\times10^9} = 855\text{ N/mm}$$

3 (b) cont.

examiners note

No candidate checked both ① and ②, and hence no candidate recieved full marks for (b)

Need to take two cuts through web

$$\tau_1 = \frac{q_1}{2 \times 15\,mm} = \underline{29\ N/mm^2}$$

Thus ② is the more critical section.

(c)(i) Use $q = \dfrac{T}{2A_e}$

examiners note: Not critical

Use enclosed area to mid-thickness – ignore stiffeners, as it is unreasonable to expect any shear 'flow' their

$A_e = 490\,mm \times 1785\,m = 875 \times 10^3\,mm^2$

Worst stress is where t is minimal, i.e. any unstiffened section in either flange, $t = 10\,mm$

$$\tau = \frac{q}{t} = \frac{200 \times 10^6\,Nmm}{2 \times 875 \times 10^3\,mm^2 \times 10\,mm} = \underline{\underline{11\ N/mm^2}}$$

(ii) Use $\phi = \dfrac{T}{G \cdot 4A_e^2 / \oint \frac{ds}{t}}$

$81 \times 10^3\,N/mm^2$ from data book

$$\oint \frac{ds}{t} = \frac{1785}{10} \times 2 + \frac{490}{15} \times 2 = 422$$

Response is dominated by more flexible unstiffened sections, hence use actual flange thickness

examiners note

Most candidates used (correctly) the actual flange thickness in (i), but the (incorrect) smeared thickness in (ii) – in fact, the reasons for using the actual thickness in (i) carry over unchanged to (ii)

$$\phi = \frac{200 \times 10^6\,Nmm \times 422}{81 \times 10^3\,N/mm^2 \times 4 \times (875 \times 10^3\,mm^2)^2}$$

$= 340 \times 10^{-9}\,rad/mm$

$= \underline{\underline{340 \times 10^{-6}\,rad/m}}$ $(= 0.02°/m)$

S.D. Guest

June 2005

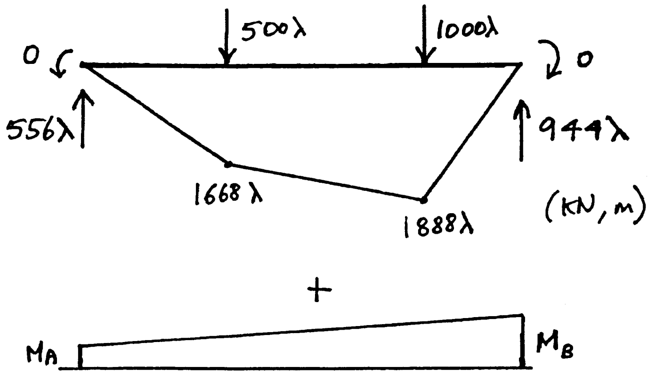
Section B

4 (a) Section 610 × 229 × 125 has $Z_p = 3676cm^3$ (from data book) Many candidates used the elastic section modulus here and some used the value for bending about the minor axis.

$$\text{So } M_p = 3676 \cdot 10^3 \cdot 300 = 1103kNm$$

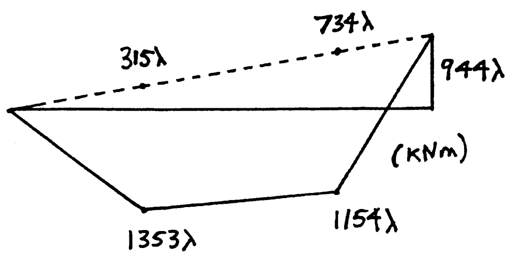
(b) The structure is twice indeterminate (ignoring the axial restraint since the loading is flexural). If the indeterminacies are taken to be the moments at the two end supports, then any valid bending moment diagram can be found from the sum of a free bending moment diagram, due to all the applied loads, and a reactant bending moment diagram due to the moments at the two supports.

This leads to:-

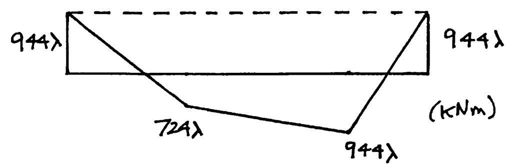


Note that there is a completely free choice for the two values M_A and M_B; *any* value will be in equilibrium with the applied loads. The two cases shown below are illustrative.

Case 1. Take $M_A = 0$ and $M_B = 1888\lambda/2 = 944\lambda$ (kNm)



Case 2. Take $M_A = M_B = 944\lambda$ (kNm)



(c) For each bending moment diagram, find the value of λ that makes the maximum moment equal M_p.

$$\text{For Case 1, } \lambda = \frac{1103}{1353} = 0.815$$

$$\text{For Case 2, } \lambda = \frac{1103}{944} = 1.168$$

(d) The answers found are lower bounds, so the higher value (1.168) is closer to the correct solution.

The Lower Bound Theorem states:-

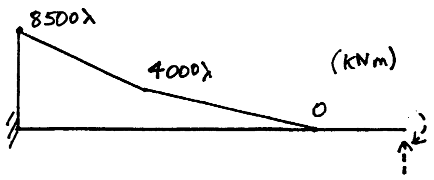
"If a set of forces and moments can be found which is *everywhere* in equilibrium with the applied load and which *nowhere* exceeds the capacity of the section, then that load can be safely carried".

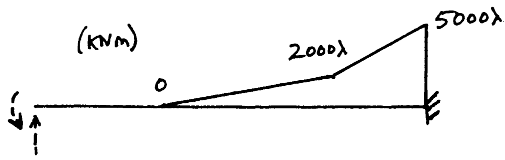
(e) The answer would not have been altered by the misalignment of the supports. No account is taken of the compatibility condition in finding the lower bound.

Examiner's comments. This question was done either very well or very badly. Many candidates could not find *any* bending moment that was in equilibrium with the loads. Some went on not to use the bending moments they had calculated but instead carried out an upper bound collapse analysis. The Lower Bound Theorem was often guessed, leaving out the critical words "everywhere" and "nowhere"; in general candidates were better able to quote the Theorem than to use it.

Note that Case 2 above leads to the moment capacity being reached in three places, which would lead to a collapse mechanism at $\lambda = 1.168$. Thus, this value represents the actual collapse load of the structure. This had not been asked-for in the question, although it was noted by the best candidates.

A few students chose to select equilibrium solutions in the form of cantilevers. In these cases the indeterminacies take the form of point loads and moments at the tips of the cantilevers (shown dotted below). These are perfectly acceptable and lead to valid lower bounds.

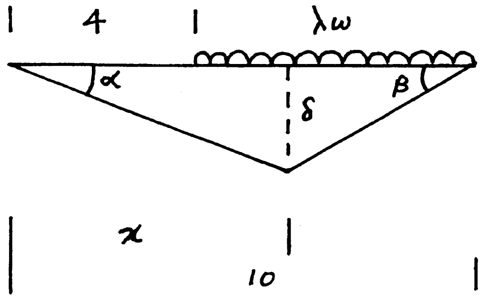




If no tip loads are applied, these lead to lower bounds of $\lambda = \dfrac{1103}{8500} = 0.130$ and $\dfrac{1103}{5000} = 0.221$ for the two cases shown here. They *are* valid lower bounds, but if used for design purposes would lead to very uneconomic structures.

Attempts 112; Average Mark 9.9/20

5 (a) The specified collapse mechanism is:-



Compatibility gives $\delta = \alpha x = \beta(10 - x)$ so $\beta = \dfrac{x}{(10 - x)}\alpha$

Average work done by load $= \lambda w(10-x)\frac{\alpha x}{2} + \lambda w\alpha\frac{(x+4)}{2}(x-4)$

$$= \frac{\lambda w\alpha}{2}[(10-x)x + (x-4)(x+4)] = \lambda w\alpha(5x-8)$$

Energy dissipated in hinges $= M_p(2\alpha + 2\beta) = 2M_p\alpha(1 + \frac{x}{10-x}) = 2M_p\alpha(\frac{10}{10-x})$

Equating WD and ED $\lambda = \frac{20M_p}{w(10-x)(5x-8)} = \frac{20M_p}{-5x^2+58x-80}$

(b) We could differentiate $\frac{d\lambda}{dx}$ to find the value of x which gives the minimum value of λ, but it is easier to rewrite as $M_p = \frac{w\lambda(-5x^2+58x-80)}{20}$ and differentiate to find the value of x which minimises the value of M_p needed to carry a given λw, which is equivalent.

$\frac{dM_p}{dx} = 0$ when $58 - 10x = 0$ so $x = 5.8$ m.

The corresponding load factor is $\lambda = \frac{200 \cdot 20}{(10-5.8) \cdot 42 \cdot 300} = 0.151$. This is an upper bound on the collapse load.

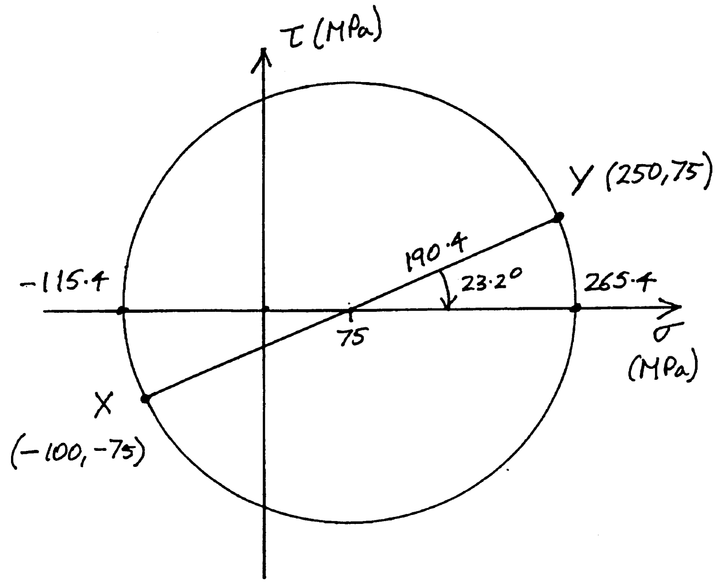
The Upper Bound Theorem states that "If the work done in *any* compatible collapse mechanism is equated to the work done by the applied load, the resulting estimate of the collapse load is an upper bound on the true collapse load."

(c) Provided the structure is sufficiently ductile, and assuming that the displacements are sufficiently small not to alter the overall geometry, misalignment of the supports will have no effect on the collapse load found here since the magnitude of the rotations at each hinge will adjust to take up the original lack of fit.

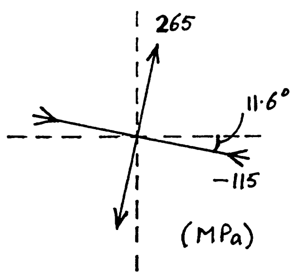
Examiner's comments. Several candidates assumed that the load deflected by an average amount of $\delta/2$, or even worse, by δ. There was a tendency to be able to apply the upper bound theorem, but not to be able to quote it.

Attempts 246; Average Mark 12.2/20

6. (a) Mohr's circle of stress



The principal stresses ($\sigma_1 = 265.4$ MPa and $\sigma_2 = -115.4$ MPa) are orientated at 11.6° clockwise from the original axes. Many candidates did not completely define these direction. "...at an angle of 11° ...". 11° in what sense, and measured relative to what axis?



(b) The third principal stress $\sigma_3 = 0$

(i) Tresca condition. Radius of Mohr's circle = 190.4 MPa.

$$\text{Thus, factor} = \frac{300}{2 \cdot 190.4} = 0.788$$

(ii) von Mises' condition.

$$\lambda^2((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2) = 2Y^2$$

$$\lambda^2(380.8^2 + 115.4^2 + 265.4^2) = 2 \cdot 300^2$$

$$\lambda = \frac{1}{\sqrt{(1.271)}} = 0.887$$

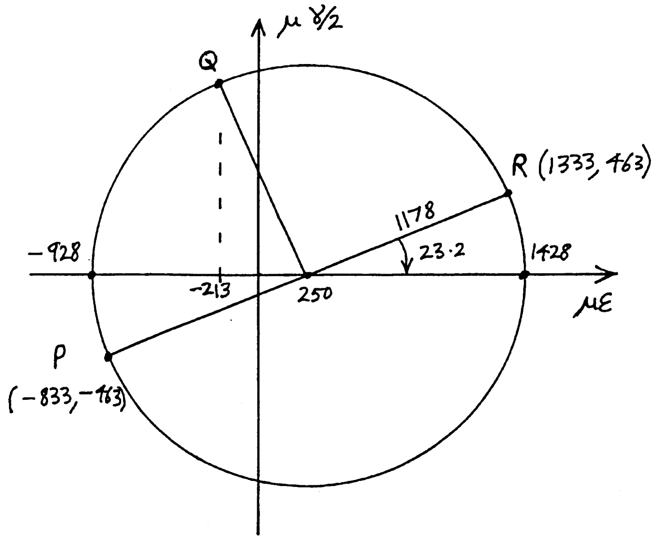
(c) On the assumption that the strains remain elastic (it was correctly pointed out by a few top candidates that this should have been stated in the question), and taking Poisson's ratio as 0.3 and Young's modulus as 210.10^3 MPa (from the Data Book), then:-

$$\varepsilon_r = \frac{250 - 0.3(-100)}{210.10^3} = 0.001333$$

$$\varepsilon_p = \frac{-100 - 0.3(250)}{210.10^3} = -0.000833$$

$$\gamma_{rp} = \frac{75}{81.10^3} = 0.000926 \text{ and } \gamma_{rp}/2 = 0.000463$$

Mohr's circle for strain



(i) $\varepsilon_q = -0.000213$

(ii) Principal strains are 0.001428 and -0.000928, in the same directions as the principal stresses.

(d) Principal stresses occur on faces where there is no shear stress, so there is also no shear strain on the same surfaces, provided the material is isotropic. It is thus no surprise that the principal directions coincide.

Examiner's comments. The question was quite well done, but many marks were lost unnecessarily by not following the sign convention for the Mohr's circles, as set out in the Data Book. Many candidates failed to take account of the fact that the stress in the x direction was negative. It does make a difference!

Very few candidates actually drew Mohr's circles, to scale, on graph paper; instead they relied on calculations. This led to many errors that would have been noticed if done by drawing. The question specifically *required* a Mohr's circle in part (c). Even when drawn, many candidates failed to mark the R and P points correctly, and thus failed correctly to locate the Q point. Many put it diametrically opposite its correct position. Almost none of the candidates who used the formula to find ε_q bothered to consider whether the gauge was at $\pm 45°$; most got it wrong.

The question could have been worded slightly better; the stresses as given were deliberately above the allowable stress, so the factor required was less than one. The intention was to see whether candidates could correctly determine whether the applied or allowable stress had to be higher to be safe. About a third correctly pointed out this inconsistency. A very small number of the brightest candidates pointed out that this called into question the calculation of strains needed in part (c) since the material would not then be elastic. Due allowance was made for any candidate who said they had wasted time on this.

Attempts 243; Average Mark 13.8/20

C J Burgoyne 16th June 2005