IB Exam Cribs 2004-2005 - PAPER 4 TBN
THERMOFLUID MECHANICS DR TB NICKELS

Question 1

(a) Starting from

$$Tds = dh - vdp$$

and noting that the pressure is constant ($dp = 0$), we have

$$Tds = dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp = \left(\frac{\partial h}{\partial T}\right)_p dT = c_p dT$$

[Part IB students may not appreciate the subtlety but $dh = c_p dT$ is NOT generally true]. Hence,

$$Tds = dh = c_p dT$$

and rearranging then integrating (noting that c_p is constant),

$$\int_1^2 ds = \int_1^2 c_p/T dT$$

$$s_2 - s_1 = c_p \ln T_2/T_1$$

Since this equation only contains thermodynamic properties (c_p, T) then the entropy change is only a function of the beginning and end states and not the path taken between these states.

(b) The arrangement must look like the one below since $T_2 > T_o$.

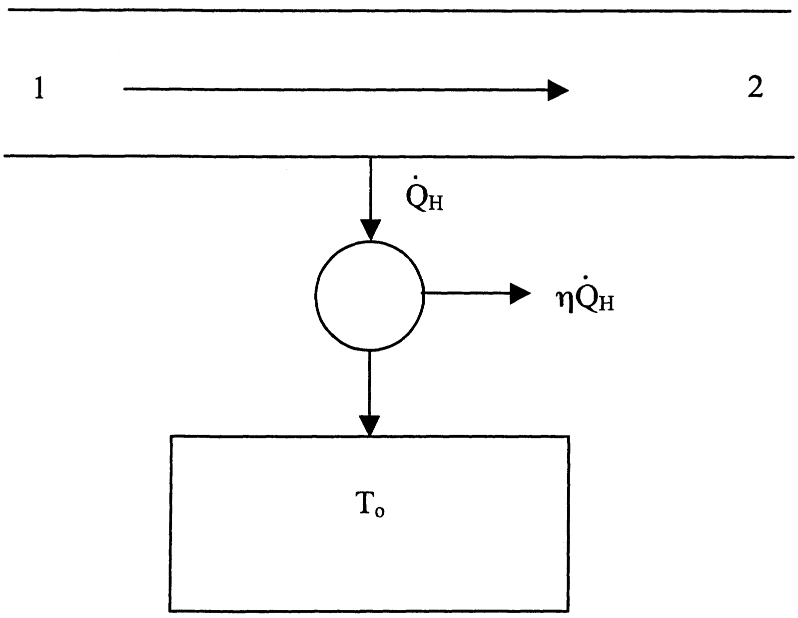
(i) Steady Flow Energy Equation for the water gives $\dot{Q}_H = \dot{m}c_p(T_2 - T_1)$, which is the change in enthalpy flowrate.

(ii) Now using the Clausius inequality for the equal case we find

$$\int_1^2 \frac{d\dot{Q}_H}{T_H} - \frac{\dot{Q}_o}{T_o} = 0.$$

But for the liquid

$$\int_1^2 \frac{d\dot{Q}_H}{T_H} = -\dot{m}c_p \ln(T_2/T_1).$$



and the term on the right hand side is the change in entropy flowrate of the liquid.

(iii) Now from the diagram

$$\eta = \frac{\dot{Q}_H - \dot{Q}_o}{\dot{Q}_H} = 1 - \frac{\dot{Q}_o}{\dot{Q}_H} \quad (1)$$

$$= 1 - \frac{T_o \dot{m} c_p \ln(T_2/T_1)}{\dot{m} c_p (T_2 - T_1)} \quad (2)$$

$$= 1 - \frac{T_o \ln(T_2/T_1)}{(T_2 - T_1)} \quad (3)$$

(iv) To maximise η the flow rate should be as high as possible. This is because the average temperature at which heat is transferred to the engine will be as high as possible.

(c)(i) Point A is downstream since, in the absence of any work transfer the pressure must drop so B must be upstream since the pressure there is higher.

(ii)S.F.E.E. gives $\dot{Q} = \dot{m}(h_A - h_B)$ and we can look up the values of the enthalpies and (in anticipation of the next part) the entropies. $h_A = 2786.5, s_A = 5.794$ and $h_B = 1343.3, s_B = 3.249$ which leads to $\dot{Q} = 10(2786.5 - 1343.3) = 14432$kW.

(iii)$\dot{s}_{irrev} = \dot{m}(s_A - s_B) - \dot{Q}/T$. Substituting in the values gives

$\dot{s}_{irrev} = 0.270\text{kW/K}$ and since this is greater than zero the second law is satisfied and hence the original assumption of the flow direction is confirmed. The irreversibility could be due to friction losses, pipe roughness or bends or fittings.

Question 2

(a) 1st law: $dq - dw = du$

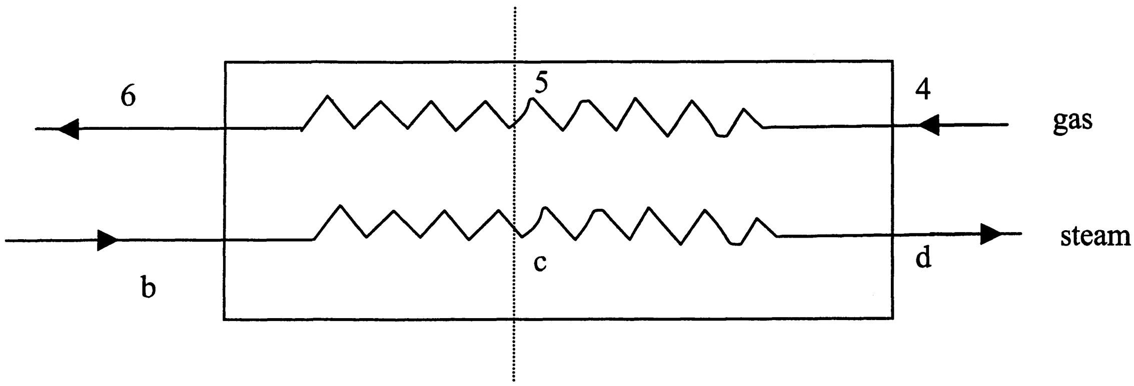
2nd law: $dq_{rev} = Tds$ and hence

$Tds - dw = du$ for a reversible process. We also know that $dw = pdv$ hence $Tds = du + pdv$. We also note that

$h = u + pv$ which leads to $dh = du + pdv + vdp$ and substitution into the above gives $Tds = du + pdv = dh - vdp$. This is valid for an irreversible process since it involves only looking at properties and changes in properties which are independent of the path taken.

(b)$Tds = dh - vdp$. If it is isentropic then $ds = 0$ and so $dh = vdp$. If it is also incompressible then v =constant and we can integrate this since it is reversible to find $\Delta h = v\Delta p$.

(c) (i) $T_5 - T_c = 10$K. Look now at the hot-side of the HRSG. SFEE consider that the



heat lost by the gas up to the pinch point is the same as that gained by the steam.

$$\dot{m}_g(h_4 - h_5) = \dot{m}_g c_p(T_4 - T_5) \quad (4)$$

$$= \dot{m}_s(h_d - h_c).$$

Now we are given $T_4 = 530^o$C, $T_d = 400^o$C and $p_d = 4$MPa so we can look up the enthalpies for the steam in the tables. $h_d = 3214.5$kJ/kg, $h_c = h_f(sat) = 1087.5$kJ/kg and we know that T_c is the saturation temperature at this pressure $T_c = 250.35^o$C. Substituting these values into equation 5 then gives

$$\begin{aligned} \frac{\dot{m}_s}{\dot{m}_g} &= \frac{c_p(T_4 - T_5)}{h_d - h_c)} \quad (5) \\ &= \frac{1.005(530 - (250.35 + 10))}{3214.5 - 1087.5} \\ &= 0.1274 \\ &= 1/7.85 \end{aligned}$$

(ii) Feed pump work per kg is $w_p = v\Delta p$. Where $v \approx v_{sat} = 0.001004$. Now need to find the pressure change.
$\Delta p = (p_b - p_a)$

$p_a = p(T_{sat}) = 0.04247$bar and $p_b = 40$bar. Hence $w_p = 4011.7$J/kg steam.
Now apply the steady flow energy equation to find the enthalpy at (b) i.e.
$h_b = h_a + w_p = 129.7$kJ/kg. Now equate the enthalpy change in the gas to the enthalpy change of the steam through the whole HRSG.

$$\dot{m}_g c_p(T_4 - T_6) = \dot{m}_s(h_d - h_b)$$

therefore

$$\begin{aligned} T_6 &= T_4 - \frac{\dot{m}_s}{\dot{m}_g}\frac{(h_d - h_b)}{c_p} \quad (6) \\ &= 530 - 0.1274\frac{(3214.5 - 129.7)}{1.005} \quad (7) \\ &= 138.95^o C \quad (8) \end{aligned}$$

(iii) Net entropy creation per kg of gas

$$s_c = (s_6 - s_4) + \frac{\dot{m}_s}{\dot{m}_g}(s_d - s_b)$$

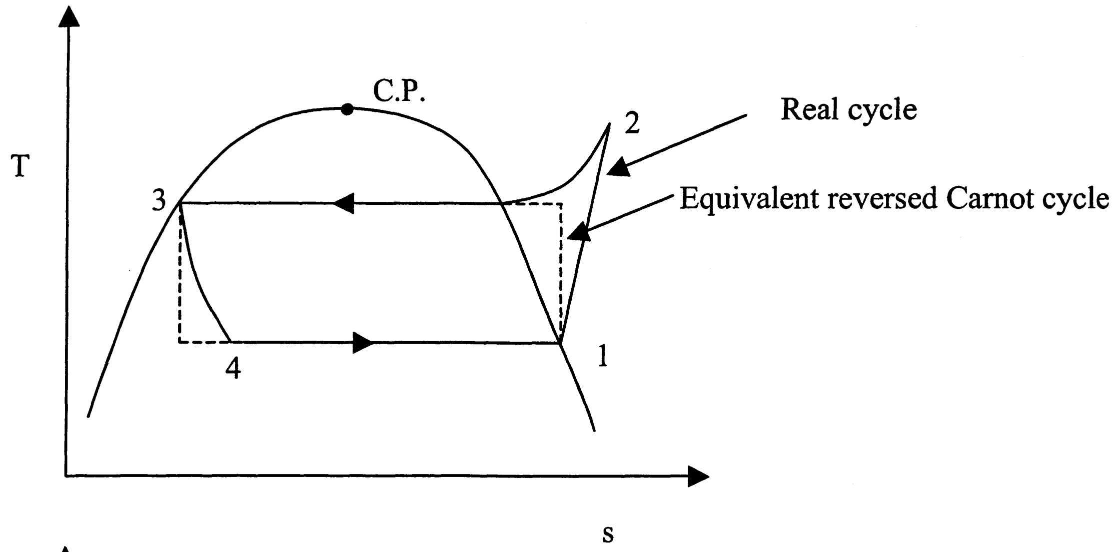
Now for the gas

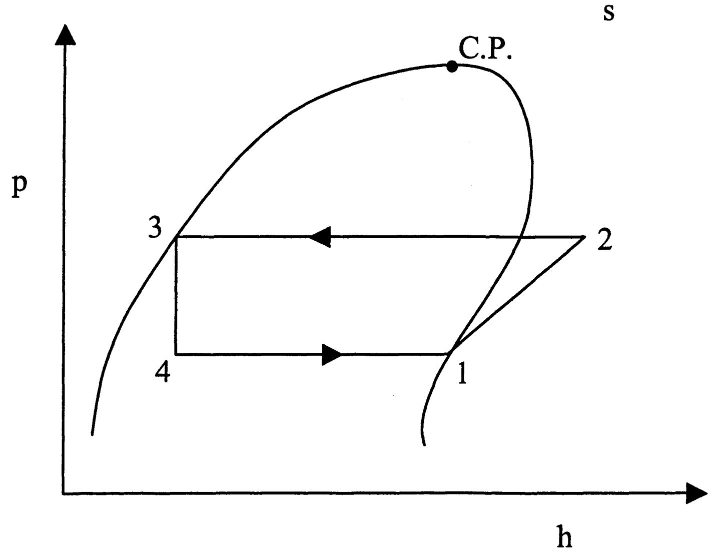
$$\begin{aligned} s_6 - s_4 &= c_p \ln(T_2/T_1) - R\ln(p_6/p_4) \quad (9) \\ &= 1.005 \ln(138.95 + 273)/(520 + 273) \\ &= -0.6708\text{kJ/kgK} \end{aligned}$$

and the second term is zero in the case of constant pressure. Hence we have a decrease in the specific entropy of the gas. Use tables for the steam
$s_d = 6.771$kJ/kgK and $s_b = s_a = 0.437$kJ/kgK since the feed pump is isentropic. Hence the entropy change of the system per kg of gas is
$\dot{s}_c = 0.1362$kJ/kg(gas)K which is greater than zero as we would expect.
(iv) For an environment at $30^oC = 303$K this represents a lost work of 41.3kJ/kg(gas). It is caused by the transfer of heat across a finite temperature difference.

Question 3

(a) The points that are easy to find are points 1 and 3 since these are on the lines (dry sat.





and wet sat.) From tables : $h_1 = 392.7\text{kJ/kg}$, $h_3 = 256.4\text{kJ/kg}$
$s_1 = 1.7331\text{kJ/kgK}$, $s_3 = 1.1906\text{kJ/kgK}$
$T_1 = -10°\text{C}$, $T_3 = 40°\text{C}$

(b) Throttle is isenthalpic so $h_3 - h_4$
At $-10°\text{C}$, $h_f = 186.7$, $h_g = 392.7$
IF x =dryness fraction, $h_4 = x(h_g - h_f) + h_f$ and hence

$$x = \frac{h_4 - h_f}{h_g - h_f} = \frac{256.4 - 186.7}{392.7 - 186.7} = 0.338$$

(c) SFEE for the evaporator
$\dot{Q}_{41} = \dot{m}(h_1 - h_4) = 10\text{kW}$
therefore $\dot{m} = 10/(392.7 - 256.4) = 0.07337\text{kg/s}$
C.O.P.$= \dot{Q}_{41}/\dot{W}_{IN} = \dot{Q}_{41}/(\dot{m}(h_2 - h_1))$.

(d)30K of superheat at the compressor exit:
From tables at 10.17bar Can interpolate for +30K. $h_2 = 451.65\text{kJ/kg}$.

	SAT	+20K	+40K
h	419.4	441.2	462.1
s	1.7112	1.7786	1.8395

$s_2s = s_1 = 1.7331$
hence

$$h_{2s} = h_{SAT} + \frac{s_{2s} - s_{SAT}}{s_{+20K} - s_{SAT}}(h_{+20K} - h_{SAT}) = 426.48$$

and so the isentropic efficiency is

$$\eta_{isen} = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{426.48 - 392.7}{451.65 - 392.7} = 0.573$$

(e) $W_{IN} = \dot{m}(h_2 - h_1) = 0.07337(451.65 - 392.7) = 4.325\text{kW}$
hence C.O.P.$= Q_{41}/W_{IN} = 2.31$

(f) Best case is reversed Carnot engine.

$$\frac{Q_H}{T_H} = \frac{Q_C}{T_C}$$

$T_H = 40^oC$ =313K, $T_C = -10^oC$=263K
C.O.P.$_{max} = Q_C/(Q_H - Q_C) = T_C/(T_H - T_C) = 5.26$
Differences are due to irreversibilities caused by
1. Superheat in condenser
2. Throttle losses
3. Irreversibilities in compressor

To overcome these

1. Use isothermal compressor once 40°C is reached in isentropic compressor
2. use isentropic turbine instead of throttle
3. use isentropic compressor up to 40°C

Question 4

(a) Continuity gives
$3Hu_1 = 2Hu_2$
$u_2 = 3u_1/2$
No losses between (1) and (2) so we can use Bernoulli
$p_1 + \rho u_1^2/2 = p_2 + \rho u_2^2/2$
$p_2 = \rho(u_1^2 - u_2^2)/2 + p_1$
$= -(5/8)\rho u_1^2 + p_1$

(b) Can't use Bernoulli here since there is significant mixing and hence losses. Consider the forces on the control volume due to pressure on each end and balance this with the rate of change of momentum.
$\dot{m}(u_2 - u_3) = (p_3 - p_2)3H$
$\rho 3Hu_1(u_2 - u_3) = 3H(P_3 - P_2)$
Continuity gives $u_1 = u_3$ and substituting gives
$p_3 = p_2 + \rho u_1^2/2$ and from the first part we can relate p_2 and p_1 leading to
$p_3 = p_1 - \frac{1}{8}\rho u_1^2$
(c) Since the velocity at 1 and 3 is the same then the loss in total pressure is simply the difference in static pressure ,$(p_3 - p_2)$ calculated in (b).
The lost mechanical energy is given by $Tds = \dot{m}\Delta P_{o13}/\rho$
$= 3Hu_1(\rho u_1^2/8)$
$= (3/8)\rho u_1^3 H$
(d) Apply the control volume analysis between (1) and (3). The momentum flux in and out of the control volume are the same since the velocities and areas are the same. The static pressure is, however, different. The forces on the control volume must be zero and this balance is equivalent to the drag on the semi-cylinder. The drag, D, is then
$D = 3H * \rho u_1^2/8$
The non-dimensional drag coefficient, C_D, is then $D/(\rho u_1^2.Area/2)$
$C_D = 3/4.$

Question 5

(a)Balancing torques on the top and bottom of the ring-shaped fluid element in the circumferential direction we get - no angular acceleration means that there is no net torque acting
$\tau 2\pi r dr.r - (\tau + \frac{\partial \tau}{\partial y} dy).2\pi r dr.r = 0$
and hence,
$\frac{\partial \tau}{\partial y} dy = 0$ which leads to
$\tau = B(r)$ where B is some unknown function of r only (i.e. not of y).
(b) Newtonian fluid so $\tau = \mu \frac{\partial V_\theta}{\partial y}$
so $\mu \frac{\partial V_\theta}{\partial y} = B(r)$ and this may be integrated to give
$\mu V_\theta = By + C$ where C is a constant of integration. Now apply boundary conditions
$V_\theta = 0$ at $y = 0$ and hence $C = 0$.
Adjacent to the cone, $V_\theta = \Omega.r$ when $y = h$. Also we know $h = r\theta$ from geometry.
$\mu \Omega r = B(r).r\theta$ which leads simply to
$B = \mu\Omega/\theta$ and so it turns out that B does not depend on r in this case (though in general it could have). This is useful practically since the shear is independent of r and so all the fluid is subjected to the same shearing- this makes the results easier to interpret and avoids different behaviour at different radii.
The final answer is then

$$V_\theta = \frac{\Omega}{\theta} y$$

(c) To find the total torque on the bottom disk we return to the expression for the shear-stress with the value of the constant we now have and we find the shear stress is
$\tau = B = \mu\Omega/\theta$. Considering the contribution to the torque from thin ring-shaped elements like that above we have,

$$\Gamma = \int_0^R (\mu\Omega/\theta) 2\pi r dr.r = \frac{2\pi\mu\Omega R^3}{3\theta}$$

(d) A measure of the circumferential, viscous shear component is τ multiplied by the area hence $\tau 2\pi r dr$ is a measure of the circumferential force on a ring-shaped element (and we have an expression for τ from the last part. The centrifugal force is $\rho V \Omega^2 r$ where V is the volume of the element which is given by $V = 2\pi r dr h$. Taking the ratio of these two gives

$$\frac{2\pi r dr h \Omega^2 r}{2\pi r dr \tau} = \frac{\rho\Omega r^2 \theta^2}{\mu}$$

Since the largest r in our rig is the outside radius, R, then a useful measure of when we may have problems due to the effect of centrifugal forces is when

$$\frac{\rho\Omega R^2\theta^2}{\mu}$$

is larger than some prescribed limit. This is of course a kind of Reynolds number since if we take ΩR as the tangential velocity of the surface then our number is of the form of (density*velocity*length)/viscosity.

Question 6

(a) $\Gamma = f_1(D, h, \omega, \mu, \rho)$, where f_1 is some unknown function.
We have 6 variables $(\Gamma, D, h, \omega, \mu, \rho)$ which may be expressed in terms of three fundamental quantities, M,L and T. Hence from Buckingham's PI theorem we should have 6-3=3 fundamental quantities. There are different possibilities here which are correct. I chose

$$\frac{\Gamma}{\rho\omega^2 D^5} = f_2\left(\frac{\rho\omega D^2}{\mu}, \frac{h}{D}\right)$$

Another possibility for the torque parameter would be

$$\frac{\Gamma}{\mu D^3 \omega}$$

It doesn't matter which ones you choose as long as there are 3 non-dimensional numbers resulting. It is possible to imagine more obtained by manipulating these. Also nice is that one of these looks like a Reynolds number.

(b) Now we want similarity. Since it is a geometrically similar model (or a scale model) then h/D must be the same for both. To get full similarity then we also want the other parameter to be the same hence (using subscript m for model and F for full-scale) we set the Reynolds numbers to be the same. (We have freedom to adjust this by varying the speed). If both h/D and $\rho\omega D^2/\mu$ are the same then our torque parameter must be also since its a function of those two only. Hence

$$\frac{\rho_m \omega_m D_m^2}{\mu_m} = \frac{\rho_F \omega_F D_F^2}{\mu_F}$$

Now, since the fluid is the same then $\rho_m = \rho_F$ and $\mu_m = \mu_F$ and so we find

$$\frac{\omega_m}{\omega_F} = \frac{D_F^2}{D_m^2}$$

and its a half-scale model so $\omega_m = 4\omega_F$.

(c) Now since we have ensured that we have similarity in (b) then we know that the torque parameter must be the same hence

$$\frac{\Gamma_m}{\rho\omega_m^2 D_m^5} = \frac{\Gamma_F}{\rho\omega_F^2 D_F^5}$$

which gives

$$\frac{\Gamma_m}{\Gamma_F} = \frac{\omega_m^2}{\omega_F^2}\frac{D_m^5}{D_F^5} = \frac{1}{2}$$

(d) There are different ways to do this but the essential fact to realise is that all the shaft work goes into heating up the fluid (changing its enthalpy). The work put in over the time t_o is the power $\times$ time and the change in enthalpy of the liquid is $mC\Delta T$. Hence

$$m_m C_m \Delta T_m = \Gamma_m \omega_m t_o$$

This can be written as a non-dimensional paramter if desired which would be

$$\frac{m_m C_m \Delta T_m}{\Gamma_m \omega_m t_o}$$

Before proceeding we note that the mass of liquid is different for the two and since the densities are the same it is the ratio of the volumes ($m_m/m_F = h_m D_m^2/h_F D_F^2$). Now, as before, we equate the non-dimensional parameter for the model and for the full-scale noting that t_o is the same for both

$$\frac{\Delta T_F}{\Delta T_m} = \frac{\Gamma_F \omega_F h_m D_m^2}{\Gamma_m \omega_m h_F D_F^2}$$

and using all the results we already have we find that the temperature change of the full-scale tank in the time, t_o, is $\Delta T_F = \Delta T_m/16 = 0.125$°C.