2005 IB paper 5 Electrical ENGINEERING

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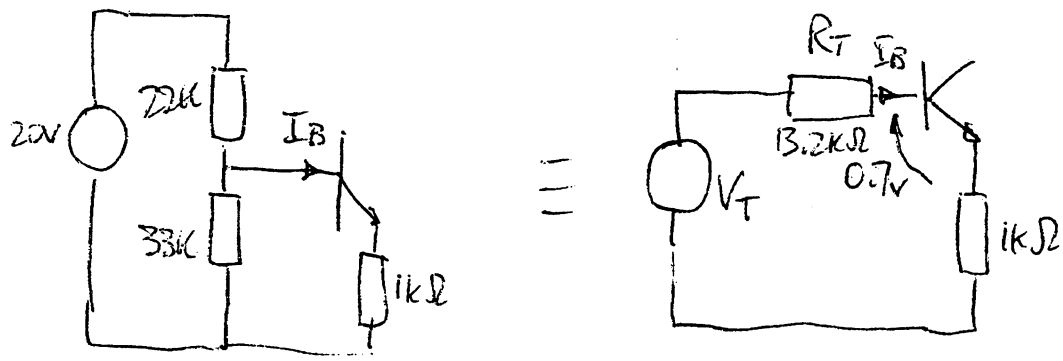
1a/ Voltage gain just below +1, input impedance very high, output impedance very low. The current gain depends on the transistor's h_{fe}. Used where a stage with high impedance input stage and low impedance output are required, but voltage amplification not needed.
For example, to drive a low impedance load like a loudspeaker.

b/ Since h_{FE} is finite, base current flows. Hence we can't treat the 22k – 33K resistors as a simple voltage divider and we should use the Thevenin equiv circuit for the bias chain. Careful application of KCL (nodal voltage analysis) should give the same result.



R_T = 22k//33k = 13.2 kΩ

V_T = 33/(33+22) × 20 = 12V

Applying Kirchhoff voltage law to the Thevenin circuit,

$$I_{B1} = 11.3 \big/ (R_T + (1 + h_{FE})R_3)$$

1. h_{FE} = 100

13.2 I_{B1} + 0.7 + 101 I_{B1} = 12

I_{B1} = 11.3/114.2 = 0.099 mA

V_{E1} = 101 (0.099) × 1 = **9.99 V**

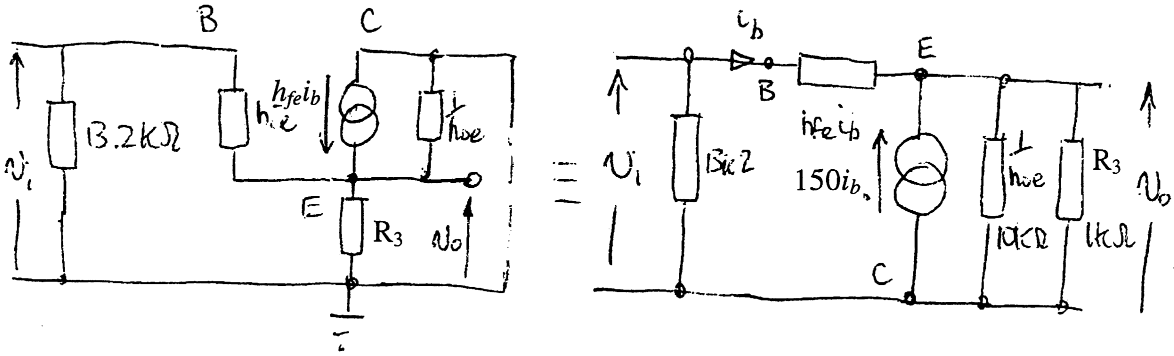
2. h_{FE} = 500

13.2 I_{B2} + 0.7 + 501 I_{B2} = 12

I_{B2} = 11.3/514.2 = 0.022 mA

$V_{E2} = 501\ (0.022) \times 1 =$ **11.02V**

c/ Small signal equivalent circuit,



Note that output is taken from the emitter, and the collector is grounded for the SS analysis. (*Common errors were to overlook the ground connection for the collector in the SSEC for this configuration, and to show* v_o *taken from the collector – should be the emitter – or to ignore the effect of* $1/h_{oe}$)

$i_b = (v_i - v_0)/h_{ie}$

Sum currents at E;

$$(1 + h_{fe})i_b - \frac{v_0}{R_3} - \frac{v_0}{1/h_{oe}} = 0$$

Eliminate i_b

$$\frac{v_0}{v_i} = \frac{1}{1 + \dfrac{\dfrac{h_{ie}}{R_3} + h_{oe}h_{ie}}{1 + h_{fe}}}$$

Insert numbers; gain $= \dfrac{1}{1 + \dfrac{1 + 1/10}{151}} =$ **0.993**

For R_{out}, we connect input to earth, hence $v_i = 0$. (*A common error was to assume that* R_{out} *was just* $1/h_{oe}$ *in parallel with* R_3)

Apply test voltage v_x, test current i_x flows. We want v_x/i_x.

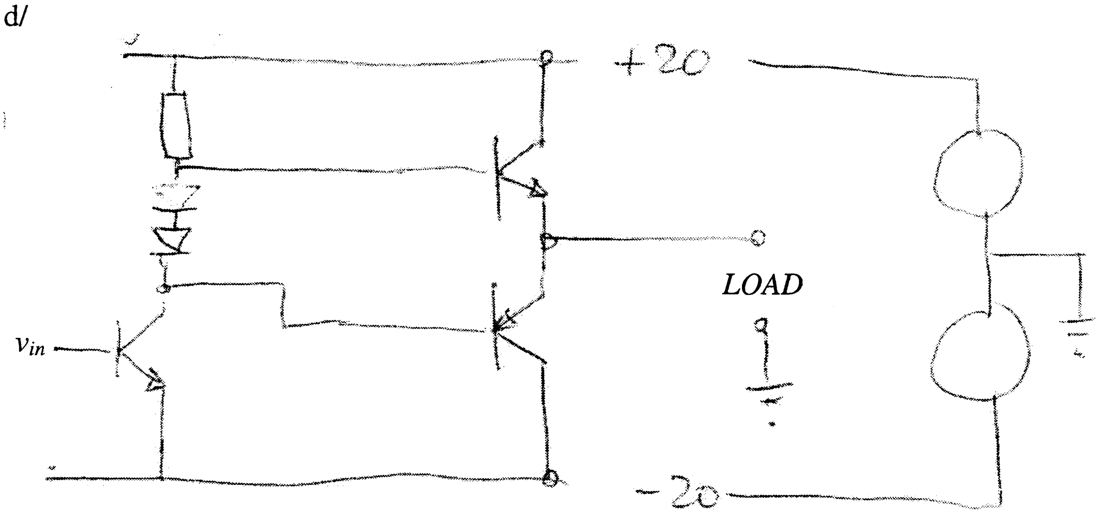
$i_b = -\ v_x/h_{ie}$

Sum currents at E: $(1 + h_{fe})\, i_b + i_x - \frac{v_x}{R_E} - \frac{v_x}{1/h_{oe}} = 0$

$$\frac{v_x}{i_x} = \frac{1}{1/R_E + h_{oe} + \frac{h_{fe}+1}{h_{ie}}}$$

Insert numbers $R_{out} = \frac{1}{10^{-3} + 10^{-4} + \frac{151}{1000}}$ **= 6.6 Ω**

d/

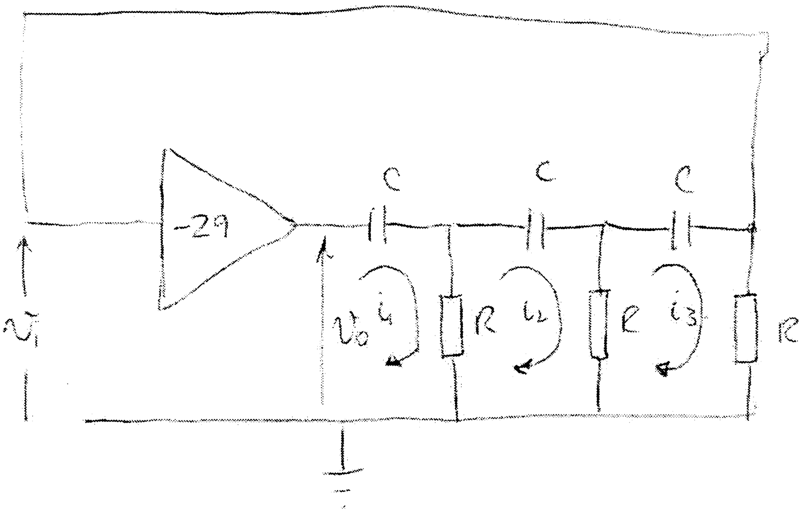


(*Common errors were to use an NPN device rather than a PNP device, or to connect the PNP with emitter to –20 V, or some other non-follower configuration*)

V_0 can now swing ±20 V, not just 0 to 20V (±10). Power is proportional to V^2.
All transistor current now passes through load rather than through a resistor. In the class A circuit the device conducts all the time whether or not a signal is applied. In Class B conduction only occurs when the input signal rises above/falls below 0, so better efficiency. However, crossover distortion will result, which can be alleviated by careful adjustment of the base bias for each device and by negative feedback.

2

a/ $|\mathbf{AB}| = 1$, $\angle(\mathbf{AB}) = 0$



$v_i = R.i_3$ eqn (0)

$(R + 1/j\omega C)i_1 - Ri_2 = v_0$ (1)

$(2R + 1/j\omega C)i_2 - Ri_1 - Ri_3 = 0$ (2)

$(2R + 1/j\omega C)i_3 - Ri_2 = 0$ (3)

From 3, $i_2 = (2R + 1/j\omega C)i_3/R$ (4)

From 1 and 4, $i_1 = \dfrac{v_0 + (2R + 1/j\omega C)i_3}{R + 1/j\omega C}$ (5)

Substitute for i_1 and i_2 in (2)

$$(2R + 1/j\omega C)(2R + 1/j\omega C).i_3/R - R\left(\frac{v_0 + (2R + 1/j\omega C)i_3}{R + 1/j\omega C}\right) - Ri_3 = 0$$

multiply thru by R and (R+1/jωC)

$$(2R + 1/j\omega C)(2R + 1/j\omega C)(R + 1/j\omega C)i_3 - R^2 v_0 - R^2 (2R + 1/j\omega C)i_3 - R^2 (R + 1/j\omega C)i_3 = 0$$

multiply out and cancel terms

$$\frac{R^2 v_0}{i_3} = (R^3 - \frac{5R}{\omega^2 C^2}) + \frac{6R^2}{j\omega C} - \frac{1}{j\omega^3 C^3}$$

Hence $\quad v_i = i_3R = \dfrac{R^3}{R^3 - \dfrac{5R}{\omega^2C^2} + \dfrac{6R^2}{j\omega C} - \dfrac{1}{j\omega^3C^3}} v_0$ $\quad$ which is the form required.

(Other methods of analysis possible, but tend to lead to more variables & messier equations).

Oscillation occurs when this is real (in phase), or $\dfrac{6R^2}{\omega_0 C} = \dfrac{1}{\omega_0{}^3C^3}$

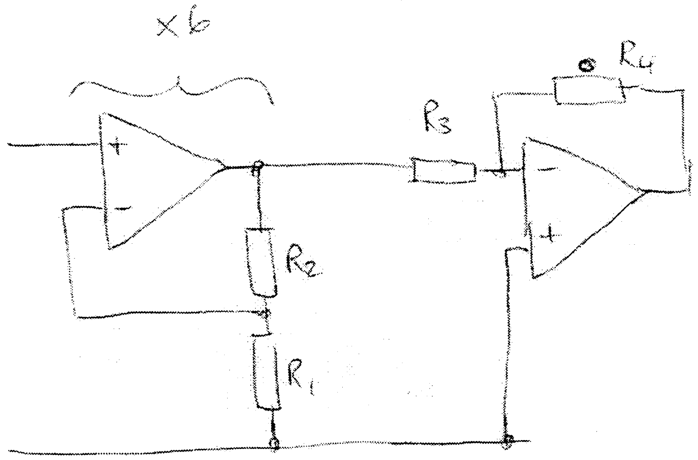
i.e. $\quad \omega_0{}^2 = \dfrac{1}{6R^2C^2}$, hence: $\quad f_0 = \dfrac{1}{2\pi RC\sqrt{6}}$

Also at this freq, $\dfrac{v_i}{v_o} = \dfrac{R^3}{R^3 - \dfrac{5R}{\omega_0{}^2C^2}} = \dfrac{1}{1 - \dfrac{5}{1/6}} = -\dfrac{1}{29}$

This result is obtained by substituting from above for ω_0. The circuit has attenuation of 29 and phase shift of 180^0. Hence a gain of just over **–29** is needed for oscillation.

If R = 10kΩ and f = 130 Hz, $\dfrac{1}{2\pi}\cdot\dfrac{1}{10^4\sqrt{6}C} = 130$ $\quad$ hence **C = 50 nF**

An inverting op-amp circuit has lowish input resistance which will load the phase shift network, reduce its output, and prevent oscillation. The best way to achieve an amplifier with high input resistance is to use non-inverting form, hence 2 stages will be needed. The second provides the necessary inversion. Other schemes are possible.



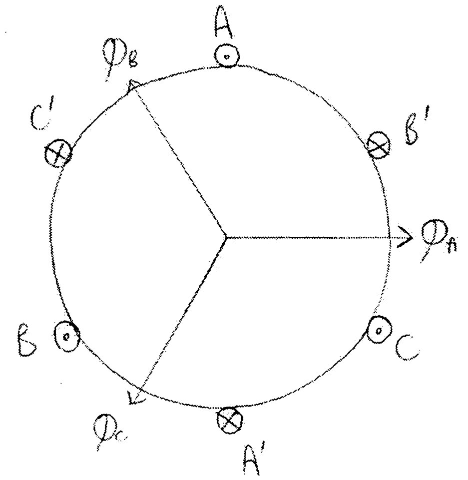
Let $R_2 = 5R_1$ so gain = 6 $\qquad$ With op-amps of ideal (infinite) gain, the input resistance of this stage will be infinite. In practice it will be many MΩ.

Let $R_4 \sim (29/6)R_3 \sim 5R_3$, so second stage gain is about –5.

R_4 may include a suitably chosen thermistor to regulate the gain and prevent clipping.

Choose R values in low kΩ to avoid conflict with moderate R_{in} of op amps.

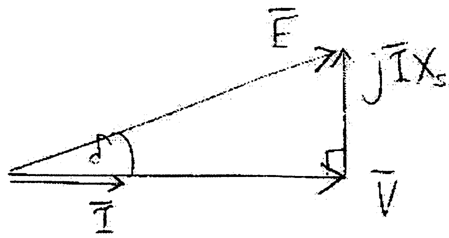
3a/ An AC generator has 3 phase windings, each located in space 120^0 electrical degrees apart.



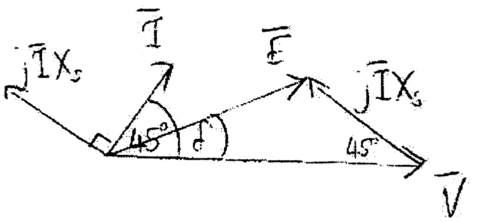
The flux axis of phase A is 0^0, B is 120^0 and C is 240^0. When each phase is excited in turn, the field distribution rotates. A balanced 3 phase supply in effect causes each phase to be excited in turn, hence producing a rotating magnetic field.

b/ The 3 different phasor diagrams are shown below.

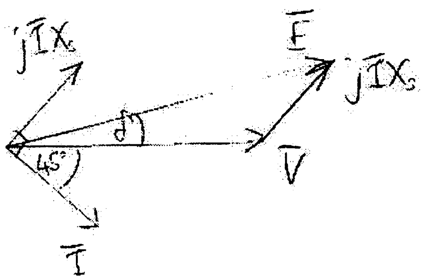
a)



b)



c)

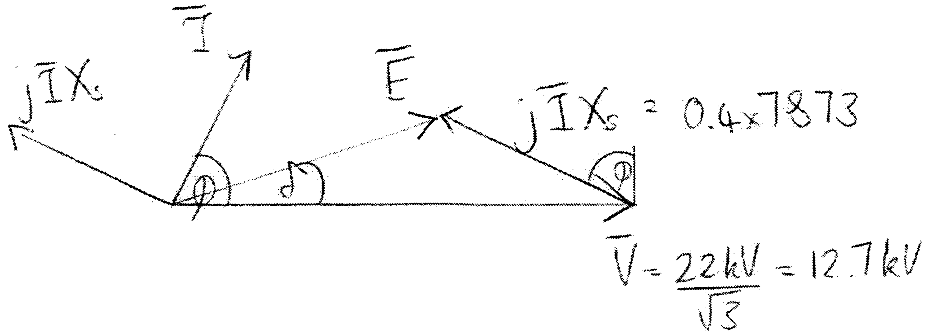


c/ 60% rated MVA = 0.6 x 500 = 300MVA,
power factor = 0.6 leading

$S = 3\ V_{ph}\ I_{ph}$

$300.10^6 = 3x\ 22.10^3\ I_{ph}\ /\sqrt{3}$ $\qquad I_{ph} = 7873A$

$\cos\varphi = 0.6$, so $\sin\varphi = 0.8$



$V_l = 22.10^3\ /\sqrt{3} = 12.7\ kV$

$I_{line} = I_{ph} =$ **7873A**

IX = 0.4 x 7873 = 3149 V

$E^2 = (12700 - 3149\cos\varphi)^2 + (3149\sin\varphi)^2 = (12700 - 3149 \times 0.6)^2 + (3149 \times 0.8)^2$

E =10.35 kV phase

$\sin(\delta) = IX\cos\varphi / E = 3149 \times 0.6/10350 = 0.182$ so $\boldsymbol{\delta = 10.5^0}$

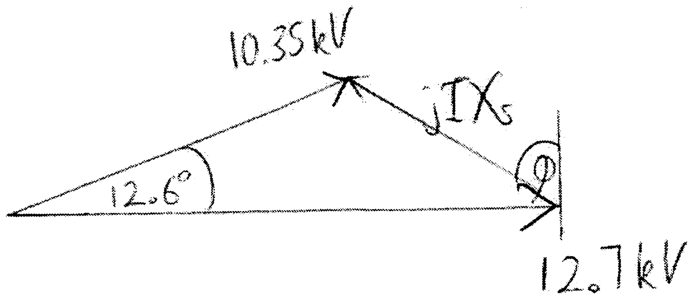
d/

$\mathbf{P_{new}} = 1.2\ P = 300.10^6 \times 0.6 \times 1.2 =$ **216 MW**

E = const

$$\frac{P_{new}}{P_{old}} = 1.2 = \frac{\sin\delta_{new}}{\sin\delta_{old}}$$

so, $\sin\delta_{new} = 1.2 \times 0.182$ $\qquad$ new $\delta = 12.6^0$



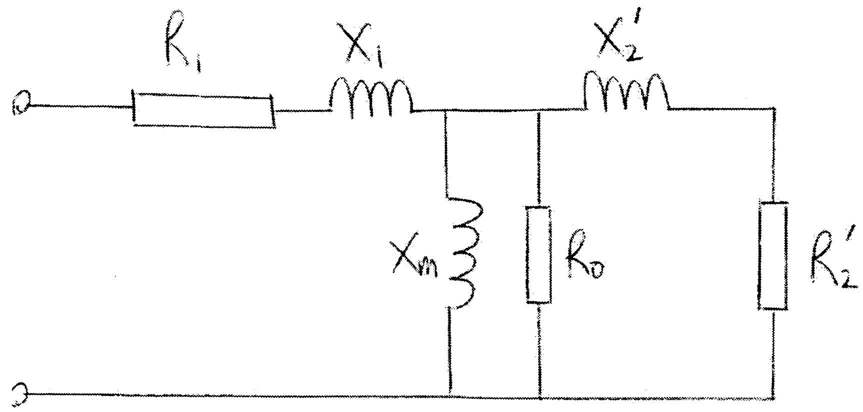
XI cosδ = 10.35 sin12.6 = 2.26 kV
XI sin δ = 12.7 - 10.35cos12.6 = 2.599 kV

tan φ = 2.599/2.26 = 1.15,   **Q** = P tanφ = **248 MVA**

4/

a/ $\omega_s = \omega/p$ p = number of pole pairs. ω = supply freq.

b/ Stationary induction motor has same form as a 3 phase transformer.



When rotor rotates at speed ω_r, define slip as $s = (\omega_s - \omega_r)/\omega_s$

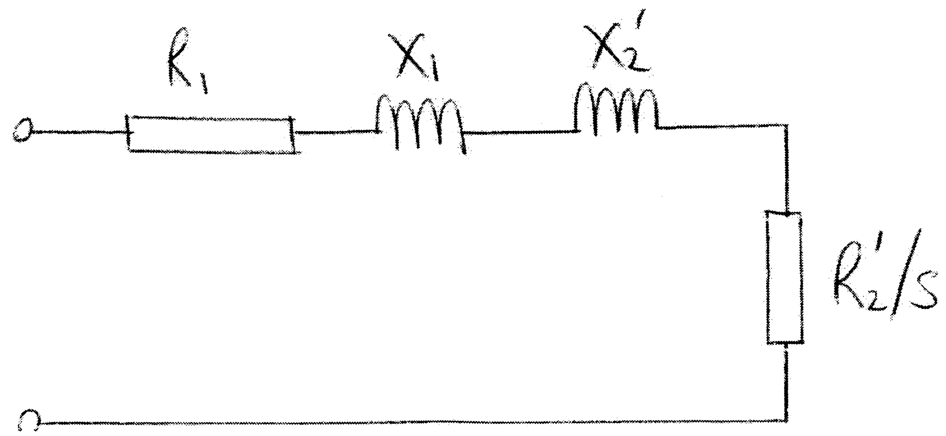
Rotor 'sees' field which induces an emf and currents at frequency $s\omega$

Hence $E_2 \rightarrow sE_2$, $X'_2 \rightarrow sX'_2$,

$sE_2 = (sX'_2 + R'_2)I'_2$

so $E_2 = (X'_2 + R'_2/s)I'_2$

so we modify the equivalent circuit by altering R'_2 to R'_2/s. We also miss out R_0 and X_m.



c/

$P_{in} = 3I^2 (R_1 + R'_2/s)$

$P_{in} = 3I^2 (R_1 + R'_2)$

$P_{out} = P_{in} - P_{loss} = 3I^2(R'_2/s - R'_2)$

$$I^2 = \frac{V^2}{(R_1 + R'_2/s)^2 + (X_1 + X'_2)^2}$$

$$P_{out} = \frac{3V^2}{(R_1 + R'_2/s)^2 + (X_1 + X'_2)^2} . R'_2 (\frac{1}{s} - 1) \qquad \text{QED}$$

d/ By max power transfer theorem, max power occurs when

$$R'_2(\frac{1}{s} - 1) = |R_1 + R'_2 + j(X_1 + X'_2)| = Z$$

$R'_2(1-s)/s = Z$

$R'_2 = s(Z+R'_2)$

$\mathbf{s = R'_2/(R'_2 + Z)}$ **QED**

e/
R1 = 5Ω, R'2 = 3.5Ω, X1 = 4Ω, X'2 = 8Ω

$Z = ((5 + 3.5)^2 + (4 + 8)^2)^{1/2} = 14.7\ \Omega$

s = 3.5/(3.5 + 14.7) = **0.192**

speed = 1500 (1-s) = 1500 (1 – 0.192) = **1212 rpm**

$$P = \frac{3.415^2}{(5 + \frac{3.5}{0.192})^2 + 12^2} x3.5(\frac{1}{0.192} - 1) = \frac{7.61x10^6}{683.5} = \mathbf{11.13\ kW}$$

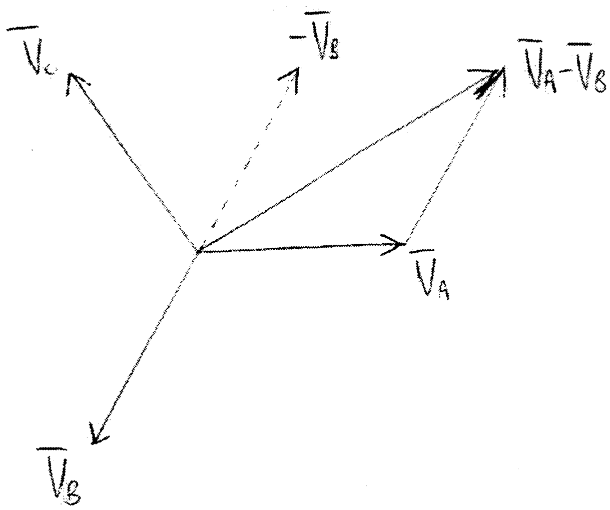
5/a/

1.Makes better utilisation of generator, eg $S_{max} = 3V_{max}I_{max}$ for 3 phase,
but = 2VI for 1-phase

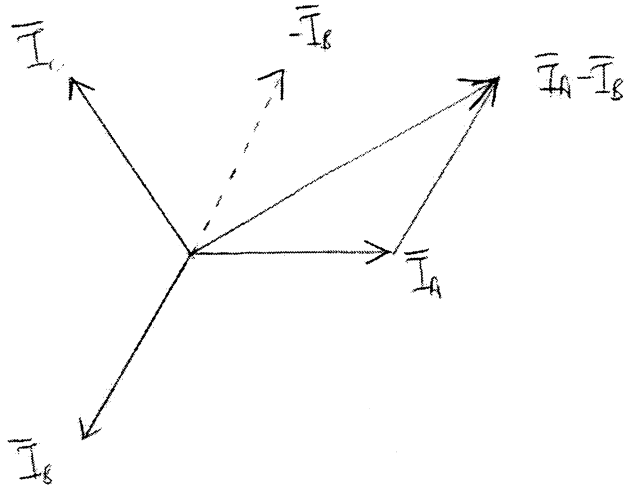
2.Generators are more efficient

3.Higher number of phases requires more volts, for diminishing returns.

b/



$V_{line} = V_1 - V_2 = |V_l| = 2V_1\cos 30 = \sqrt{3}.V_1$ in magnitude.



$I_{line} = I_1 - I_2 = \sqrt{3}I_1$ in magnitude

c/ delta load 40 + j30
star load 15 + j20

delta

$I_{ph} = 415/(40+j30) = 8.3A$

$P = 3I^2R =$ **8267 W**

$Q = 3I^2X = 6700$ VAR

$\cos \phi =$ **0.8** lagging

star load

$V_{ph} = 415/\sqrt{3} = 239.6$ V

$I_{ph} = V_{ph}/(15+j20) = 9.58$ A

$P = 3I^2R =$ **4130 W**

$Q = 3I^2X = 5507$ VAR

$\text{Cos}\ \varphi =$ **0.6 lagging**

Ptotal = P1 + P2 = **12397 W**

Qtotal = Q1 + Q2 = 11707 VAR

Stotal = 17051 VAR

$\text{Cos}\ \varphi = P/S =$ **0.727 lag**

d/ $S = \sqrt{3}\ VI$, $I_l = 23.7A$

power loss in lines = $3I_l^2\ R = 3\ (23.7)^2R =$ **675 W**

If PF is corrected to unity, S = P = 12397 W

$I_l = 12397\ /\ (\sqrt{3}.415) = 17.25A$

P 3. $17.25^2\ 0.4 =$ **357 W**

6/ a/

The current flows along the inner conductor and back down the outer conductor.

The magnetic field is the same as that due to a long wire, and at a radius r is $H = \frac{I}{2\pi r}$

The flux linked is

$$\psi = \int \mu_0 H.dr = \int_a^b \mu_0 \frac{I}{2\pi r} dr = \frac{\mu_0 I}{2\pi} \ln(\frac{b}{a})$$

Thus the inductance per unit length is ψ/I or

$$L = \frac{\mu_0}{2\pi} \ln(\frac{b}{a})$$

b/

$Z = \sqrt{\frac{L}{C}} = 50$ ohms, $v = \frac{1}{\sqrt{LC}} = 2.10^8$ m/s.

Hence $C = 1/(Zv) = 1/(50.2.10^8) = \mathbf{10^{-10}}$ farads/m

$L = Z/v = 50/2.10^8 = \mathbf{2.5 \times 10^{-7}}$ H/m

$\mu_0 = 4\pi.10^{-7}$ H/m

So $\ln(b/a) = L2\pi/\mu_0 = 2.5\text{x}10^{-7}.2\pi/4\pi.10^{-7} = 2.5/2 = 1.25$

$b/a = \mathbf{3.49}$

c/

source $Z_s = 10$ ohms, line $Z_0 = 50$ ohms, load $Z_2 = 100$ ohms

voltage transmission coef into the line = $\frac{Z_0}{Z_0 + Z_s} = \frac{50}{10+50} = 0.833$

power transmission coef = 0.833^2 **= 0.694**

power reflection coef at load end, R1 = $\left(\frac{Z2-Z0}{Z2+Z0}\right)^2 = \left(\frac{100-50}{100+50}\right)^2 = 0.111$

power reflection coef2 at source end, R2 = $\left(\frac{Zs-Z0}{Zs+Z0}\right)^2 = \left(\frac{10-50}{10+50}\right)^2 = 0.445$

So, Initial value inside line = 0.694

After R1	V =0.077
After R2	V = 0.0343
After R3	V = 0.00381
After R4	V = 0.00169
R5	V = 0.0001915

Hence only after 5 reflections is power below 0.1%.

7/a/

Radial electric field around a point charge $E = \dfrac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}$

Electric flux density

$D = \varepsilon_0\varepsilon_r E = \dfrac{Q}{4\pi r^2}$

Integral of D across a spherical surface at radius r is given by

$\oint D.ds = \int 4\pi r \dfrac{Q}{4\pi r^2}.dr = Q$

Thus the surface integral of D equals the charge enclosed, which is a statement of Gauss' Law in integral form.

b/ the wave must be transverse, with E normal to direction of propagation.

So $E_y = E_0 \exp(j(wt - \beta.x))$ is correct.

c/

The Maxwell eqn for free space is $\dfrac{\partial D}{\partial t} = \dfrac{\partial E_y}{\partial x}$

$\partial E_y/\partial x = -j\beta E_0 \exp(j(\omega t - \beta.x)) = \partial D/\partial t$

So integrating,

$D_z = (-j\beta/jw) E_0 \exp(j(\omega t - \beta.x)) = -(E_0/c) \exp(j(\omega t - \beta.x))$

And

$H_z = -(E_0/c\mu_0) \exp(j(\omega t - \beta.x))$

$c = 1/\sqrt{(\mu_0\varepsilon_0)}$, so this gives $\mathbf{H_z = -E_0/Z \exp(j (\omega t - \beta.x))}$ where $\mathbf{Z = \sqrt{(\mu_0/\varepsilon_0)}}$

The power flux is given by the Poynting vector

$P = E\wedge H$, so $\mathbf{P_x = E_0^2/Z}$ **in x direction**

d/

4kW is radiated over an area of $4\pi r^2$ at a distance r. It is intercepted over an area of 0.1 m^2.

So,

$10^{-9} = 4000 \text{ x } 0.1/4\pi r^2$

$r = 4000 \text{ x } 0.1 /4\pi.10^{-9} = 10^{11}/\pi = 1.8.\mathbf{10^5\ m}$.

180km is about right.