ENGINEERING TRIPOS PART IB

Monday 6 June 2005 9 to 11

Paper 1

MECHANICS

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments to this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

1 A slender brickwork chimney of length L is shown in Fig. 1(a). It has uniform cross section and its total mass is m. It is to be demolished by firing a small explosive charge at one side of the base O so that the chimney begins to fall over. Initially, the chimney rotates about the fixed point O as a rigid body. After the chimney has rotated through an angle θ, as shown in Fig. 1(b), the bending moment becomes too large at some point along the chimney and it begins to break in two.

For the initial rigid-body motion of the chimney, find in terms of m, g, L and θ:

(a) the angular velocity of the chimney; [4]

(b) the angular acceleration of the chimney; [8]

(c) the maximum bending moment in the chimney and show that this maximum bending moment occurs at distance $L/3$ from O. [8]

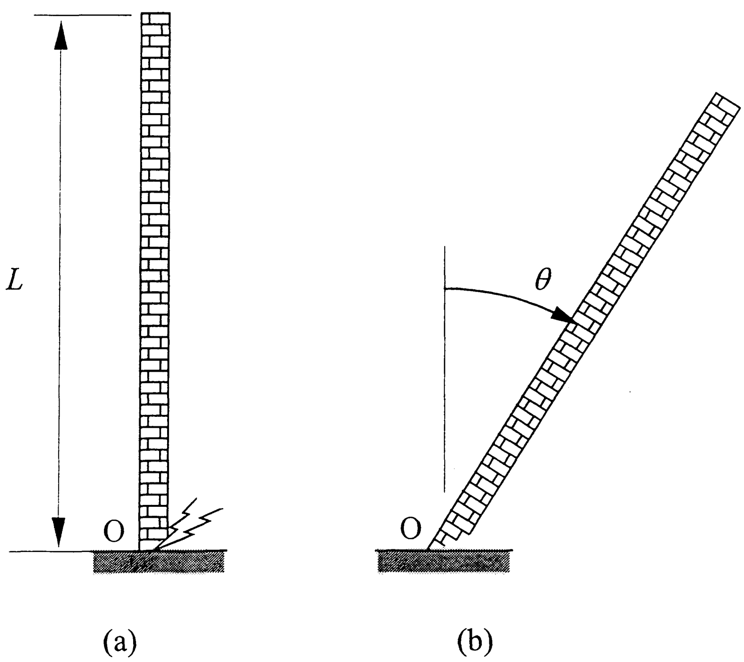

Fig. 1

2 A uniform circular cylinder of radius R and mass m, whose centre is at O, has a cylindrical hole cut through it to form the rocker shown in Fig. 2. The radius of the hole is $R/2$ and its centre is at a distance $R/2$ from O. The rocker therefore has mass $3m/4$ and its centre of mass is at G. The rocker rolls without slip on a smooth horizontal table and its displacement from equilibrium is described by the angle θ between OG and the vertical.

(a) Find the distance OG and the moment of inertia of the rocker about G. [6]

(b) Obtain expressions for the potential energy and kinetic energy of the rocker as functions of θ and the angular velocity $\dot{\theta}$. [8]

(c) Determine the natural frequency of small oscillations. [6]

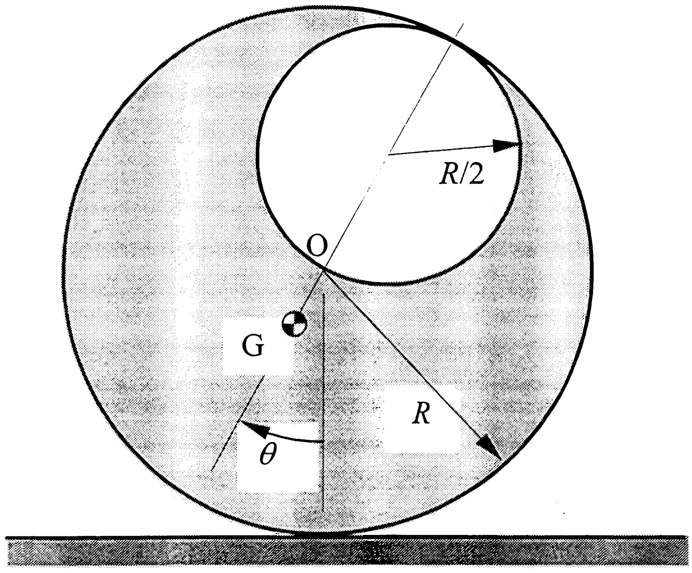

Fig. 2

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3 Two uniform rods AB and BC, each of mass m and length $2L$, hang vertically under the influence of gravity from a fixed pivot A as shown in Fig. 3. They are freely hinged at A and B so that they can swing in a vertical plane. A horizontal impulse P, acting in this plane, is delivered to the free end at C. Let v_1 and v_2 denote the subsequent velocities of the centres of rods AB and BC respectively and let the corresponding angular velocities be ω_1 and ω_2.

(a) Express ω_1 and ω_2 in terms of v_1 and v_2. [4]

(b) Find an expression for the impulsive reaction at A. [8]

(c) Find the velocities v_1 and v_2. [8]

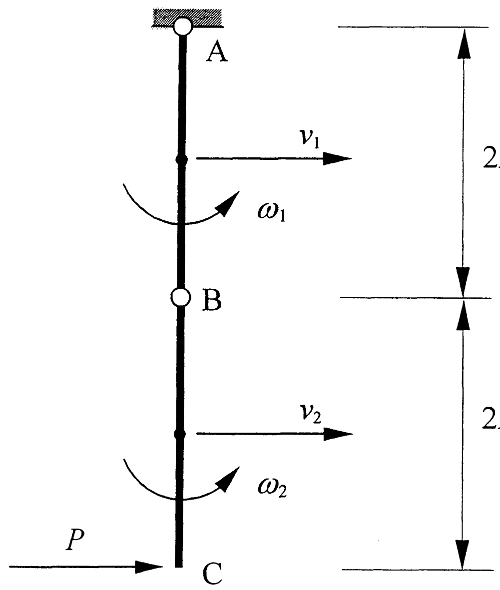

Fig. 3

SECTION B

4 Figure 4 shows two flywheels with moments of inertia 3J and J connected by a friction clutch C and a gearbox with a 2:1 gear ratio. The shafts and gears are supported in frictionless bearings and can be considered to possess negligible inertia. Initially the clutch is disconnected and the smaller flywheel is at rest. At a moment when the larger flywheel is rotating with angular velocity Ω the clutch is engaged.

(a) What are the angular velocities of the two flywheels after all slipping in the clutch has ceased? [8]

(b) During the period of slip the clutch transmits a constant torque T. Find the time that elapses before slipping ceases. [6]

(c) How much energy is dissipated by the clutch? [6]

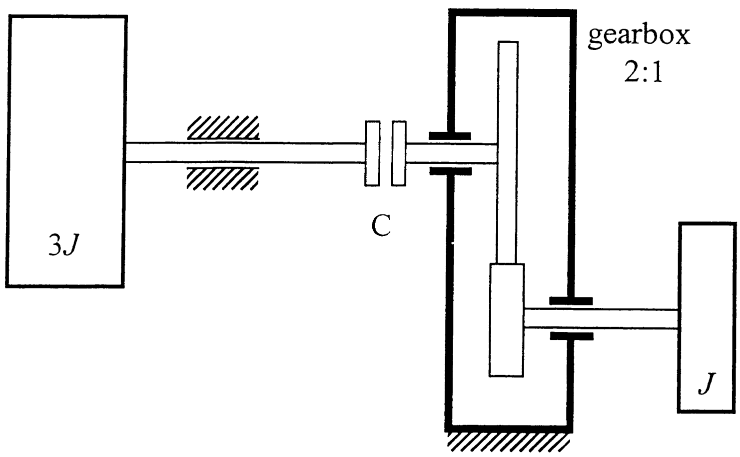

Fig. 4

5 Figure 5 shows a thin uniform circular disc of mass m and radius a which spins freely on a shaft CG also of length a. The centre of mass of the disc is at G. The shaft is fixed at C to a light string OC of length b. The string is fixed to the ceiling at O. The disc is spinning at a constant rate ω clockwise as viewed from C to G.

In steady-state precession the plane OCG is vertical and rotates at a steady rate Ω about a vertical axis through O. The shaft CG is horizontal and the angle θ between OC and the vertical axis is constant.

(a) Draw a free-body diagram of the shaft and rotor and hence:

(i) find the tension in the string in terms of m, g and θ; [5]

(ii) show that the string angle $\theta \approx \dfrac{a\Omega^2}{g - b\Omega^2}$ for small θ. [5]

(b) Find the couple acting on the rotor and hence find an expression for the precession rate Ω in terms of g, a and ω. [6]

(c) Viewed from above, is the precession clockwise or anticlockwise? Outline your reasoning with reference to the laws of mechanics. [4]

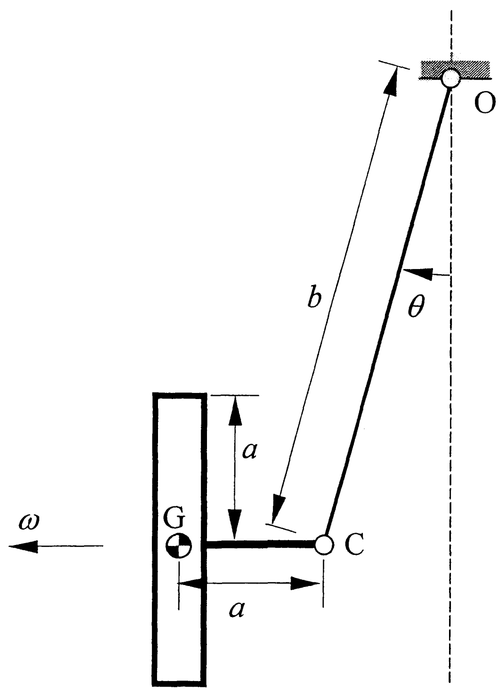

Fig. 5

6 The mechanism shown in Fig. 6 comprises three rigid links AB, AC and DE, and two sliders, one at B and another at C. The slider at C slides on link DE. Link AC is driven at constant angular velocity ω about a fixed point O midway between A and C as shown. Links AB and AC are of lengths $2\sqrt{2}a$ and $2a$ respectively. At the instant shown all angles between links are multiples of $45°$ and the distance DC is $2\sqrt{2}a$.

(a) By means of instantaneous centres or velocity diagram find:

(i) the angular velocities of links AB and DE; [3]

(ii) the sliding velocities at B and at C. [3]

(b) By means of an acceleration diagram or otherwise find:

(i) the acceleration of the slider at B; [6]

(ii) the angular accelerations of links AB and DE. [8]

Notes for both parts (a) and (b):
marks will be deducted unless both magnitude *and* direction are given in your answers;
the recommended scale for velocities is $a\omega = 5$cm and for accelerations is $a\omega^2 = 5$cm.

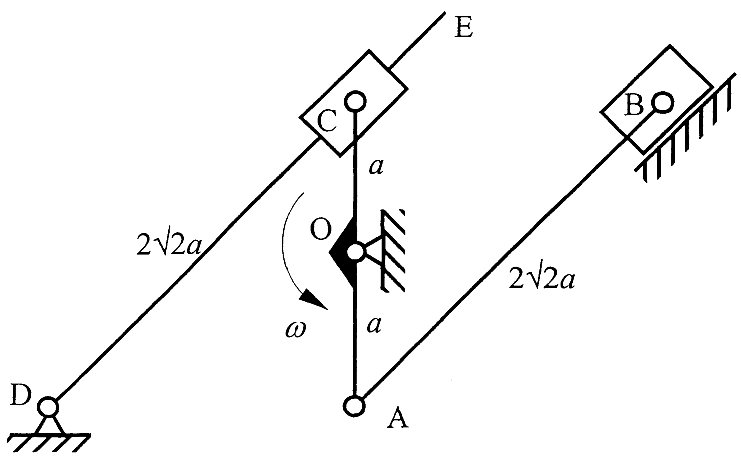

Fig. 6

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