ENGINEERING TRIPOS PART IB

Monday 6 June 2005 2 to 4

Paper 2

STRUCTURES

Answer not more than ***four*** *questions, which may be taken from either section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments to this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

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SECTION A

1 Figure 1 shows a pin-jointed structure, where only node D is not pinned to a foundation. The structure when unloaded is unstressed. All bars have a cross-sectional area A, are made from material with a Young's Modulus E, and may be considered to be light.

(a) A vertical load W is applied to the structure at D.

(i) Find a particular equilibrium solution for the forces in the bars. [3]

(ii) Calculate the single state of self-stress in the structure. [3]

(iii) Find the elastic solution for the forces in bars due to the applied load. [4]

(b) With the load W still applied, the temperature of bar AD alone is increased by T. It has a coefficient of thermal expansion α.

(i) Find the forces carried by each of the bars. [3]

(ii) By Virtual Work, or otherwise, find the total displacement of the free node from its original position due to the combined effects of loading and heating. [7]

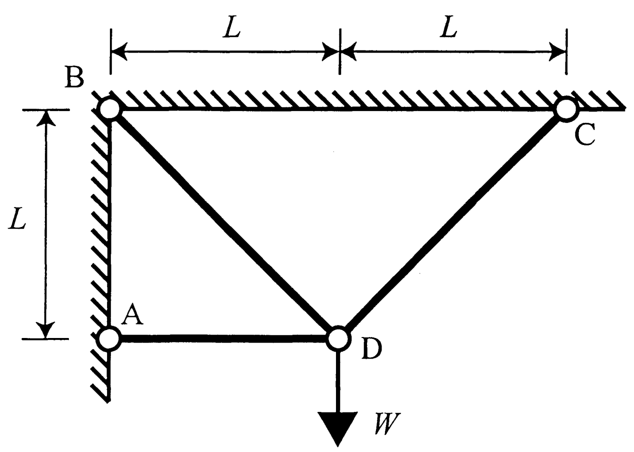

Fig. 1

2 Figure 2(a) shows a light curved cantilever taking the form of one quarter of a circle of radius R. It has a bending stiffness EI, and is axially rigid. A horizontal load H and a vertical load V are applied at the tip.

(a) Find the bending moment distribution in the cantilever as a function of θ. [4]

(b) By Virtual Work, or otherwise, show that the horizontal deflection at the tip of the cantilever is $\frac{R^3}{EI}(0.356H + 0.5V)$ and find the vertical deflection of the tip due to the applied loads. [8]

$$\left[\text{Note: } \int_0^{\pi/2} (1-\cos\theta)^2 \, d\theta = \frac{3\pi - 8}{4}, \quad \int_0^{\pi/2} \sin^2\theta \, d\theta = \frac{\pi}{4}, \quad \int_0^{\pi/2} \sin\theta(1-\cos\theta) \, d\theta = \frac{1}{2} \right]$$

(c) Figure 2(b) shows a light circular arch of radius R, built-in at both ends, with a central pin. It has a bending stiffness EI, and is axially rigid. Calculate the deflection at the centre of the arch due to an applied load W as shown. [8]

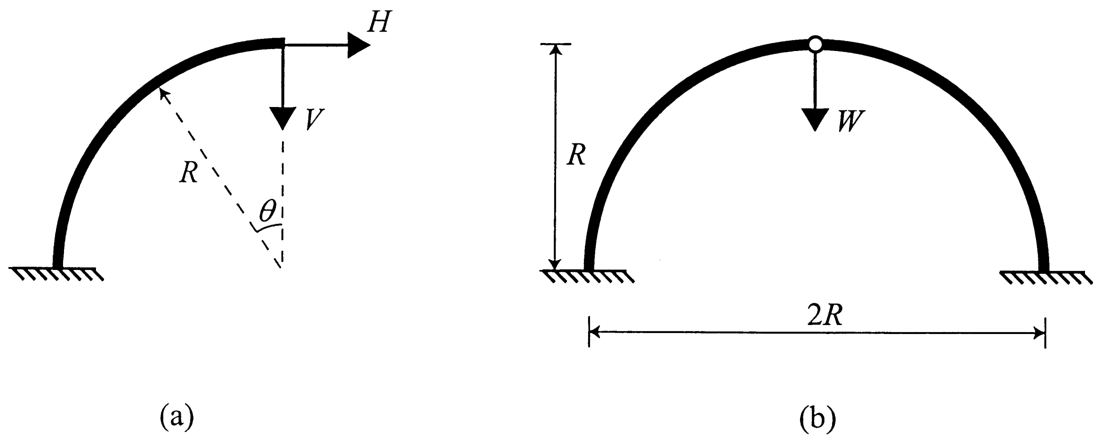

Fig. 2

3 Figure 3 shows a cross-section through the steel deck of a footbridge. The two flanges have a width of 1800 mm, have thickness 10 mm, and are each reinforced with eight 50 mm stiffeners, also of thickness 10 mm. The two webs have a height of 500 mm and have a thickness 15 mm.

(a) Show that the cross-sectional area of *each* flange, with its stiffeners, is equal to that of a rectangle 1800×12.2 mm. Give two reasons why the bridge is constructed as shown in the figure, rather than using unstiffened flanges of thickness 12.2 mm. [4]

(b) Due to vertical loads the bridge is subjected to a peak bending moment of 1000 kNm, and a peak shear force of 400 kN. Calculate the magnitude and position of the peak longitudinal normal stress, and peak shear stress on the cross-section, due to these applied loads. [8]

(c) Due to an off-centre load, the bridge deck carries a torque of 200 kNm. Calculate:

(i) the position and magnitude of the peak shear stress on the cross-section due to this torque alone; [4]

(ii) the twist per unit length due to this torque alone. [4]

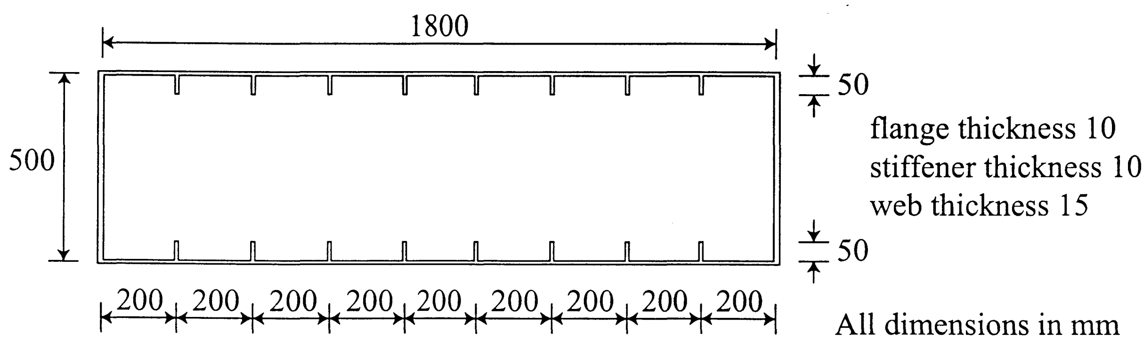

Fig. 3

SECTION B

4 A steel Universal Beam $610 \times 229 \times 125$ is built-in at both ends and has a span of 9 m, as shown in Fig. 4. The web of the beam is vertical. It is subjected to loads of 500λ kN and 1000λ kN at two points, as shown. The beam is unstressed when unloaded. Ignore the self-weight of the beam in all calculations.

(a) Calculate the fully-plastic moment capacity of the beam if the yield stress of the steel is 300 MPa. [4]

(b) Without carrying out an elastic analysis, draw two *different* valid bending moment distributions, and show that each satisfies the equations of equilibrium. [4]

(c) For each of the equilibrium systems considered in (b), calculate the value of the load factor λ which would cause the bending moment to exceed the fully-plastic capacity somewhere in the beam. [4]

(d) Which of the two load factors found in (c) is closer to the collapse load factor of the beam? Have you found an upper bound or a lower bound on the true collapse load of the beam? Quote the relevant bound theorem. [5]

(e) Without doing further calculations, explain how your answer would have altered if the supports had been misaligned, so that the structure *was* stressed when unloaded? [3]

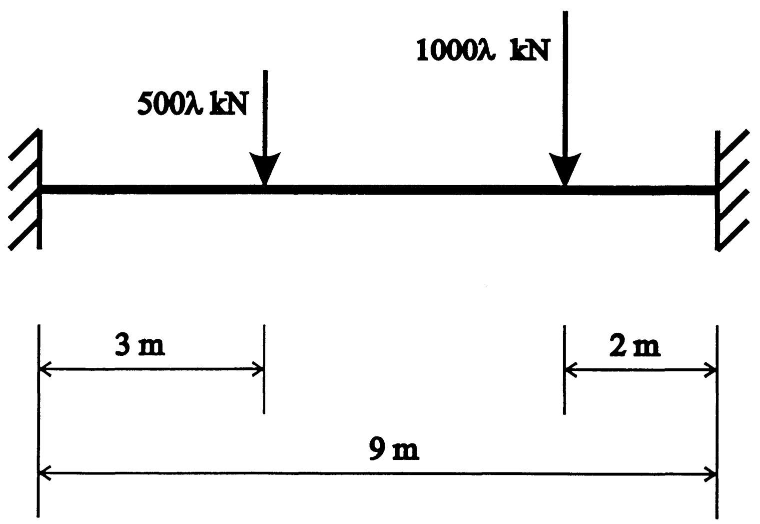

Fig. 4

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5 A beam is built-in at both ends and subjected to a distributed load of intensity 300λ kN/m over part of its length, as shown in Fig. 5. The beam has a fully-plastic moment capacity of 200 kNm. The beam is unstressed when unloaded. Ignore the self-weight of the beam in all calculations.

(a) Consider a collapse mechanism which has hinges at the end supports and another hinge at a distance x from the left-hand support ($x > 4$ m). Find an expression for the collapse load factor λ as a function of x. [8]

(b) Using the result of (a), obtain your best estimate of the collapse load of the beam. Have you found an upper bound or a lower bound on the true collapse load of the beam? Quote the relevant bound theorem. [9]

(c) Without doing further calculations, explain how your answer would have altered if the supports had been misaligned, so that the structure *was* stressed when unloaded? [3]

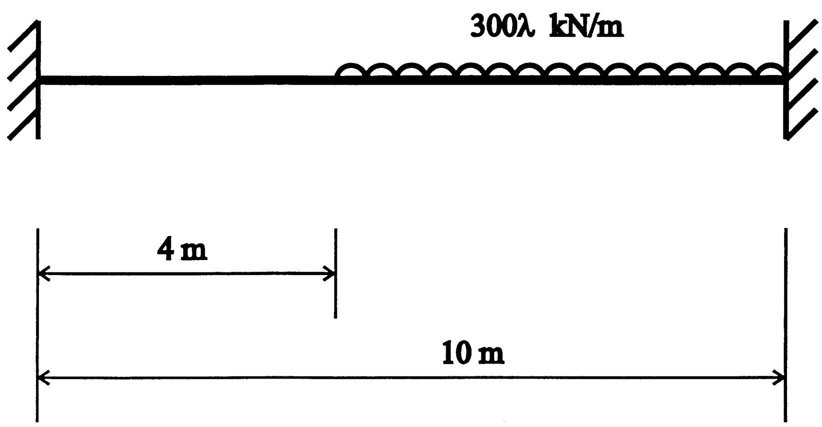

Fig. 5

6 An element of a thin steel plate is loaded by stresses shown in Fig. 6. It is fitted with a strain gauge rosette (PQR). Gauge Q is at 45° to P and R.

(a) By drawing a Mohr's circle of stress, or otherwise, determine the principal *stresses* and their orientation. [5]

(b) If the yield stress in tension of the material is 300 MPa, calculate the factor by which the stresses could be increased if the material obeys

(i) the Tresca yield condition;

(ii) the von Mises yield condition. [5]

(c) Calculate the strains in gauges P and R and the shear strain γ_{xy}. Hence draw a Mohr's circle of strain. Use this to determine

(i) the strain in gauge Q ;

(ii) the magnitude and direction of the principal *strains*. [7]

(d) Comment on the relationship between the orientation of the principal stresses and that of the principal strains. [3]

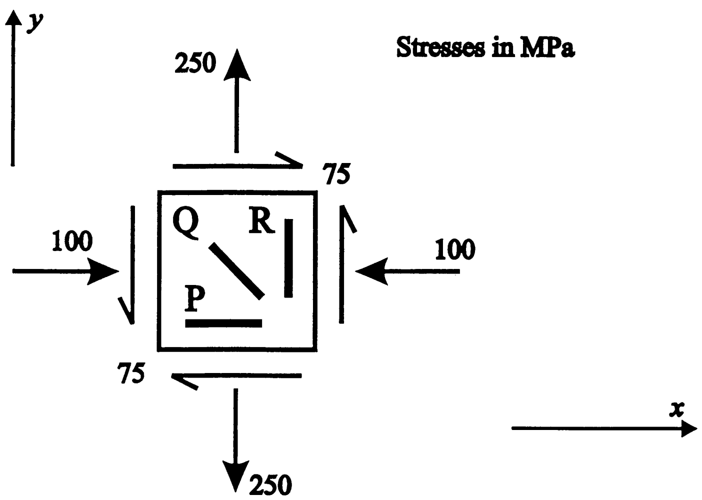

Fig. 6

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