ENGINEERING TRIPOS PART IB

Thursday 9 June 2005 2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than ***four*** *questions.*

Answer at least ***one*** *question from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Additional copy of Fig. 3
Additional copy of Fig. 5

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

*Answer at least **one** question from this section.*

1 (a) Define asymptotic stability for a system with impulse response $g(t)$. Given a linear ordinary differential equation that describes the behaviour of a particular system, explain how you would decide if the system is asymptotically stable, marginally stable or unstable. [4]

(b) Consider a system with impulse response $g(t)$ as shown in Fig. 1(a).

(i) Find the system's transfer function $G(s)$.

(ii) Is the system asymptotically stable, marginally stable or unstable? [6]

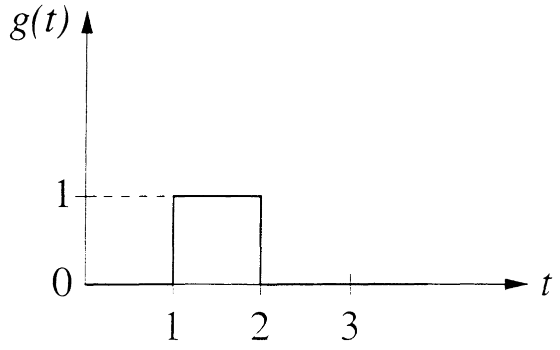
(c) Calculate the steady-state response of the system with transfer function $G(s)$, as in Part (b), to the following inputs:

(i) $x(t) = u(t)$ (unit step function)

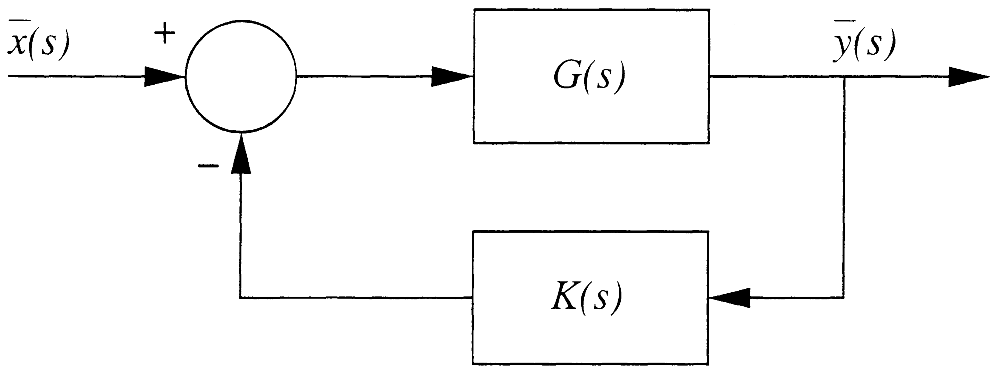
(ii) $x(t) = \sin \pi t$

(iii) $x(t) = \sin \pi t$ if $0 \leq t \leq 1$ and $x(t) = 0$ otherwise. [6]

(d) Consider the feedback system shown in Fig. 1(b). If $K(s) = 2/s$ and $G(s)$ is the function calculated in Part (b), sketch the impulse response of the return ratio $K(s)G(s)$. Calculate the steady-state response of the closed-loop system to a unit step input. [4]



(a)



(b)

Fig. 1

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2 (a) Consider the feedback system shown in Fig. 2. How might the Bode diagram of $G(s)$ be determined experimentally? [4]

(b) The Bode diagram of an asymptotically stable $G(s)$ is shown in Fig. 3.

(i) Is the closed-loop system stable if $K = 1$? Estimate the phase margin and comment on the result.

(ii) Given that

$$G(s) = \frac{k(s+\alpha)}{s^n(s^2+\beta s+\gamma)}$$

estimate the parameters k , n , α , β and γ from Fig. 3. [7]

(c) For $G(s)$ as in Part (b), and $K(s)$ given by the equation

$$K(s) = \frac{s+1}{5(s+A)}$$

find A so that the modulus of $K(j\omega)G(j\omega)$ is 1 at the frequency $\omega = 2$ rad/s. [2]

(d) On the additional copy of Fig. 3 supplied, sketch the Bode diagram of $K(j\omega)G(j\omega)$ with K as in Part (c). Estimate the gain margin and the phase margin of the closed-loop system, and comment on the likely difference in performance of the feedback systems of Parts b(i) and (c). [7]

The additional copy of Fig. 3 should be handed in.

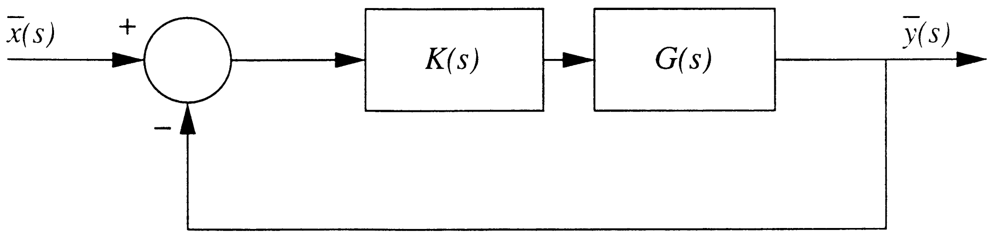

Fig. 2

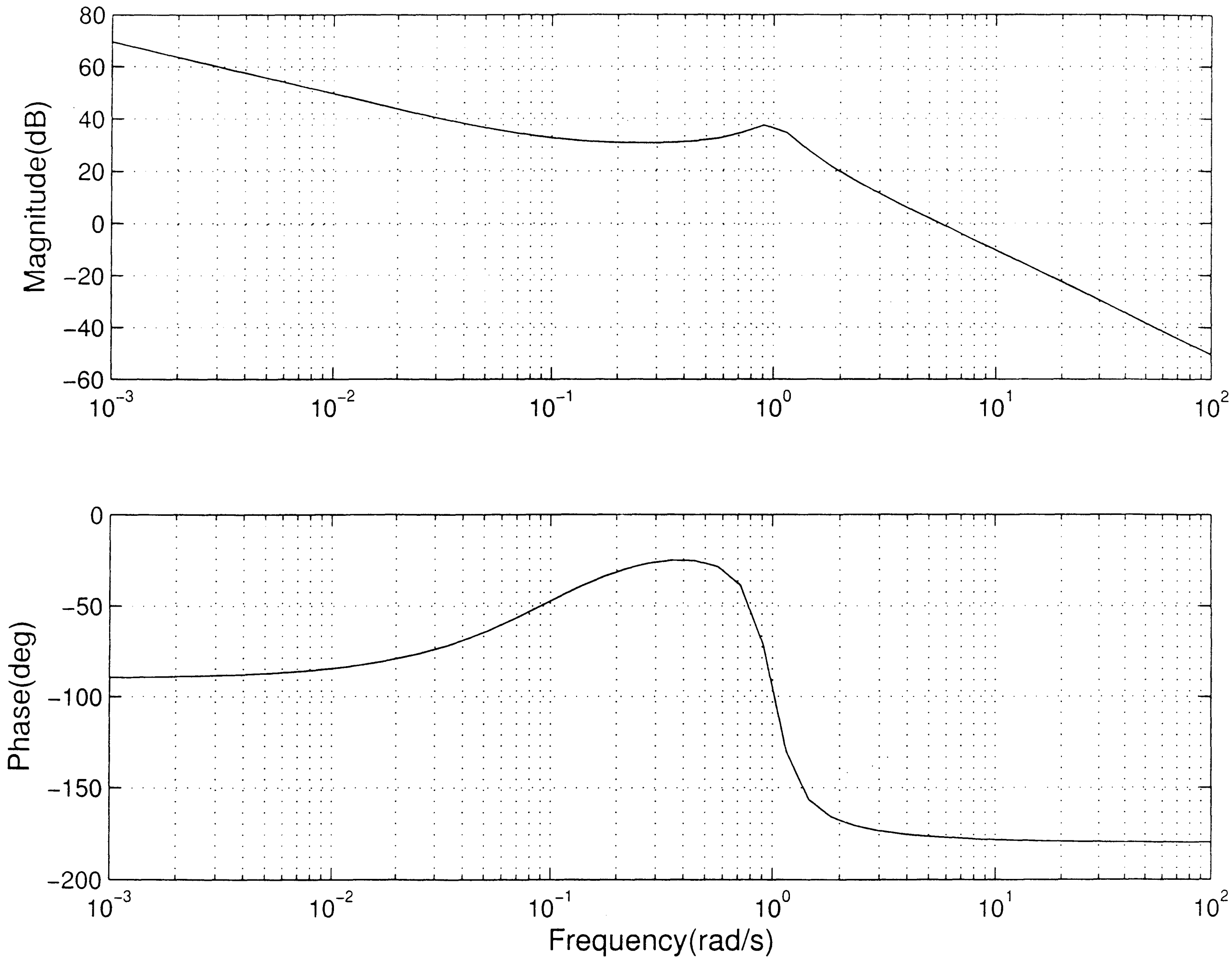

Fig. 3

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3 (a) Consider the feedback system shown in Fig. 4. Given the Nyquist diagram of $G(s)$, itself assumed to be asymptotically stable, and given an expression for $K(s)$, how could you determine if the closed-loop system is asymptotically stable? [4]

(b) The Nyquist diagram for an asymptotically stable system with transfer function $G(s)$ is given in Fig. 5.

(i) If $K = 1$, is the closed-loop system asymptotically stable or not?

(ii) For what range of K is the closed-loop system stable?

(iii) Calculate the gain margin and the phase margin if $K = 0.3$. [6]

(c) Now let $K(s) = 0.1s + 0.3$. Sketch the appropriate Nyquist diagram on the additional copy of Fig. 5 provided and estimate the new gain margin. [6]

(d) Calculate the maximum magnitude of $1/[1+K(j\omega)G(j\omega)]$ for the two choices $K(s) = 0.3$ and $K(s) = 0.1s + 0.3$, and comment on the difference between the two feedback systems. [4]

The additional copy of Fig. 5 should be handed in.

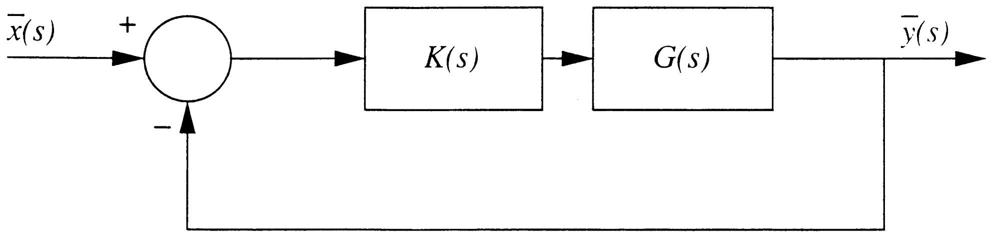

Fig. 4

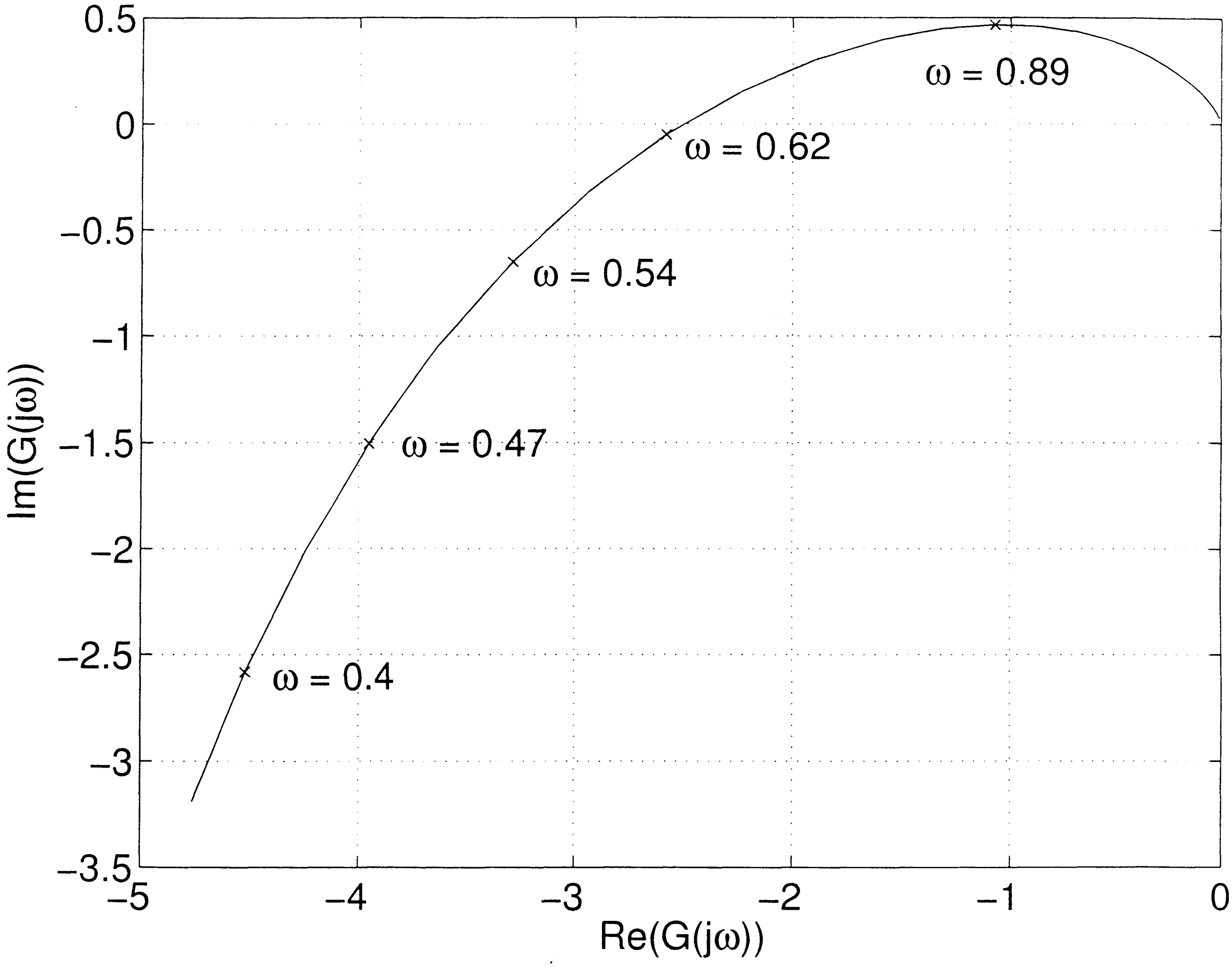

Fig. 5

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4 A system for electric traction motor control includes a difference amplifier with gain G_1 , an armature-controlled motor and a tachometer.

(a) First consider the armature-controlled motor alone. The armature current is given by the equation

$$\bar{I}_a(s) = \frac{1}{R + Ls}(\bar{V}_a(s) - \bar{V}_b(s))$$

where V_a is the applied voltage, R and L are the armature resistance and inductance respectively and the back emf V_b is proportional to the motor angular velocity $V_b = K_b\omega$.

The torque generated by the motor is given by $T_m = K_m I_a$, and the load is characterised by inertia and friction, so that to a first approximation

$$J\dot{\omega} + b\omega = T_m - T_d$$

where J and b are constants and the disturbance torque T_d can vary with time.

Draw the block diagram of the armature-controlled motor, and obtain expressions for the transfer functions from $\bar{V}_a$ to $\bar{\omega}$ and from $\bar{T}_d$ to $\bar{\omega}$. [10]

(b) In the complete system, the reference voltage V_a for the motor of Part (a) is supplied by a difference amplifier. The motor angular velocity is ω and the tachometer generates a voltage $K_t\omega$. This voltage is fed to the inverting input of the difference amplifier. The non-inverting input is fed a voltage which is proportional to the desired angular velocity ω_d , so that

$$V_a = G_1 K_t(\omega_d - \omega)$$

Draw a block diagram of the electric traction motor control system and find an expression for the transfer function from $\bar{\omega}_d$ to $\bar{\omega}$. [6]

(c) Given $G_1 = 10$, $J = 2$, $b = 0.5$, $K_m = 10$, $K_b = 0.1$, $K_t = 0.1$, $R = 1$ and $L = 1$, calculate the natural frequency of oscillation and the damping ratio for the systems described in Parts (a) and (b), and comment on the effect of the controller added in Part (b). [4]

SECTION B

Answer at least ***one*** *question from this section*

5 (a) Explain what is meant by aliasing and quantising distortions in analogue to digital conversion. [4]

(b) Consider an n-bit ADC (with range -1V to 1V) and an input signal of

$$v(t) = 0.8 \sin \omega t \text{ volts}$$

Modelling the quantisation error as noise with a uniform probability distribution function, show that the RMS error noise voltage is $\delta v / \sqrt{12}$, where δv is the quantiser step size. Hence calculate the signal to noise (S/N) voltage ratio for this ADC and show that $n = 4$ ensures that the S/N ratio is greater than 23 dB. [6]

(c) The signal in Part (b) is ideally sampled with a periodic train of impulses having a unit mean value and a period $T = 1/400$s. Given that $n = 4$ and $\omega = 100\pi$, calculate the binary values of the sampled signal $v(kT)$ over one period of the input signal, where k is the sampling index. Assume that the ADC outputs integer code values from 0 to $(2^4 - 1)$. Sketch the waveform of the most significant bit of the binary code. [6]

(d) (i) Estimate the maximum signal bandwidth that can be digitised without excessive aliasing distortion if the low-pass filter preceeding the ADC has a transition band that is 40% of its passband bandwidth.

(ii) How might the maximum signal bandwidth of Part d(i) be raised, while maintaining a constant sampling period? What would be the disadvantages of your proposed method? [4]

6 The UK standard for analogue TV is 625 lines/frame, 25 frame/s, aspect ratio 4:3.

(a) Describe the interlaced scanning method and discuss the advantages and the extra requirements introduced. [4]

(b) Calculate the approximate pixel rate, stating clearly what assumptions are made in the calculation. [4]

(c) An image consists of one vertical line of width one pixel. Evaluate the Fourier series of the signal and hence estimate the 3 dB bandwidth necessary for transmission. [4]

Note: $\mathrm{sinc}(1.392) = 0.707$

(d) Consider another image that consists of two vertical lines, each one pixel wide, situated at 100 pixels from each other. By modelling the signal as the sum of two pulse trains and considering its Fourier expansion, discuss how the addition of a second line to the image in Part (c) affects the bandwidth of the transmission. [8]

END OF PAPER

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Candidate number:

Extra copy of Fig. 3 which should be annotated and handed in with your answer to Question 2.

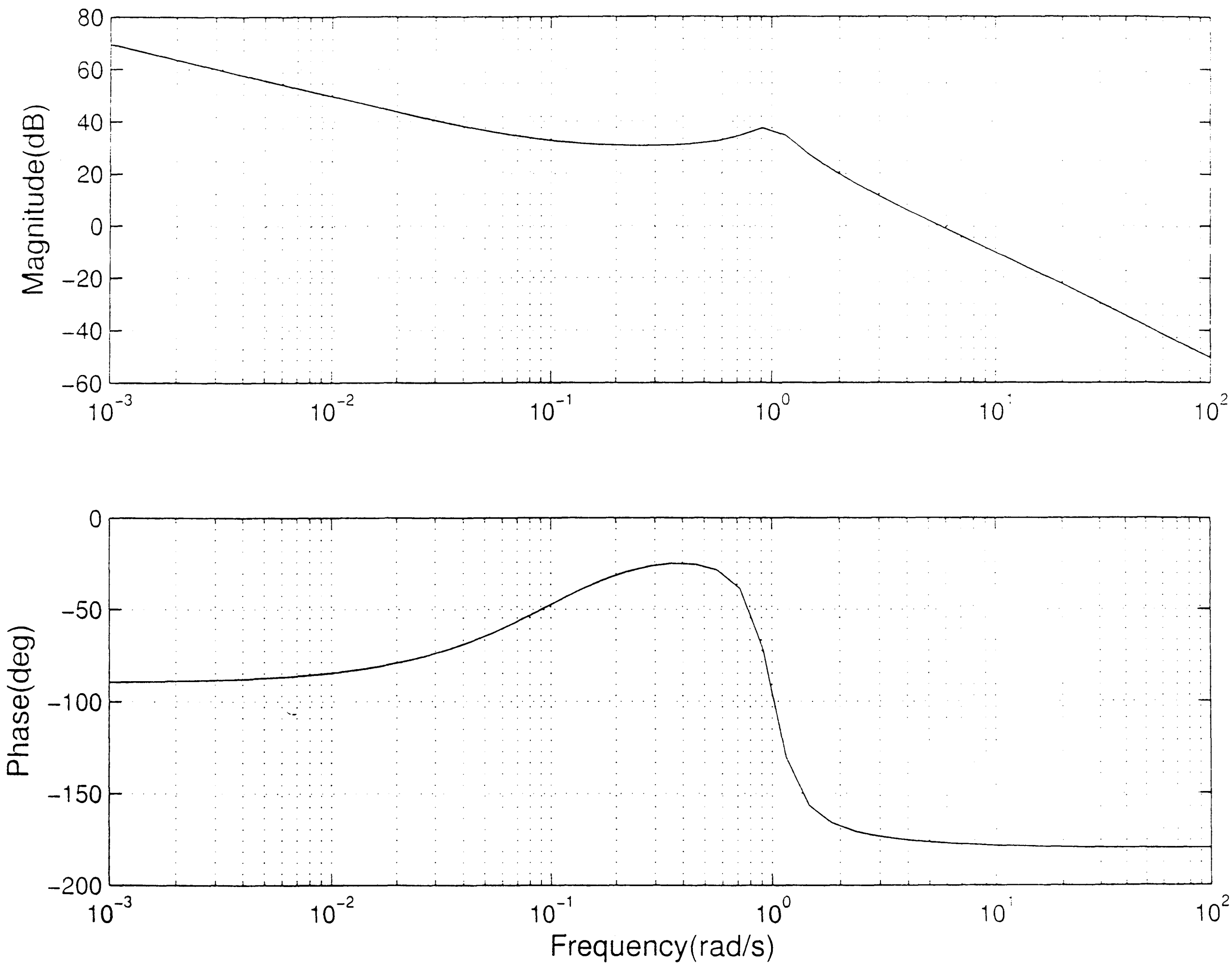

Fig. 3

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Candidate number:

Extra copy of Fig. 5 which should be annotated and handed in with your answer to Question 3.

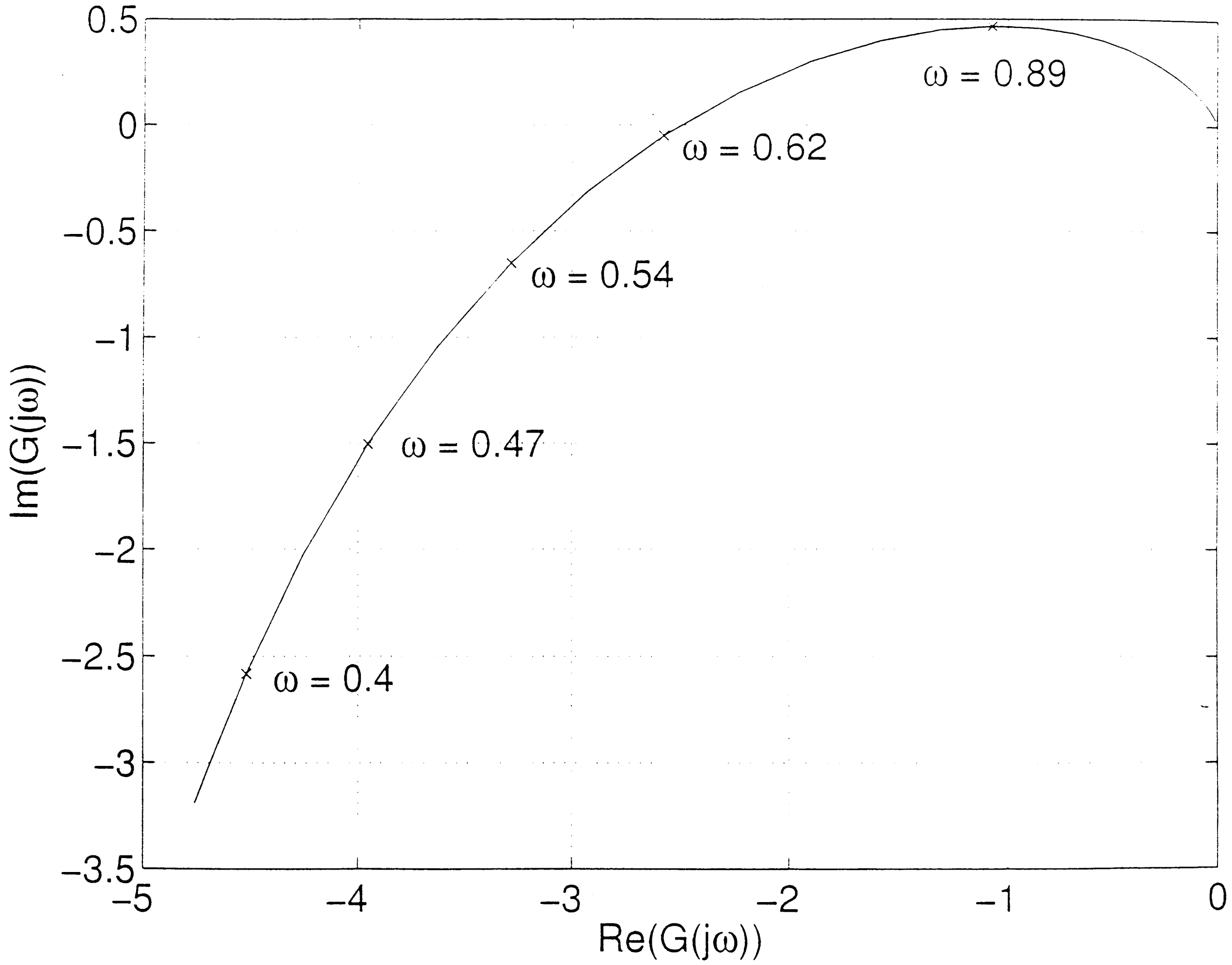

Fig. 5