ENGINEERING TRIPOS PART IB

Friday 10 June 2005 2 to 4.30

2 to 3.30 Foreign Language Option

Paper 8

SELECTED TOPICS

Answer **one** *question from Section A. In addition:*

> *If you are not taking the Foreign Language option, answer* **four** *questions, taken from only* **two** *of Sections B – H. Not more than* **two** *questions from each section may be answered.*
> *If you are taking the Foreign Language option, answer* **two** *questions from* **one** *of Sections B – H.*

All questions carry the same number of marks.

The **approximate** *number of marks allocated to each Part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Data Sheet for Section B (6 pages)
Data Sheet for Section E (2 pages)

> You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A *Introductory Business Economics*
*Answer not more than **one** question from this section*

1 (a) Describe the model of perfect competition. [6]

(b) Why is perfect competition so rare? [4]

(c) Under what circumstances would you expect a monopoly to charge a lower price than if the industry was operating under perfect competition? [6]

(d) What impact would a decrease in interest rates have on the level of investment in an economy? [4]

2 (a) What is an oligopoly? [4]

(b) Why might oligopolists wish to collude with each other? [4]

(c) Explain the kinked demand curve theory. [6]

(d) Describe the likely effects on the UK economy if house prices were to *fall significantly* over the next two years. [6]

SECTION B *Civil and Structural Engineering*
*Answer not more than **two** questions from this section*
You may refer to the data sheet as necessary at the end of the examination paper

3 A client is considering construction in a congested city, of a new metro comprising stations and bored tunnels. The typical depth of the tunnels will be 20 m. The ground conditions along the length of the proposed route are highly variable and include rocks, stiff clays, soft clays and sands. The water table is close to the ground surface. The stiff clays and soft clays have undrained shear strengths in the range 150 – 250 kPa and 20 – 40 kPa respectively.

(a) Explain the key engineering issues relating to possible construction methods for bored tunnelling in the different ground conditions, highlighting the significance of stability ratio in clays and of preventing water in-flow in the case of the sands. [6]

(b) Describe the distinguishing features of segmental linings and sprayed concrete linings, and identify for which of the anticipated ground conditions they would be suitable. [3]

(c) Explain the significance of ground movements caused by bored tunnelling. What kind of buildings are most vulnerable and why? Describe the difference between the effects of differential and total settlement on buildings. [3]

(d) One of the stations will be constructed beneath a busy railway that cannot be interrupted, and tunnel jacking is being considered for this. Explain the principles of tunnel jacking and suggest how the soft clays and sands could be stabilized. [3]

(e) Explain the purposes of instrumentation and monitoring during construction of the proposed metro, giving examples of three types of instrumentation that would be used. [5]

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4 A tunnel structure is to be constructed beneath a 10 m deep river estuary. In order to do this, a temporary wall is driven into the river bed, as shown in Fig. 1. Dewatering is then undertaken (by pumping) to lower the water level temporarily by 10 m on one side of the wall to river bed level. Before dewatering, a prop is installed near the top of the wall. The river bed comprises of a 4 m sand layer overlying a 4 m layer of stiff clay, which is underlain by gravel. The wall is driven 2 m into the stiff clay.

The sand has a unit weight of 20 kN m^{-3} and a critical state angle of friction of 35°. The stiff clay has a unit weight of 20 kN m^{-3} and an undrained shear strength of 75 kPa. The unit weight of the river water is 10 kN m^{-3}. The wall is smooth and watertight. Assume undrained conditions apply in the clay when the water level is temporarily lowered on one side.

(a) Assuming that the wall moves sufficiently to mobilize active and passive pressures, calculate the prop force. Sketch the pressure distributions on the wall. [8]

(b) Calculate the factor of safety against failure of the wall by rotation about the prop. [4]

(c) To increase the factor of safety against failure by rotation about the prop, 1m of loose rockfill is placed on the river bed on the dewatered side of the wall (prior to dewatering). Assuming the unit weight of the rockfill is 20 kN m^{-3}, and treating it simply as a surcharge (ignoring its strength), calculate the effect on the prop force if active and passive pressures are still fully mobilized. [6]

(d) Comment on the significance of assuming undrained conditions in the clay. [2]

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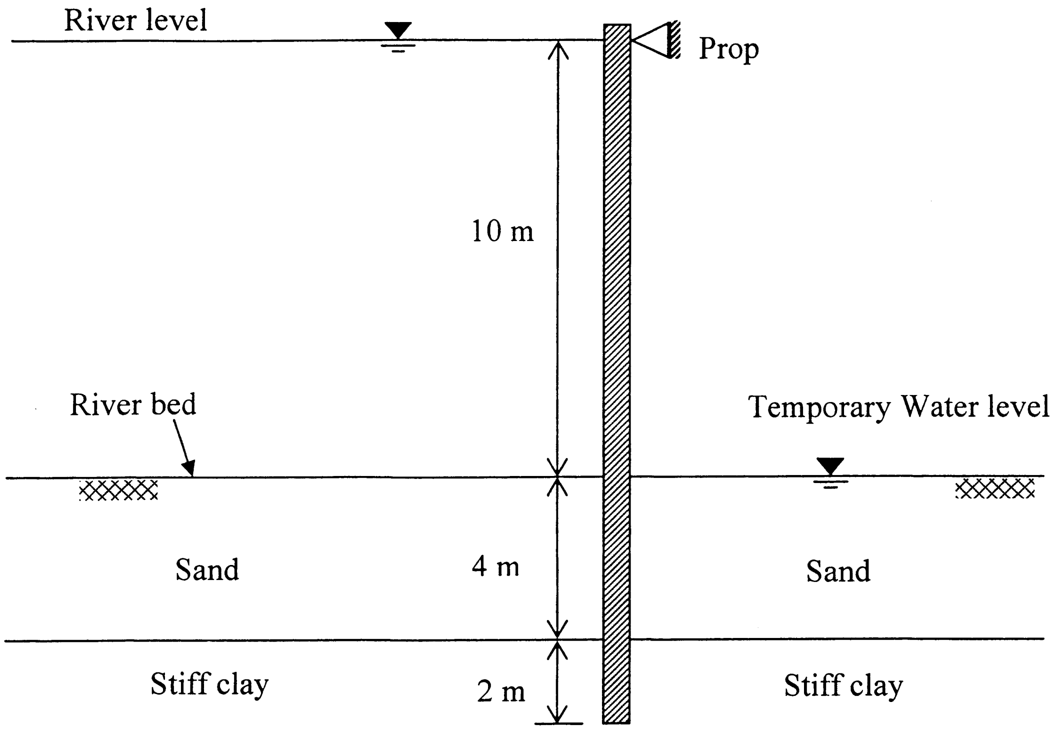

Fig. 1

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5 A road is to run below the ground surface in a concrete trough, as shown in Fig. 2. The critical loading case for the slab AB is when the hydrostatic pressure is fully active beneath the slab and the piles at A and B are in tension.

The degree of fixity between the slab and the walls cannot be guaranteed, so the designer must ensure that the slab is adequate if there is full fixity at A and B, or if the slab is simply supported at those points. The design upward pressure on the slab, which makes an allowance for the weight of the concrete slab and the partial safety factors on load, is 150 kPa. The bending moment at the end of a fully fixed beam of length L under a distributed load of w per unit length may be taken as $wL^2/12$.

(a) Draw the bending moment diagrams for the two limiting conditions of full fixity and simply supported conditions at points A and B respectively, and hence determine the design values of the bending moment at point A and at point C (at the mid span). [6]

(b) Determine the minimum effective depth of the slab, if it is to be singly reinforced. Take $f_y = 460 \text{ N mm}^{-2}$ and $f_{cu} = 30 \text{ N mm}^{-2}$. [4]

(c) On the assumption that the slab provided is 1600 mm thick, design suitable reinforcement at A and C. [4]

(d) Suggest a reinforcement layout that should be provided in the slab, and in the junction to the walls. Take due account of the fact that the maximum length for a reinforcing bar is 12 m and give details of how you would lay out the reinforcement in the area of any laps. [4]

(e) Comment on the factors that would cause the designer to choose a slab thickness larger than the value you calculated in Part (b) above. [2]

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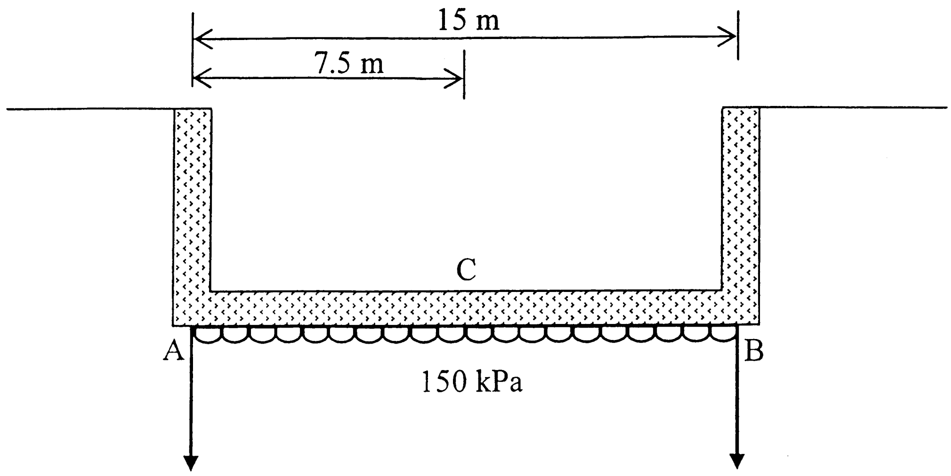

Fig. 2

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SECTION C *Mechanics, Materials and Design*
Answer not more than **two** *questions from this section*

6 (a) Outline the primary considerations in determining whether investment in a new material is justified. [5]

(b) Draw and explain a market assessment diagram, and discuss the conclusions that can be drawn from such a diagram. [5]

(c) Use the market assessment diagram from Part (b) above to judge the market potential of metal foams for energy absorption in automobiles. [3]

(d) With the help of a cash-flow diagram, describe the pattern of cash-flow in establishing production of a part made from a new material. Identify critical points on the cash-flow curve and describe what determines these points. [7]

7 The indentation of a metal foam sandwich, of length L and unit depth into the plane of the paper, by a cylindrical roller of radius R is sketched in Fig. 3. The sandwich comprises a face sheet made from a rigid ideally plastic metal with a tensile yield strength σ_Y and thickness h, perfectly bonded to a foam of thickness H. The foam may be assumed to be a rigid ideally plastic solid with a plateau strength σ_P.

(a) Employing the collapse mechanism sketched by the dashed lines in Fig. 3, calculate an upper bound to the indentation collapse load P in terms of the length parameter λ. [7]

(b) Obtain an expression for λ such that P as calculated in Part (a) is minimised. Hence, determine the optimum upper bound indentation load P_I. [4]

(c) Derive an expression for the critical value of L/h below which uniform compression of the foam sandwich is expected. [6]

(d) Briefly discuss the advantages of a low strength foam in the energy absorbing sandwich sketched in Fig. 3. [3]

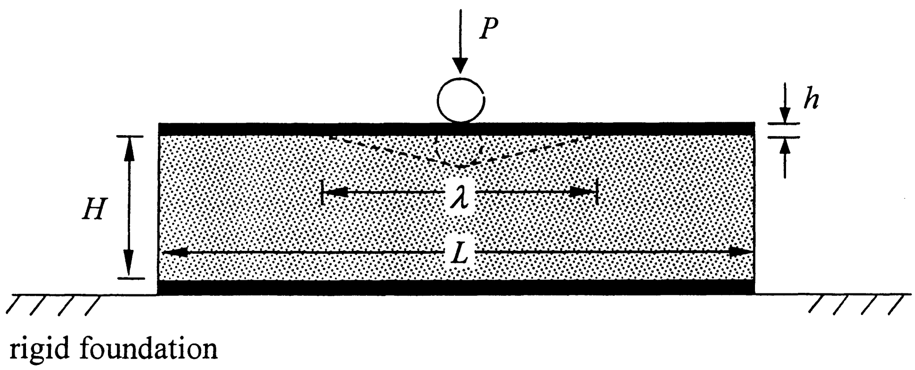

Fig. 3

8 A two-dimensional lattice material has a periodic microstructure comprising square cells of side l and thickness t, such that $t/l \ll 1$. Each cell is reinforced by a diagonal member, also of thickness t, see Fig. 4. The walls of the lattice material are made from a rigid ideally plastic solid with a tensile yield strength σ_Y.

(a) Define the relative density $\bar{\rho}$ of a cellular material and obtain an expression for $\bar{\rho}$ of the lattice material sketched in Fig. 4 in terms of l and t. [4]

(b) The lattice material is now subjected to a uniform engineering shear strain γ along the x_1 direction as shown in Fig. 4.

(i) Use Mohrs circle to derive a relation between γ and the direct strain component ε at $+45^{\circ}$ to the x_1 axis (i.e. 45° anticlockwise). [5]

(ii) Using an upper bound approach or otherwise, obtain an expression for the shear yield strength τ_Y of the lattice material in terms of $\bar{\rho}$ and σ_Y. You may assume that the cell walls are pin-jointed at the nodes and that deformation occurs with the horizontal and vertical cell walls remaining rigid. [7]

(c) Explain why the stiffness and strength of foams are more sensitive to relative density than they are for lattice materials. [4]

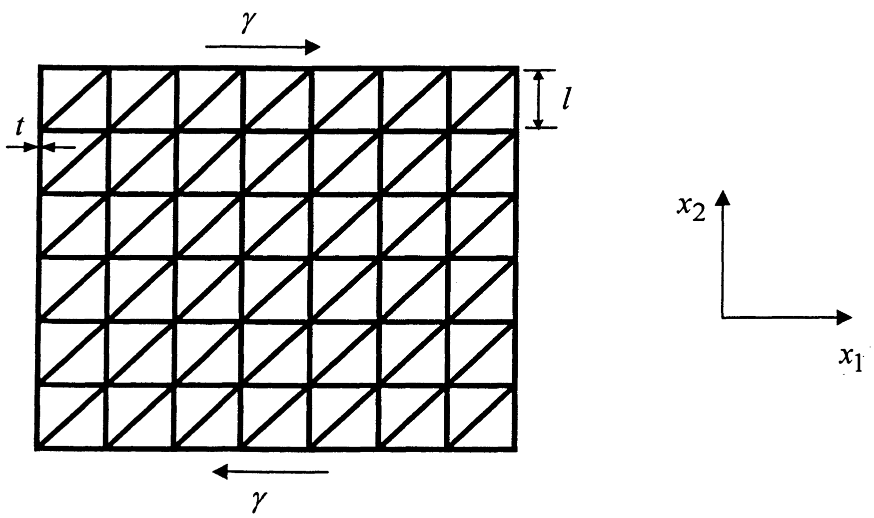

Fig. 4

SECTION D *Aerothermal Engineering*
*Answer not more than **two** questions from this section*

9 An aircraft fitted with *turbojet* engines cruises at 400 m s^{-1} at an altitude where the atmospheric pressure is 26.5 kPa and atmospheric temperature is 225 K.

(a) The stagnation pressure and stagnation temperature at inlet to the compressor are 76.5 kPa and 305 K respectively. If the compressor pressure ratio is 10 and it has an isentropic efficiency of 85%, show that the stagnation temperature at compressor exit is approximately 638 K. [4]

(b) The compressor blade speed is 280 m s^{-1}. Given that the design stage loading factor $\Delta h_0 / U^2$ is less than 0.4, calculate the minimum number of compressor stages required. [2]

(c) Given that the stagnation temperature at turbine entry is 1300 K and the turbine isentropic efficiency is 90%, calculate the stagnation temperature and stagnation pressure at the exit from the turbine. [4]

(d) If the turbine blade speed is 300 m s^{-1} and the turbine stage loading factor $\Delta h_0 / U^2$ is less than 2.0, calculate the number of turbine stages. [2]

(e) Calculate the jet velocity at the exit from the propelling nozzle assuming that the flow is isentropic. Comment on the type of nozzle geometry that would be required. [4]

(f) Calculate the propulsive efficiency, $\eta_p = 2V/(V_j + V)$, of the turbojet and the engine mass flow rate required to produce a net thrust of 12 kN per engine. Comment on the suitability of a turbojet for this application. [4]

(Assume that the properties of air are given by: $\gamma = 1.4$, R = 287 J kg^{-1} K^{-1}, c_p = 1005 J kg^{-1} K^{-1} throughout and the mass flow rate of fuel can be ignored.)

10 (a) An aircraft is cruising at 250 m s^{-1} at an altitude of 11 km. If the aircraft has total mass (including fuel and payload) of 100 tonne and the lift coefficient is 0.5, calculate the required wing area and the stagnation temperature and stagnation pressure at entry to the engines. [4]

(b) The aircraft has two engines each operating with a mass flow rate of 100 kg s^{-1} . If the lift-to-drag ratio for the aircraft is 18, calculate both the net thrust F_N and the gross thrust F_G that each engine must produce. [4]

(c) Due to an engine failure, the pilot descends to an altitude of 6 km. Calculate the flight speed required to produce the necessary lift at this altitude, assuming that the lift coefficient for the aircraft remains the same. [2]

(d) Given that the engine exit nozzle has area $A_N = 0.3 \text{ m}^2$ and assuming that the one remaining engine is operating at the same non-dimensional conditions as for cruise, calculate the mass flow rate through this engine at the reduced altitude. [5]

(e) By calculating the gross thrust, and hence the net thrust at the reduced altitude, discuss whether the aircraft can maintain steady cruise at the new altitude. [5]

Note: The properties of air are: $\gamma = 1.4, R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$, $c_p = 1005 \text{ J kg}^{-1} \text{ K}^{-1}$ throughout and the mass flow rate of fuel can be ignored.

The properties of the standard atmosphere are listed below:

Altitude	Pressure	Temperature	Density
6 km	47.2 kPa	249 K	0.660 kg m^{-3}
11 km	22.7 kPa	217 K	0.365 kg m^{-3}

You may assume that suitable non-dimensional expressions for the air mass flow rate through an engine and the gross thrust produced are respectively:

$$\frac{\dot{m}_{air}\sqrt{c_p T_{02}}}{p_{02} A_N} \quad \text{and} \quad \frac{F_G + p_a A_N}{p_{02} A_N}$$

where all the terms have their usual meaning.

11 There are many aspects to achieving an optimum design for a large civil aircraft and its aeroengines. In answering the following questions, it is more important to identify the physical constraints that influence the design rather than the precise values of the quantities involved.

(a) What determines the wing area for a large civil aircraft? [2]

(b) How are the design cruise Mach number and altitude chosen? [4]

(c) Describe how the overall efficiency can be decomposed into the cycle efficiency and the propulsive efficiency. Explain how these affect the basic design of an aeroengine for a large civil aircraft. [6]

(d) For the core compression system, explain why a single turbine stage can drive several compressor stages. [4]

(e) Why are several low-pressure turbine stages required to drive the fan in a high bypass-ratio civil aeroengine? [4]

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SECTION E *Electrical Engineering*

Answer not more than ***two*** *questions from this section*

You may refer to the data sheet as necessary at the end of the examination paper

12 (a) With the aid of diagrams, describe the operation of an active matrix liquid crystal display based on amorphous silicon technology. [6]

(b) A 300 mm diameter Si wafer containing 10^5 chips is to be processed in a class 10 clean room. The minimum feature size throughout the process is to be $0.3\ \mu m$.

(i) Explain with the aid of diagrams what is meant by positive resist technology and negative resist technology. Discuss the criteria which would lead to a choice between them for the above process. [4]

(ii) Compare the merits of contact, proximity and projection exposure and indicate which lithographic process is best to use for the above. [4]

(iii) If a UV source of 0.25 μm is utilised, determine the value of the Numerical Aperture required for the optical system and comment upon the subsequent depth of field if the wafer is flat to within ±5 μm. [4]

(iv) For an optical system with a power density of $0.06\ \text{W cm}^{-2}$, estimate the required exposure time if the resist used has an exposure energy density of $150\ \text{mJ cm}^{-2}$. [2]

13 (a) Explain with the aid of diagrams how the gate voltage controls the source-drain current in a field effect transistor FET. [3]

(b) Explain the difference between an accumulation mode and a depletion mode FET. [2]

(c) What is the role of silicon dioxide in a FET? Compare the advantages and disadvantages of Si and GaAs based FETs, using diagrams to illustrate your answer. [5]

(d) Derive, from Gauss' Law, the gate voltage necessary to switch off a Si depletion mode FET. [4]

(e) Calculate the ~~minimum~~ maximum gate threshold voltage if the dopant density is 10^{25} m^{-3} so that the gate field does not exceed the breakdown field for Si of 2.0×10^{7} V m^{-1}. The dielectric constant of Si is 10^{-10} F m^{-1}. [6]

14 (a) Explain, with the aid of diagrams, how conduction occurs in a doped semiconductor. What controls the number of carriers? [3]

(b) In an n-type semiconductor, explain what is meant by the *electron mobility*, *conductivity* and *effective mass* of a charge carrier. [3]

(c) What is meant by scaling in electronic device design? How do the number of transistors per chip, gate length, circuit speed, and power per chip scale with scaling parameter? [4]

(d) Derive the transit time for an InAs FET with source-drain length of $0.1\ \mu m$, if the carrier mobility is $3.3\ m^{2}\ V^{-1}\ s^{-1}$ and V_{SD} is 0.8 V. Comment on the result. [6]

(e) Optimise the dopant density so that the transistor of Part (d) will operate with a gate voltage swing of 0.1 V. Assume a dielectric constant for InAs of 1.5×10^{-10} F m^{-1} and a channel depth of $0.1\ \mu m$. [4]

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SECTION F *Information Engineering*
*Answer not more than **two** questions from this section*

15 (a) Describe what a Lamport hash chain is, how it works and how it could be used to implement a login mechanism in an operating system. Compare advantages and disadvantages with the password-based system that is traditionally used. [6]

(b) Explain what 'password salting' is and how it works. What attacks does it prevent and to what attacks is it still vulnerable? [4]

(c) The palmtop of a terrorist has been captured. If switched on, it asks for a userid and password. Explain what you would do to extract as much information as possible from the palmtop. Illustrate all the counter-measures the terrorist may have taken and explain how you would attempt to bypass them. [10]

16 (a) What are the basic properties of the Session Initiation Protocol (SIP) and where is it used? [5]

(b) What are SIP messages and how does the SIP message system work? [5]

(c) What are the functions of a SIP proxy and where are such proxies found? [5]

(d) Give an illustration of a SIP based network layout suitable for connecting multimedia terminal devices. [5]

17 (a) Provide one programming example of multithreading that improves performance over a single-threaded solution, and two examples that do not. [3]

(b) A scheduling algorithm favours processes that have used the least processor time in the recent past. How will this algorithm perform on I/O bound processes and CPU-bound processes? Would you consider it a good or bad scheduling algorithm? Why? [3]

(c) Explain briefly how the following file allocation strategies work, drawing a clear diagram for each:

(i) indexed allocation;

(ii) contiguous allocation;

(iii) linked allocation. [6]

(d) Which of the three file allocation strategies mentioned in Part (c) is sometimes improved by the addition of a File Allocation Table (FAT), as used in MS-DOS? Briefly explain how FAT works. [2]

(e) A file system has a block size of 512 bytes and uses one-block clusters. File pointers are 4 bytes wide. There is no FAT. The last logical block that was accessed was block 18. We now want to access block 5. How many blocks will be read from disk, for each of the following three cases:

(i) indexed allocation;

(ii) contiguous allocation;

(iii) linked allocation. [6]

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SECTION G *Biological and Medical Engineering*
*Answer not more than **two** questions from this section*

18 (a) Explain why it is desirable for a visual target

(i) to be located on the fovea;

(ii) to have an angular velocity that is very similar to that of the eye itself. [4]

(b) What kinds of eye movement have evolved to achieve the goals listed in Part (a). [4]

(c) Sketch typical eye movements that might be observed when, after some practice, a subject tracks a stimulus moving sinusoidally with amplitude 5 degrees in a horizontal line normal to the line of sight

(i) at 0.2 Hz;

(ii) at 2.2 Hz. [4]

(d) In what way would the eye movements recorded in Part (c) differ if measured before rather than after a period of practice? [2]

(e) What experiments imply that smooth pursuit is not primarily generated by a simple direct feedback system? [3]

(f) How can it be demonstrated that in the control of saccades the oculo-motor system uses target velocity, but not acceleration, to determine the size of a future saccade? [3]

19 Write brief notes on each of the following:

(a) Describe the roles of 'smooth swimming' and 'tumbling' in the navigational strategy of bacteria such as *salmonella*. [4]

(b) How can a protein 'building block' which is equipped with a bi-stable conformational 'switch' construct a family of flagellar filaments with distinct helical forms? [4]

(c) How can an organism's cells 'specialise', in order to construct different limbs, organs, etc., even though every cell contains the full DNA prescription for the entire organism? [4]

(d) Explain how it is possible to trace evolutionary relationships between different animals, plants, insects, etc. by means of a knowledge of their DNA sequences. [4]

(e) Why is it useful for protein molecules to be able to recognise:

(i) specific short sequences of DNA, and [2]

(ii) specific chemical species? [2]

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20 (a) (i) For an animal in a steady glide, draw the force balance and show that the glide speed U is given by

$$U \approx \sqrt{\frac{2mg}{\rho S C_L}}$$

where mg is the weight, ρ is the density of air (1.2 kg m^{-3}), S is the planform area and C_L is the lift coefficient. [4]

(ii) For a wandering albatross of mass 12 kg, wing span 3.5 m, and wing chord 0.24 m, its best glide angle and corresponding glide speed were recorded to be 2.5° and 19 m s^{-1}, respectively. What is the value of C_L? What is the drag force acting on the bird? [2]

(iii) Ultralight aircraft have a similar wing loading and glide speed to those of the albatross and fly in the same Reynolds number regime. Why might their glide angles be steeper than that of the albatross? [4]

(b) In order to explore a region in the ocean that is hard to access with manned vehicles, you are tasked to design a biomimetic autonomous underwater vehicle. You have decided to model your vehicle on the Bluefin tuna, a fast swimmer with good manoeuvrability.

(i) Describe the main locomotive mechanism of the Bluefin tuna. How does the body motion generate thrust? Use a sketch if necessary. [2]

(ii) List the relevant non-dimensional groups that should be considered when designing your vehicle. Define each non-dimensional group in terms of the length of the fish L, the frequency of fin motion f, amplitude of fin motion A, the chord-length C of the propulsive fin, forward swimming speed U, the fluid density ρ, dynamic viscosity μ and gravity g. Sketch a diagram of the swimming motion to indicate how you have defined length, amplitude and frequency. [4]

(iii) Describe the physical significance of these non-dimensional groups, and explain why their values should be closely matched between the robotic vehicle and the fish. [4]

SECTION H *Manufacturing, Management and Design*
*Answer not more than **two** questions from this section*

21 (a) What information about a company's finances is contained in:

(i) the profit and loss account, and

(ii) the balance sheet?

How are the two related? [5]

(b) An inventor has found a new way to produce bio-diesel from wheat. He formed a new company with £15,000 of his own money, owning all the shares in the company. On the same day, the company paid £3,000 in lawyers' fees. Later in the first financial year, the company paid £8,000 for a reaction vessel, £1,000 for chemicals which were used in experiments and £2,000 for rent of premises. In the second financial year, it paid £8,000 for a marketing study and £2,000 for rent of premises, and received an interest-free loan of £20,000. No other financial transactions occurred in these periods. You may ignore any depreciation of the company's assets or interest received on cash.

Generate:

(i) a balance sheet for the day after the company was formed; [3]

(ii) a balance sheet and a profit and loss account for the end of the first financial year; [3]

(iii) a balance sheet and a profit and loss account for the end of the second financial year. [3]

(c) What further information, apart from these profit and loss accounts and balance sheets, might a potential investor in the company described in Part (b) require? [6]

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22 You have developed an initial concept for an 'induction kettle' as sketched in Fig. 5. This uses the principle of electro-magnetic induction to heat a conducting plate inside the water vessel. Your initial concept comprises the seven 'modules' numbered in the Figure.

(a) Outline the main stages of a design process to take this idea from initial concept to product launch. Describe how this design process differs from the wider process of 'New Product Development'. [4]

(b) Explain what is meant by a value proposition and outline its importance. Suggest an 'elevator pitch' for this new product. [6]

(c) Summarise the characteristics of an effective specification to be used by a designer. Give examples of the elements of the specification that would be required for this product. [6]

(d) Sketch two alternative product configurations and comment briefly on the relative merits of each. [4]

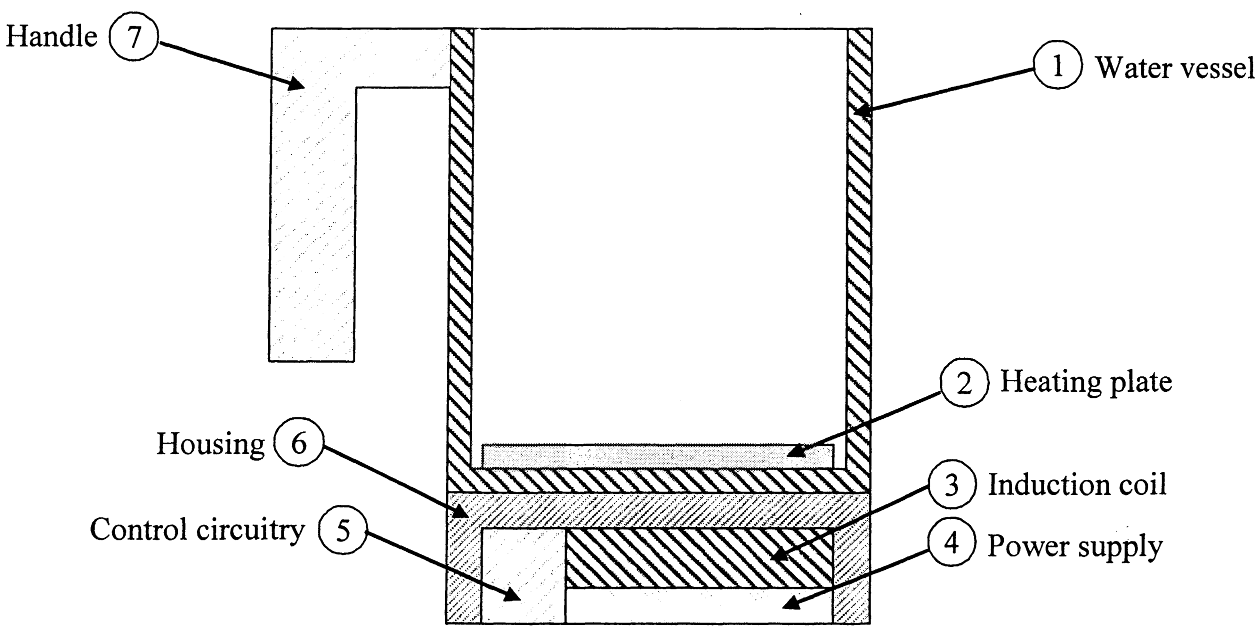


Fig. 5

23 You have invented a novel bicycle light which uses new technology.

(a) List four possible business models for making money from this idea. [4]

(b) List one advantage and one disadvantage for each of the models listed in your answer to Part (a), in exploiting this invention. [5]

(c) Briefly describe the options available to protect the intellectual property in your invention, and compare their merits for each of the business models listed in your answer to Part (a). [5]

(d) Describe the types and sources of funding that would be appropriate for each of the following stages of development of this invention:

(i) initial idea and feasibility testing;

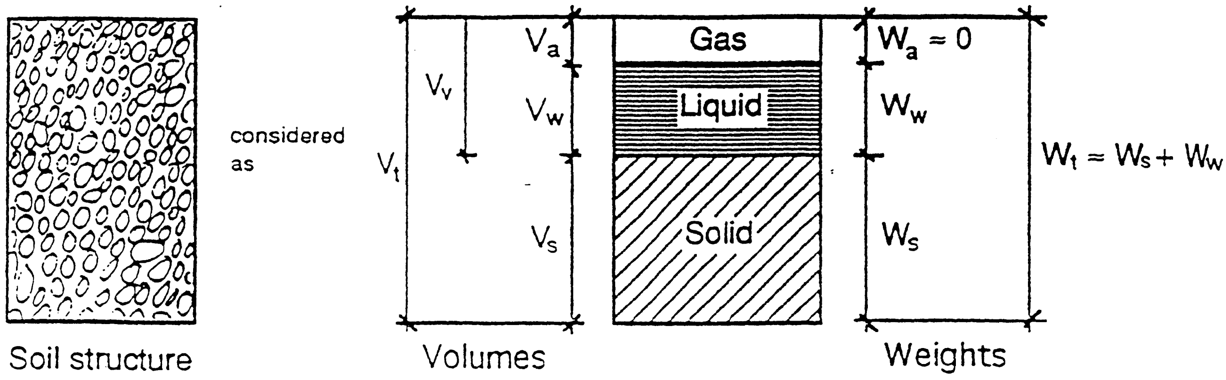
(ii) initial commercialisation;

(iii) steady growth of sales. [6]

END OF PAPER

Data sheet – Soil Mechanics

General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w\, G_s / e)$
Unit weight of water	$\gamma_w = 9.81\ \text{kN/m}^3$ (although we assume $10\ \text{kN/m}^3$)
Unit weight of soil	$\gamma = W_t / V_t = \left(\dfrac{G_s + S_r e}{1+e}\right)\gamma_w$
Buoyant (effective or submerged) unit weight	$\gamma' = \gamma - \gamma_w = \left(\dfrac{G_s - 1}{1+e}\right)\gamma_w$ (soil saturated)
Unit weight of dry soil	$\gamma_d = W_s / V_t = \left(\dfrac{G_s}{1+e}\right)\gamma_w$
Relative density	$I_d = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})}$

where e_{max} is the maximum voids ratio achievable in the quick tilt test (for sands), and
e_{min} is the minimum voids ratio achievable by vibratory compaction (for sands).

Classification of particle sizes

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D — equivalent diameter of soil particle

D_{10}, D_{60} etc — particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

Stress components

Principle of effective stress (saturated soil):

total stress	σ	=	effective stress $\sigma' +$	pore water pressure	u
	τ	=	$\tau' +$		0

and

σ_v = vertical stress

σ_h = horizontal stresss

τ = shear stress

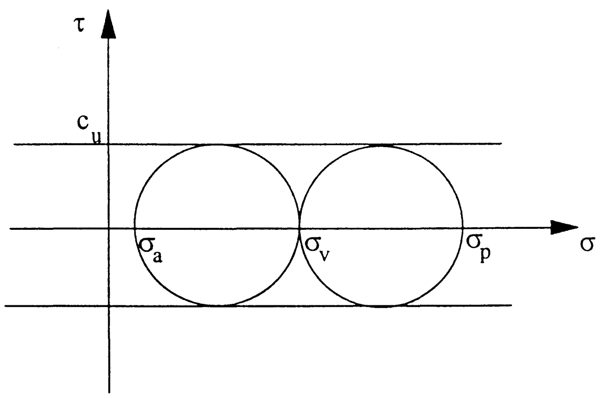
Strength of clays (undrained behaviour only)

Under constant volume (undrained) conditions only, the strength of clays can be characterised by the *undrained shear strength* c_u which is mobilized when the shear stress $\tau = c_u$. This conforms to Tresca's criterion, and the active and passive total horizontal stresses, σ_a and σ_p respectively, are given by

$$\sigma_a = \sigma_v - 2\,c_u$$

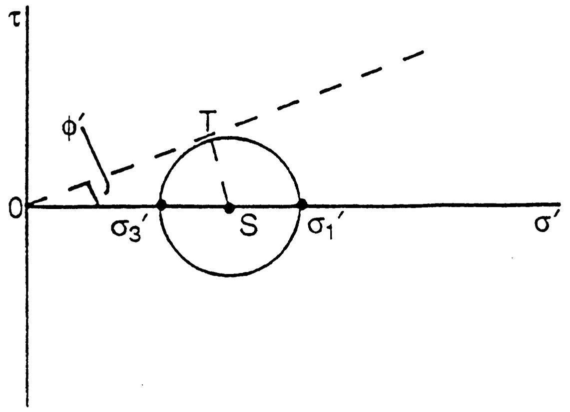
$$\sigma_p = \sigma_v + 2\,c_u$$

where σ_v is the total vertical stress.



Strength of sands

Mobilised angle of shearing ϕ'
where $\tau = \sigma' \tan \phi'$



$$\sin \phi' = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

$$\therefore \phi' = \sin^{-1}\left[\frac{\left(\sigma_1'/\sigma_3'\right) - 1}{\left(\sigma_1'/\sigma_3'\right) + 1}\right]$$

Earth pressure coefficient K: $\sigma_h' = K\sigma_v'$

Active pressure: $\sigma_v' > \sigma_h'$ $\therefore$ $\sigma_1' = \sigma_v'$

$\sigma_3' = \sigma_h'$

$K_a = (1 - \sin\phi'/1 + \sin\phi')$

Passive pressure: $\sigma_h' > \sigma_v'$ $\therefore$ $\sigma_1' = \sigma_h'$

[We assume principal stresses are horizontal and vertical]

$\sigma_3' = \sigma_v'$

$K_p = (1 + \sin\phi')/(1 - \sin\phi') = \dfrac{1}{K_a}$

Angle of shearing resistance:

at peak strength ϕ'_{max} at $\left(\sigma_1'/\sigma_3'\right)_{max}$

at critical state ϕ'_{crit} after large strains.

Sand strength data: friction hypothesis

In any shear test on soil, failure occurs when $\phi' = \phi'_{max}$ and

$$\phi'_{max} = \phi'_{crit} + \phi'_{dilatancy}$$

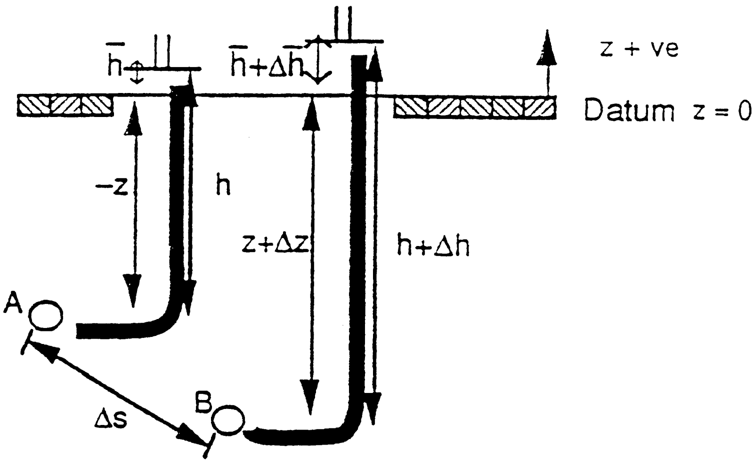
where ϕ'_{crit} is the ultimate angle of shearing resistance of a random aggregate which deforms at constant volume, so the dilatancy, which indicates an increase in volume during shearing, approaches zero $(\phi'_{dilatancy} -> 0)$ as $\phi'_{max} -> \phi'_{crit}$.

ϕ'_{crit} is a function of the mineralogy, size, shape and distribution of particles. For a particular soil it is almost independent of initial conditions. Typical values (±2°):

	ϕ'_{crit}	ϕ'_{max}	
feldspar	40°		
quartz	33°	53°	($I_d = 1$, and mean effective stress OS < 150 kPa)
mica	25°		

Seepage

Excess pore water pressure



Head $h = u/\gamma_w$

Potential $\bar{h} = h + z$

Total pore water pressure head at A: $u = \gamma_w h = \gamma_w\left(\bar{h} - (-z)\right)$

B: $u + \Delta u = \gamma_w\left(h + \Delta h\right) = \gamma_w\left(\bar{h} + z + \Delta\bar{h} + \Delta z\right)$

[Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w\left(\bar{h} + \Delta\bar{h}\right)$]

Hydraulic gradient A–B $i = -\dfrac{\Delta\bar{h}}{\Delta s} = -\dfrac{\Delta\bar{u}}{\gamma_w \Delta s}$

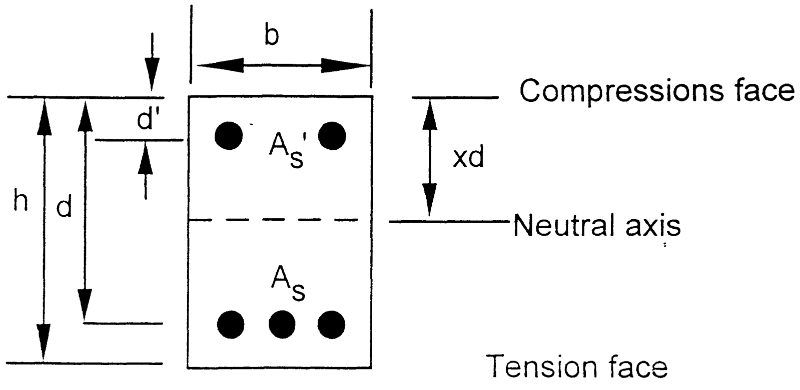
Darcy's law v = ki
v = average or superficial seepage velocity
k = coefficient of permeability

Typical permeabilities

$D_{10} > 10\text{mm}$	:	non-laminar flow
$10\text{ mm} > D_{10} > 1\mu\text{m}$	:	$k \cong 0.01(D_{10}\text{in mm})^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Design of reinforced concrete

Data sheet for use in Part IB Civil Engineering Elective Course.



Design Stresses

Cube strength for concrete f_{cu}. At failure in bending, stress in concrete $= 0.4f_{cu}$ over whole area of concrete in compression.

Tensile yield stress of steel f_y. At failure in bending, stress in bars in tension $= 0.87f_y$, stress in bars in compression $= 0.75f_y$.

Design Equations

Moment capacity of singly reinforced beam

$$M \leq 0.15\, f_{cu} bd^2$$
$$M = 0.87 f_y A_s d(1 - x/2)$$
$$x = 2.175(f_y / f_{cu}).(A_s / bd) \quad (\leq 0.5 \text{ to avoid over reinforcement})$$

Moment capacity of doubly reinforced beam

$$M = 0.15 f_{cu} bd^2 + 0.75 f_y A'_s (d - d')$$
$$0.87 f_y A_s = 0.75 f_y A'_s + 0.2 f_{cu} bd$$

Shear capacity of all beams

Total shear capacity $V = (v_c + v_s)bd$

Where, $v_c = 0.68(100A_s/bd)^{0.33}.(400/d)^{0.25}$ (N/mm^2)

and $v_s = 0.87f_y A_{sq}/(bs)$

in which s = shear link spacing, A_{sq} is total area of all shear bars in a link and A_s is the total area of effective longitudinal *tension* steel at the section.

Standard bar sizes

Diameter (mm)	6	8	10	12	16	20	25	32	40	50
Area (mm^2)	28	50	78	113	201	314	491	804	1256	1963

Available steel types

Deformed high yield steel $f_y = 460$ N/mm^2

Plain mild steel $f_y = 250$ N/mm^2

Lap and anchorage lengths 40 bar diameters

Density of reinforced concrete: 24 kN/m^3

Reinforcement areas per metre width

	Spacing of bars (mm)									
	75	100	125	150	175	200	225	250	275	300
Bar Dia. (mm)										
6	377	283	226	189	162	142	126	113	103	94.3
8	671	503	402	335	287	252	224	201	183	168
10	1050	785	628	523	449	393	349	314	285	262
12	1510	1130	905	754	646	566	503	452	411	377
16	2680	2010	1610	1340	1150	1010	894	804	731	670
20	4190	3140	2510	2090	1800	1570	1400	1260	1140	1050
25	6550	4910	3930	3270	2810	2450	2180	1960	1790	1640
32	10700	8040	6430	5360	4600	4020	3570	3220	2920	2680
40	16800	12600	10100	8380	7180	6280	5580	5030	4570	4190
50	26200	19600	15700	13100	11200	9820	8730	7850	7140	6540
Areas calculated to 3 significant figures according to B.S.I recommendations										

9 May 2005

Part IB Data Sheet: Fast Transistor Elective
Transistor Design Summary Sheet

Gauss's Theorem

$\varepsilon_0\varepsilon_r E_1 - \varepsilon_0\varepsilon_r E_2$ = charge per unit area enclosed between upper surface 1 and lower surface 2.

FET Design Summary

- $\tau_t \rightarrow$ switching time as 1st approx. (scattering limited transit time).
- $\tau_{eff} = \tau_t + R_{load}C_{eff(output)} \rightarrow$ switching time as 2nd approx.
- $L = v_s \tau_t$ (source-drain spacing).
- $I_{sat} = e N v_s W d_s = e N W L d_s/\tau_t$
- Aspect ratio W/L (technology?).
- $(1/2)eN(d_s)^2/\varepsilon_0\varepsilon_r$ = (Max Gate Voltage)
- $E'_{peak} = eN d_s/\varepsilon_0\varepsilon_r < E_{breakdown}$
- Minimum Drain Source Voltage $\sim E_s L$ (E_s is the field required to reach limiting velocities).

Mutual Conductance

$g_{mo} \sim I_{sat}/V_{gate(max)}$

Mutual conductance reduces with frequency as $g_m(\omega) \approx g_{mo}/(1 + j\omega\tau_t)$;

$$v_{out} = g_m(\omega)\, R\left(1 + j\omega R C_{eff(out)}\right) \approx g_{mo}R/[1 + j\omega(\tau_t + RC_{eff(out)})]$$
$$= g_{mo}R/[1 + j\omega\tau_{eff}]$$

Capacitances for FET

Parallel Plate Capacitance: $\varepsilon_0\varepsilon_r$ Area/spacing
Used for rough estimates of parasitic capacitance.

Effective Capacitances for FET

$C_{eff(out)} \rightarrow C_{gate/drain} + C_{drain/source} + C_{load};$

$C_{eff(in)} \rightarrow MC_{gate/drain} + C_{gate/source\,(proximity)} + C_{gate/source\,(electronic)};$

$C_{electronic} = g_{mo}\tau_t$; $M = (1 + g_{mo}R_{load})$.

Time Constants for FET

$\mu = e\tau/m^*$ relates mean free time τ and mobility.

Transit time τ_t over distance L and scattering limited velocity v_s are related by $\tau_t = L/v_s$.

$v_{out} \approx g_{mo}R/[1 + j\omega\tau_{eff}] = g_{mo}R/[1 + j\omega/(2\pi f_t)]$

$1/(2\pi f_\tau) = \tau_t + RC_{eff(out)} = \tau_{eff}$ The transition frequency is f_t.

10% to 90% rise time is $T = 2.2\,\tau_{eff} = (2.2/2\pi)(1/f_t) = 0.35/f_t$.

Fast Transistor Elective:
Tunnel Barrier Design Summary Sheet

Schrodinger's Equation

Complex Wave $\Psi = A\ \exp(-j2\pi f\ t + j2\pi x/\lambda) = A\exp(-j\omega t)\exp(jkx);$

<momentum> $\Psi = p\Psi = (h/\lambda)\Psi = -j(h/2\pi)\partial\Psi/\partial x;$

<Total energy> $\Psi = E\Psi = hf\Psi = j(h/2\pi)\partial\Psi/\partial t$

$(h/2\pi) \rightarrow \hbar;\ \ h = 6.625 \times 10^{-34}$ J/s.

Schrodinger's equation:-

$$E\Psi = (1/2m)[-j\hbar\partial/\partial x]^2\Psi + e\phi\Psi$$

Tunneling (Rectangular barrier $\varepsilon\phi$)

Propogating waves outside barrier with incident kinetic energy $U_{incident} = (\hbar k_i)^2/2m^*$

Evanescent waves inside barrier: $-(\hbar k)^2/2m^* = (\hbar k_i)^2/2m^* - e\phi$

Technology Design Summary

Diffusion

Constant Surface Concentration:

$$C(x,t) = C_s\ \mathrm{erfc}\left[\frac{x}{2(Dt)^{1/2}}\right]$$

Constant Total Dopant:

$$C(x,t) = \frac{S}{(\pi Dt)^{1/2}}\exp\left[\frac{-x^2}{4Dt}\right]$$

W I Milne
21.3.05