2006 Part IB Paper 2

Answers

1 a) $\underline{s}$ =[1, −√3, 1]

b) $\underline{t}$ = [W, 0, −W]

c) (i) $\underline{t}$ = [4W, −2√3W, 0]
(ii) $\underline{t}$ = [2W, −2√3W, 2W]

2 a) (i) δ_Q = 5.3 mm (down), δ_S = 5.3mm (up), θ_R=2.0×10^{-3} rads
(ii) M_R = 15.0 kNm
b) M_R = 10 kNm, θ_R=1.0×10^{-3} rads

3 a) (i) τ_A = 0 MPa, τ_B = 60.3 MPa
(ii) σ_A = 113.2 MPa, σ_B= 67.9 MPa
b) (i) Tresca λ_A = 2.16, λ_B = 1.77
(ii) Von Mises λ_A = 2.16, λ_B = 1.97

4 c) (i) δ_C = 8.1 mm
(ii) σ_{A1} = 0 MPa, τ_{A1} = 102 MPa
(iii) σ_1 = 102 MPa @ 45 degs clockwise from x, σ_2= −102 MPa @ 45 degs anticlockwise from x, σ_3= 0 MPa

5 a) (ii), (iv)
b) α = 0.936, w = 13.7m/L^2
c) (i) Steel
(ii) t = 25.8 mm

6 a) P = 4Mp/3L
b) P = 4Mp/3L
c) Mp = 123.6 kNm, P = 164.8 kN
d) No effect
e) No

IB Exam ANSWERS 2005-2006 - PAPER 4 T. B. Nickels and J. B. Young

1 a) 13.1 MW, 34.9%

b) 18.1 MW, 41.9%

2 a) F,F,F,F

b) (i) 0.91kW (ii) 43°C

3 a) (i) 2081 kJ/kg (ii) 352 kg/s

b) (i) 16237 kg/s (ii) 38.4 km (iii) 1.21 MW/K

4 a)

$$\frac{P_m}{P_r} = 8\alpha^2 \frac{3/2}{\alpha + 1/2} ,$$

where $\alpha = T_m/T_r$

b) $P_m = 354\text{kN/m}^2$, $V_m = 305\text{m/s}$.

c) 0.164

d) $T_m = 117\text{K}$.

5 a)

$$U_2 = U_1 \frac{D_1^2}{D_2^2}$$

b)

$$p_2 = p_1 + \tfrac{1}{2}\rho U_1^2 (1 - (D_1/D_2)^4)$$

c)

$$p_3 = p_1 - \tfrac{1}{2}\rho U_1^2 \left[\left(\frac{D_1}{D_2} \right)^2 - 1 \right]^2$$

d)

$$\rho U_1^3 \frac{\pi D_1^2}{4} \left(\frac{D_1^2}{D_2^2} - 1 \right)^2$$

6 b)

$$\tau = \rho_2 g \sin\theta (h_2 - y)$$

$$\tau(y = h_1) = \rho_2 g \sin\theta (h_2 - h_1)$$

c)

$$\tau = \rho_1 g \sin\theta (h_1 - y) + \rho_2 g \sin\theta (h_2 - h_1)$$

d)

$$u = \frac{\rho_1 g \sin\theta}{\mu}(h_1 y - \tfrac{1}{2}y^2) + \frac{\rho_2 g \sin\theta}{\mu}(h_2 - h_1)y$$

and at $y = h_1$,

$$u = \frac{\rho_1 g \sin\theta}{\mu}\tfrac{1}{2}h_1^2 + \frac{\rho_2 g \sin\theta}{\mu}(h_2 - h_1)h_1$$

ENGINEERING TRIPOS, Part IB 2006
Paper 6 - INFORMATION ENGINEERING
Answers

1 (b) G_1 continuous line, G_2 dashed line

GM_1=19.5, PM_1=50 degrees; GM_2=2.3, PM_2=19.4 degrees

(c) new gain margin = 4, new phase margin = 59 degrees

2 (b) $\omega \to 0 \quad \to \quad G(j\omega) \to -j\infty - 2$ (angle $-\pi/2$)

$\omega \to \infty \quad \to \quad G(j\omega) \to 0$ (at angle $\pi/2$)

Imaginary part of $G(j\omega)$ is 0 when $\omega = 1$

Gain margin = 2, phase margin = 20 degrees, $0 < K < 2$

(c) 1.45; (d) $\omega < 0.5$rad/s

3 (b) $G(s) = \frac{1}{s^2+4s+\alpha}$

(i) $g(t) = \frac{1}{2}\left[\exp(-t) - \exp(-3t)\right]$

(ii) $g(t) = t\exp(-2t)$

(iii) $g(t) = \frac{1}{2}\exp(-2t)\sin(-2t)$

(c) $G_{new}(s) = \frac{k_1+sk_2}{s^2+(4+k_2)s+(\alpha+k_1)}$; k_1 increases bandwidth, k_2 increases damping

1B Paper 6 Section B

Answers

4 (b) $$-\frac{4j}{\omega}\sin^2\left(\frac{\omega T}{2}\right)$$

(c) $$F_2(\omega) = \begin{cases} \frac{j}{2} & -2a < \omega < 0 \\ -\frac{j}{2} & 0 < \omega < 2a \end{cases}$$

(d) $\frac{\pi}{2}$

5 (b) $$U(\omega) = \frac{\pi}{2}\left\{ \delta\left(\omega - \frac{5}{4}\omega_0\right) + \delta\left(\omega - \frac{3}{4}\omega_0\right) + \frac{\delta}{j}\left(\omega - \frac{9}{8}\omega_0\right) + \frac{\delta}{j}\left(\omega - \frac{7}{8}\omega_0\right.\right.$$

(c) $\phi = 0 \Rightarrow g_1(t)$

$\phi = \frac{\pi}{2} \Rightarrow g_2(t)$

6 (d) $$Y_k = \frac{1}{8}[0, 6, 4, 6, 0, 6, 4, 6]$$

IB Paper 7

SELECTED ANSWERS

SECTION A

1.

(a) See the Vector Calculus notes.

(b) $V = \pi R^2 H_1 \left(2 - \frac{H_1}{H_2}\right)$.

(c) $\nabla \cdot \mathbf{u} = \frac{RH_1U_2 - 2H_2^2U_1}{R(2H_2^2 - H_1H_2)}$.

(d) 0.

2.

(a) $\phi = fyz + C$, where C is a constant.

(b) (i) 2 (ii) 0.

(c) $d^2f/dx^2 = 0$ or df/dx is symmetric about the plane $x = 0$.

(d) -

3.

(a) $k^2 = \omega/c$.

(b) $y(x,t) = \sum_n Y_n \sin\left(\frac{n\pi x}{L}\right) \cos\left(\left[\frac{n\pi}{L}\right]^2 ct\right)$.

IB paper 7

SECTION B

4.

(a) $a = 2$.

(b) $a = 1$. The eigenvalues are $0, 1, 4$. The eigenvector corresponding to $\lambda = 4$ is $[1, 3, 1]^T$.

(c) $a = -1, -1/2$. For $a = -1$, $\lambda = 1, 2, 2$ with eigenvectors $[1, 0, -2]^T$ and $[1, -1, -1]^T$. For $a = -1/2$, $\lambda = 1, 1, 3$ with eigenvectors $[1, 0, -2]^T$ and $[-1, 4, 2]^T$.

(d) The result of applying A four times is $[21, 64, 22]^T$. The accuracy is good, correct to 1 decimal place. The convergence factor is 1/4.

5.

(a)

$$LU = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Column space: $[1, -1, 1]^T$ and $[0, 1, -1]^T$ (or any other two linearly independent vectors in the plane defined by these two vectors). Row space: $[2, -1, 0, 3]^T$ and $[0, 1, 1, -1]^T$ (or any other two linearly independent vectors in the subspace defined by these two vectors). Null space: $[-1/2, -1, 1, 0]^T$ and $[-1, 1, 0, 1]^T$ (or any other two linearly independent vectors in the subspace defined by these two vectors). Left null space: $[0, 1, 1]^T$.

(b) (i) $\alpha = 1/12$. (ii) $E[X] = E[Y] = 10/9$, $E[XY] = 34/27$, $E[X^2] = 14/9$. (iii) - .

6.

(b) (i) 0.070. (ii) 0.080.

(c) (i) 0.058. (ii) 0.080.

IB 2006 Paper 8 Section D Aerothermal Elective

Answers

10 (b) Max take-off weight 175.4 tonnes. Fuel 50.4 tonnes

(c) 46.2 tonnes

(d) 36.0 tonnes

11 (c) 67.5 kg s^{-1}, 40.5 kN

(d) 21,700 rpm

Numerical Answers for
Section E Electrical Option paper 8 Part 1B

12 (c) 0.018 Volts
(d) 9×10^{5} V/m for Si and 2.8×10^{6} V/m for $Si0_2$

13 (d) 6.8×10^{-13} s, 0.041 eV, 8.2×10^{-8} m

14 (d) ~ 1 micron

2006 Engineering Part 1B Paper 8

Section G

List of Numerical Answers

18. No numerical answers.
19. No numerical answers.
20. 6.74 Hz, $f \propto m^{-\frac{1}{6}}$.

Part IB 2006 Paper 8 section H numerical answers

Question 23

(b)

(i)	Breakeven volume = 400
(ii)	Q_{max} = 550, Profit = £625
(iii)	£68.18 per unit.