IB PAPER 2

STRUCTURAL MECHANICS

EXAMINER C.R.MIDDLETON

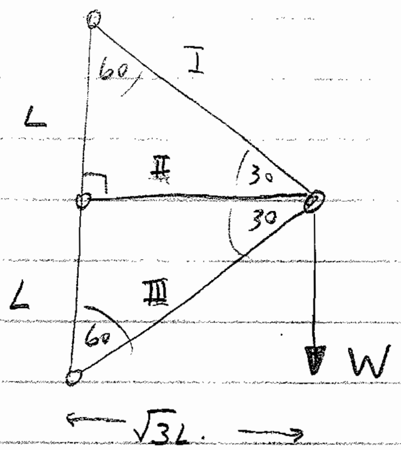
ASSESSOR S.D.GUEST

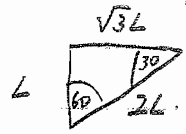
CRIB: Q1-6.

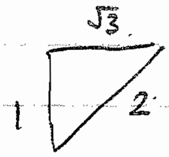
CRIB

IB STRUCTURES 2006.

Q1.

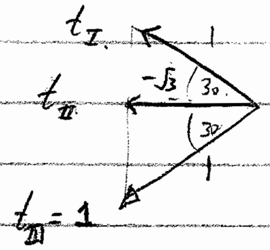






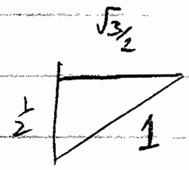
(a) Find state of self-stress in the structure.

Assume bar III is redundant bar with $t_{III} = 1$.

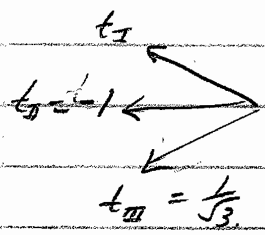


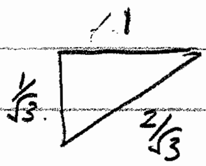
For equilibrium $\quad t_I = 1$

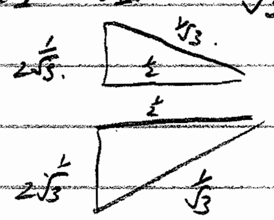
$$t_{II} = -\frac{\sqrt{3}}{2} \times 2 = -\sqrt{3}$$



Alternatively if assume $t_{II} = -1 \Rightarrow t_I = t_{III} = +\frac{1}{\sqrt{3}}$



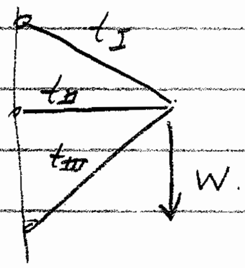




$$\frac{1}{4} + \frac{1}{4 \cdot 3}$$

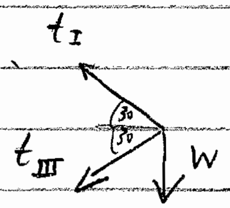
$$\frac{3+1}{12} = \frac{4}{12} = \frac{4}{4 \cdot 3} = \frac{1}{\sqrt{3}}.$$

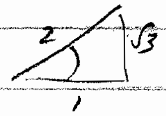
(b)



Antisymmetric loading so $t_I = -t_{III}$

hence $t_{II} = 0$.





$\Sigma V = 0 \uparrow +$ $\quad t_I \cos 60 = W + t_{III} \cos 60.$

$$\frac{t_I}{2} = W + \frac{t_{III}}{2} \quad (1)$$

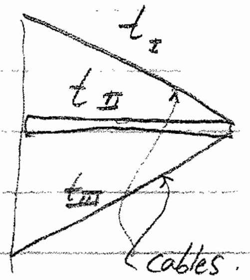
$\Sigma H = 0 \leftrightarrow +$ $\quad t_I \cancel{\cos 30} + t_{III} \cancel{\cos 30} = 0.$

$$t_I = -t_{III}. \quad (2)$$

(2) in (1): $\quad t_I = 2W + t_I \qquad t_I = W \qquad \therefore t_{III} = -W. \qquad t_{II} = 0$

NOTE: Sol^n can also be found by usual elastic approach

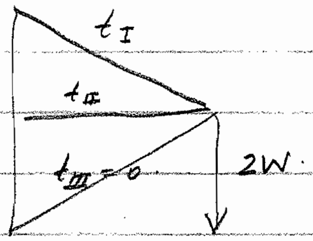
Q.1 (c)

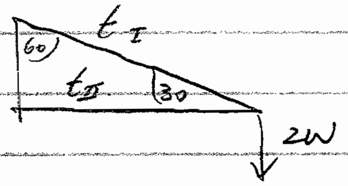


Cable III slackens with load 2W.

(i)

Initially prestressed such that $t_{III} = 0$ when load = 2W.





$\Sigma V = 0$ $\quad t_I \cos 60 = 2W.$

$$\therefore \; t_I = 4W.$$

$\Sigma H = 0.$ $\quad t_{II} + t_I \cos 30 = 0.$

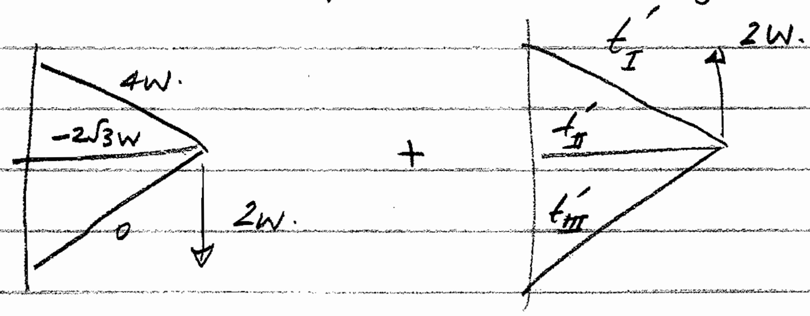
$$t_{II} = -t_I \cos 30 = -4W \frac{\sqrt{3}}{2}$$

$$= -2\sqrt{3}W$$

$$t_{III} = 0.$$

(ii) Have loading with 2W applied. To get initial tensions when no load add 2W upwards: this unloading takes place elastically.

i.e.



From part (b): $\quad t_1' = -2W \quad t_{III}' = +2W. \; t_{II}' = 0$

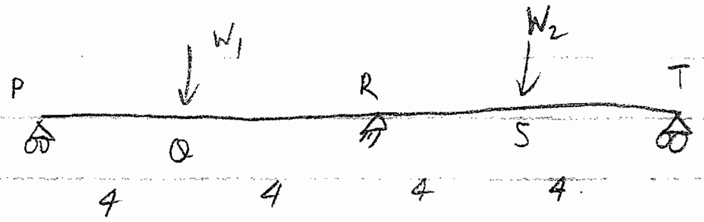
Initially

$\Rightarrow \; t_I = 4W - 2W = 2W.$

$t_{II} = -2\sqrt{3}W + 0 = -2\sqrt{3}W.$

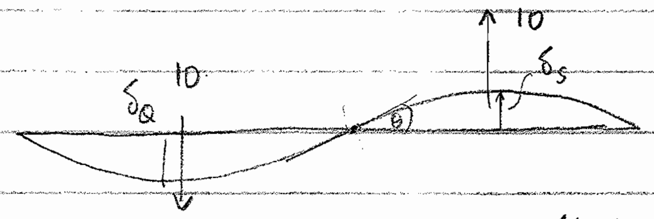
$t_{III} = 0 + 2W = 2W$

Q2./ UB 305 x 127 x 48

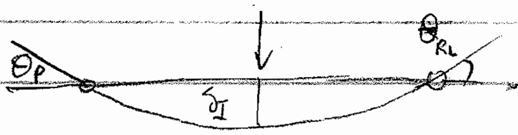


$I_{xx} = 9575\,cm^4$.

(a)(i) $W_1 = 10$ $W_2 = -10\,kN$ Want δ_Q, δ_S + θ_R



Antisymmetric mode give. $M_R = 0$.



Deflections at Q + S.

$$\delta_Q = \frac{Wl^3}{48EI} = \frac{10 \cdot 10^3 \times 8^3}{48 \times 210 \times 10^9 \times 9575 \times 10^{-8}}$$

$$= 5.3 \times 10^{-3}\,m$$

$\delta_Q = 5.3\,mm$ down (∴ $\delta_S = -5.3\,mm$ up)

Rotation at R (from relationship to δ_Q above)

$$\theta_R = \frac{Wl^2}{16EI} = \frac{\delta_Q \times 3}{l} = \frac{5.3 \times 3}{8} \times 10^{-3} = 1.99 \times 10^{-3}\,\text{rads}$$

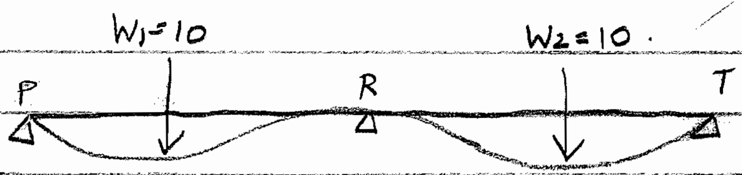
$\simeq 2 \times 10^{-3}$ rads

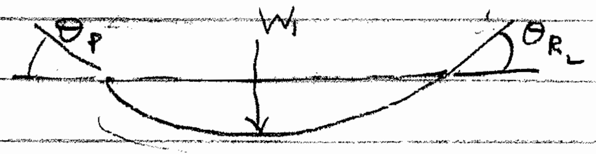
(0.114°)

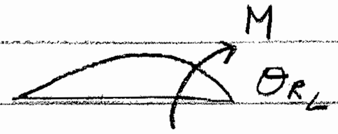
π rad = 180 d

1 rad = $\frac{180}{\pi}$

(ii) Symmetric load. Find M_R? From symmetry $\theta_R = 0$.







From databook

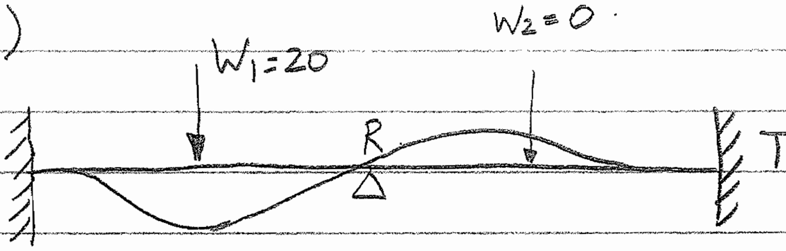
$$\theta = \frac{Ml}{3EI} = \frac{W\ldots}{16E\ldots}$$

Apply M such that beam has zero rotation at R.

$$M = \frac{3Wl}{16} = 15\,kNm$$

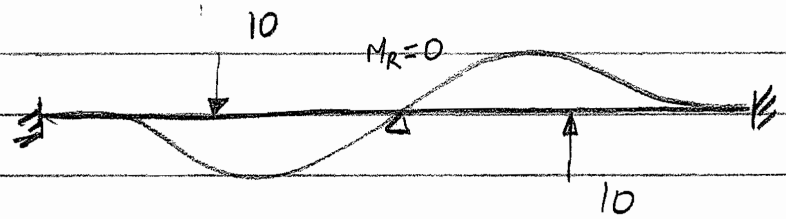
Q2 (cont.)

(b)



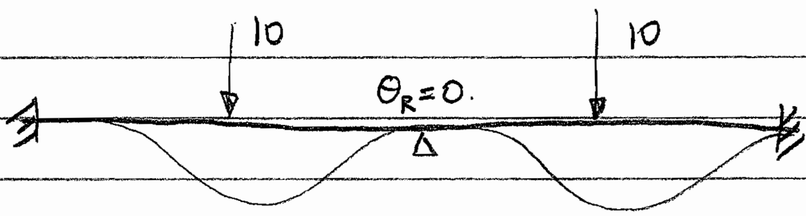
Find M_R & θ_R?

Consider using symmetric & antisymmetric cases.



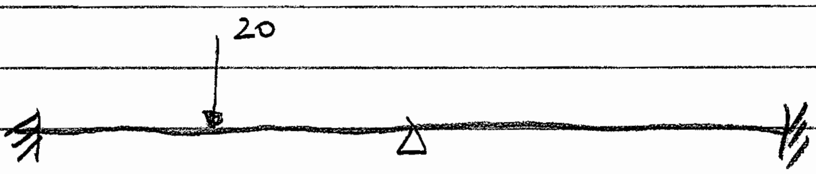
(I) Anti symmetric

+

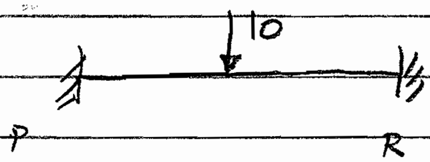


(II) Symmetric

=



Bending Moment at R is moment caused by symmetric loading in II (since $M_R = 0$ in case I antisymmetric loading). So case is same as:

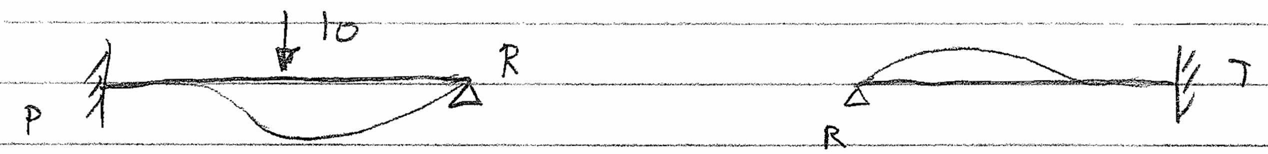


From data book $M_P = M_R = \frac{Wl}{8} = \frac{10 \times 8}{8} = 10k$

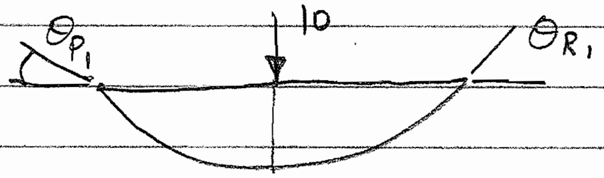
$M_R = 10$ kNm $(= M_P)$

Rotation at R is solely due to antisymmetric loading.

Antisymmetric case can be represented as: (since R is equivalent to p.



|||



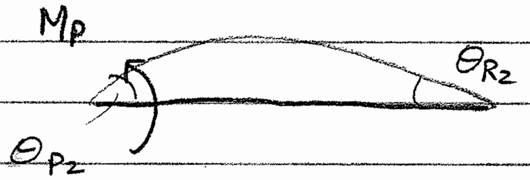
$$\theta_{P_1} = \theta_{R_1} = \frac{Wl^2}{16EI} \quad \text{from data book}$$

$$= \frac{10 \times 10^3 \times 8^2}{16 \times 210 \times 10^9 \times 9575 \times 10^{-8}}$$

$$= 1.99 \times 10^{-3} \simeq 2 \times 10^{-3} \text{ rads.}$$

(same as in Part(a)(i))

+



$M_P = 15.0$ kNm (same as in Part (a)(ii) – Moment required such that $\theta_{P2} = \theta_{P1}$)

$\therefore$ Since $\theta_{P2} = \theta_{P1} = 2.0 \times 10^{-3}$ rads.

But $\theta_{R_2} = \frac{\theta_{P2}}{2} \Rightarrow \theta_{R2} = \frac{2 \times 10^{-3}}{2} = 1.0 \times 10^{-3}$ rads

Rotation at R will be $\theta_R = 1 \times 10^{-3}$ rads

Q3(a)

(i) At Support vertical shear $S = 50kN$. No torque so no torsional stresses.

At A $\quad \tau_A = 0$ since in formula $\tau = \frac{SA\bar{y}}{I_{xx} b}$ $\quad A_c$ is zero.

At B

$$\tau_B = \frac{50\times10^3 \times (20\times200)\times 40}{13.25\times10^6 \times 10}$$

$$I_{xx} = \frac{bd_1^3}{12} - \frac{bd_2^3}{12}$$

$$= \frac{200\times100^3}{12} - \frac{190\times60^3}{12}$$

$$= 16.67\times10^6 - 3.42\times10^6$$

$$= 13.25\times10^6\ mm^4$$

$$\tau_B = 60.34\ N/mm^2$$

(ii) $\sigma = \frac{My}{I}$ $\qquad M = 50\times0.6 = 30\,kNm$

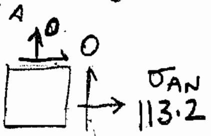
$$\sigma_A = \frac{30\times10^3\times0.050}{13.25\times10^{-6}} = 113.2\times10^6\ N/m^2 = 113.2MPa$$

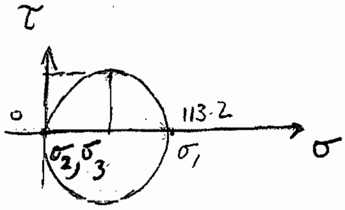
$$\sigma_B = \sigma_A\cdot\frac{y_B}{y_A} = 113.2\times\frac{30}{50} = 67.9\ MPa$$

(b) $\sigma_y = 245\ MPa$ $\qquad$ Load. 50λ.

At A $\quad \sigma_{A_N} = 113.2\ MPa$

$\tau_A = 0$.





$\tau_{max} = \frac{\sigma}{2} = 56.6\ MPa$.

Tresca

Tresca when $\tau_{max} = \frac{Y}{2} = \frac{245}{2} = 122.5\ MPa$.

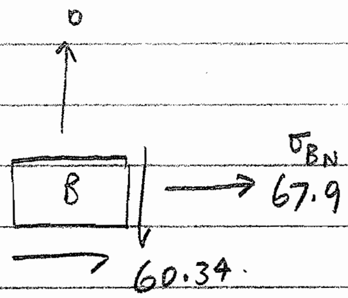
$$\therefore \lambda = \frac{122.5}{56.6} = 2.164$$

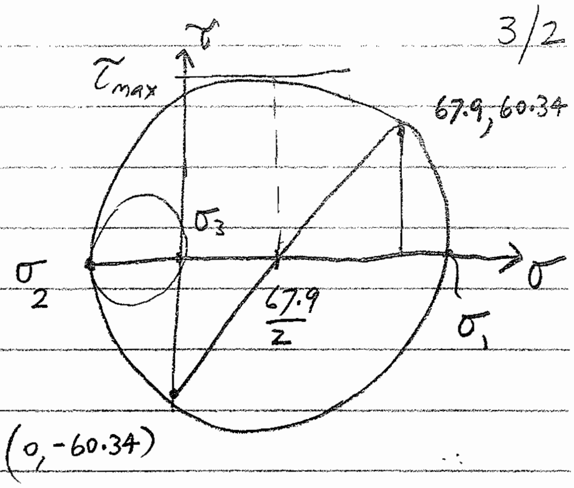
Von Mises $\lambda^2\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right] = 2Y^2$ $\qquad (\sigma_2 = \sigma_3 = 0)$

$\lambda^2 2\sigma_1^2 = 2Y^2$ $\qquad \lambda = \frac{Y}{\sigma_1}$ $\quad \frac{245\ MPa}{113.2}$ $\qquad \lambda = \frac{245}{113.2} = 2.164$

Q3(b) cont.

$\boxed{\text{At B}}$





Centre at $\sigma = \frac{67.9}{2} = 33.95$ MPa.

Radius $R = \sqrt{60.34^2 + 33.95^2} = 69.24$ MPa

$\sigma_1 = 33.95 + 69.24 = 103.2$ MPa.

$\sigma_2 = 33.95 - 69.24 = -35.29$ MPa.

$\sigma_3 = 0$.

Tresca $\tau_{max} = \frac{Y}{2} = \frac{245}{2} = 122.5$ MPa.

$\tau = R = 69.24$ MPa.

$\lambda = \frac{122.5}{69.24} = 1.77$

Von Mises $\lambda^2\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] = 2Y^2$.

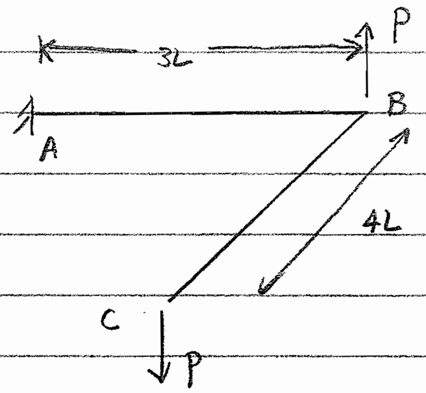
$\therefore \lambda^2\left[(\underbrace{103.2 + 35.29}_{138.5})^2 + (-35.29 - 0)^2 + (0 - 103.2)^2\right] = 2 \times 245^2$

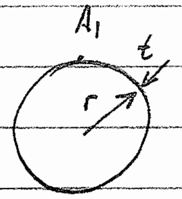
$\lambda^2 = \frac{120.05 \times 10^3}{31.08 \times 10^3} = 3.86$

$\lambda = 1.97$

Q4/

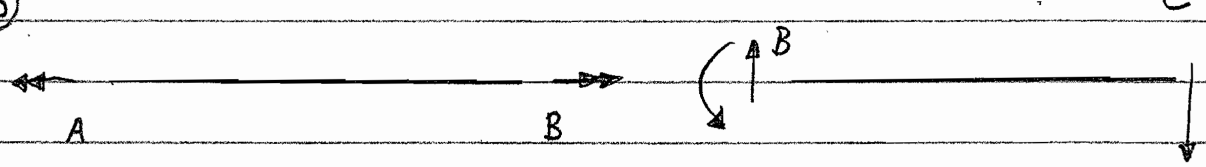
(a)



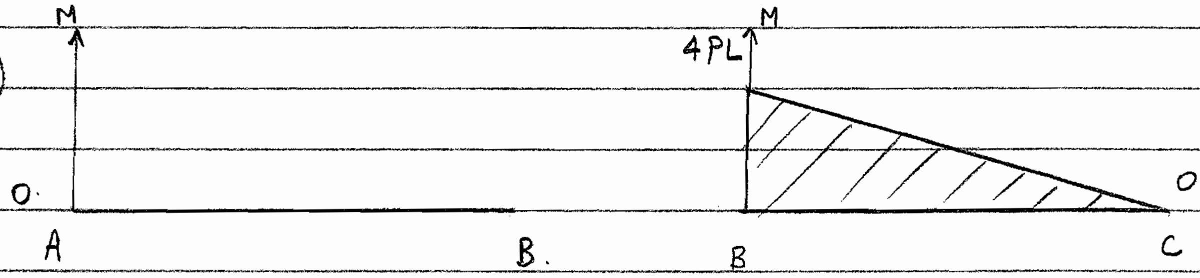


RH screw rule for tor

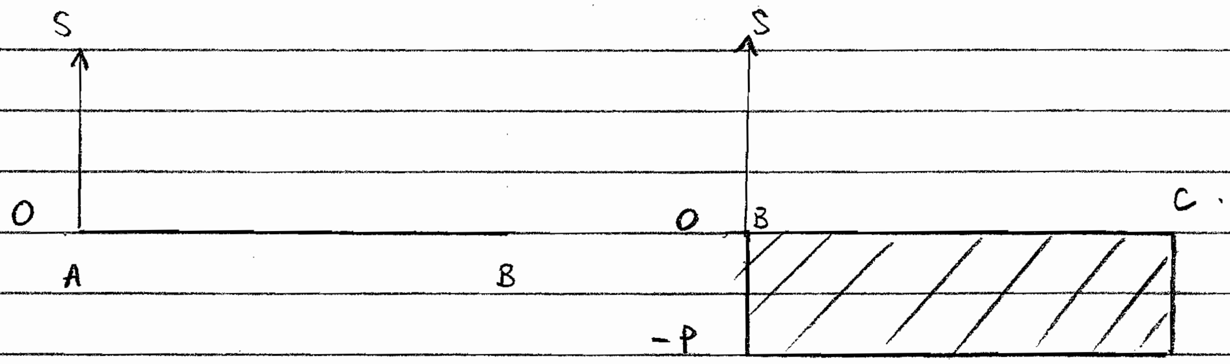
FBD



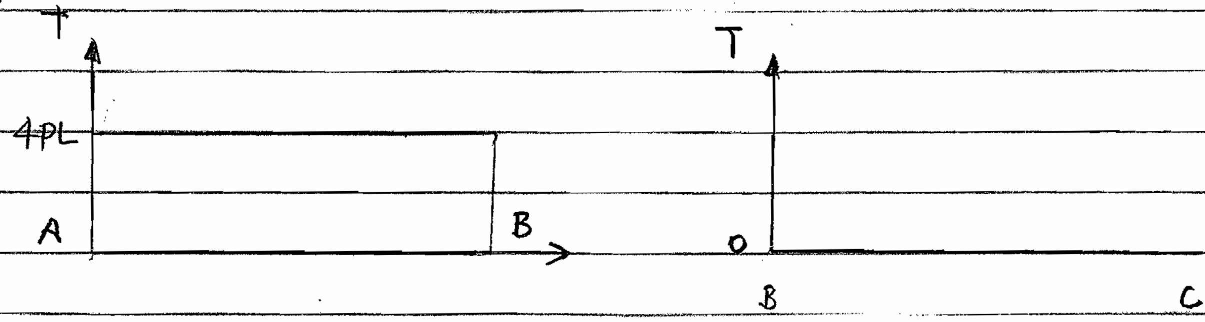
BMD



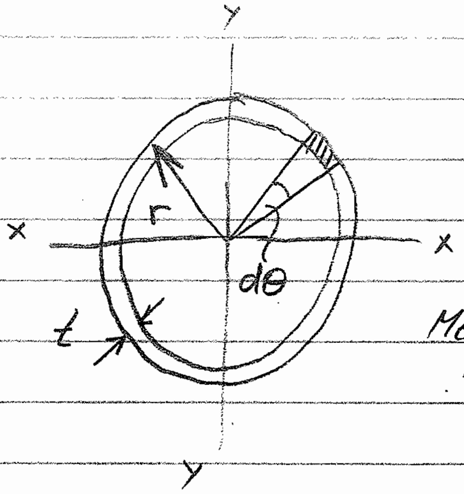
SFD



T



Q4(b)



Polar second moment of area, J.

By definition $J = \int r^2 \, dA$

Method(i) $dA = r\,d\theta . t$

$$\therefore J = \int_0^{2\pi} r^2 (r\,d\theta\,t) = r^3 t \left[\theta \right]_0^{2\pi}$$

$$= \underline{2\pi r^3 t}$$

Method(ii) r = constant. $\therefore J = r^2 \int dA$ where $\int dA = A$.

Have $A = 2\pi r . t$ $\therefore \underline{J = 2\pi r^3 t}$

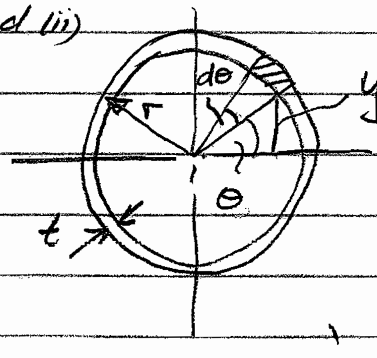
Second moment of area, I.

By definition $I = \int y^2 \, dA$.

Method(i) $J = I_{xx} + I_{yy} = 2I$ here.

$$\therefore I = \frac{J}{2} = \underline{\pi r^3 t} \text{ from 1st part.}$$

Method (ii)



$I_{xx} = \int y^2 \, dA$ $\quad dA = r . d\theta . t$

$y = r \sin\theta$.

$$\therefore I_{xx} = \int_0^{2\pi} (r \sin\theta)^2 \, r t . d\theta$$

where r = const; t = const.

$$= r^3 t \int_0^{2\pi} \sin^2\theta \, d\theta$$

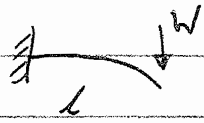
where $\int_0^{2\pi} \sin^2\theta \, d\theta = \pi$

$$\therefore I_{xx} = \underline{\pi r^3 t}$$

4/3

Q4(c) $P = 2000N$ $r = 25mm$ $t = 2mm$ $L = 100mm$

(i) Bending of BC $\quad \delta_{c_1} = \dfrac{P(4L)^3}{3EI}$



δ

$\dfrac{Wl}{3EI}$

Rotation of AB. $\quad \delta_{c_{11}} = 4L.\theta_{AB}.$

$T = GJ\phi.$

$$\phi = \frac{T}{GJ} = \frac{4PL}{GJ}$$

$\theta = \phi.l.$

$$\theta_{AB} = \phi.3L.$$

$$\delta_{c_{11}} = 4L.3L.\frac{4PL}{GJ} = \frac{48PL^3}{GJ}$$

$$\therefore \delta_c = \delta_{c_1} + \delta_{c_{11}} = \frac{64PL^3}{3EI} + \frac{48PL^3}{GJ.} = 16PL^3\left(\frac{4}{3EI} + \frac{3}{GJ}\right)$$

$$= 16 \times 2000 \times (0.1)^3 \left(\frac{4}{3 \times 210 \times 10^9 \times \pi(0.025)^3 \times 0.002} + \frac{3}{81 \times 10^9 \times 2\pi(0.025)^3 \times 0.002}\right)$$

$$= 32 \times (64.67 \times 10^{-6} + 188.6 \times 10^{-6})$$

$$= 8.1 \times 10^{-3} \ m$$

$$\delta_c \equiv 8.1mm$$

Q4 (c)(ii)

Bending stress at A_1. $\sigma = \frac{My}{I}$

$\therefore \sigma_{A_1} = 0$ since $M = 0$.

Shear stress at A_1 due to vertical shear $q = \frac{S A_c \bar{y}}{I}$; $\tau = q/a$.

$q = 0$ since $S = 0$. $\therefore \tau = 0$

Shear stress at A_1 due to torque $q = \frac{T}{2A_e}$; $\tau = q/a$.

$$q = \frac{4PL}{2.\pi r^2} \quad \therefore \tau = \frac{2PL}{\pi r^2 t} = \frac{2 \times 2000 \times 0.1}{\pi (0.025)^2 \cdot 0.002}$$

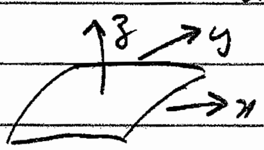
$$= 101.9 \times 10^6 \ N/m^2$$

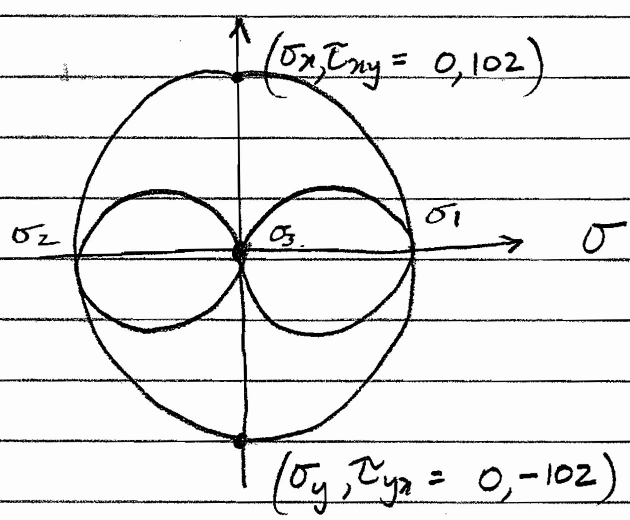
$$\equiv 102 \ MPa$$

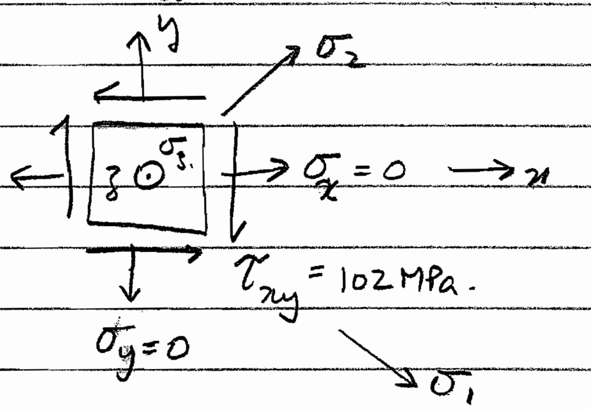
τ +ve

Q 4 (c)(iii) Tube subjected to pure torsion at A.

Consider element of tube at A_1.







$\sigma_1 = +102$ MPa.

$\sigma_2 = -102$ MPa.

$\sigma_3 = 0$ MPa

σ_1 at 45° clockwise from x.

σ_2 at 45° anticlockwise from x

σ_3 through plane of tube.

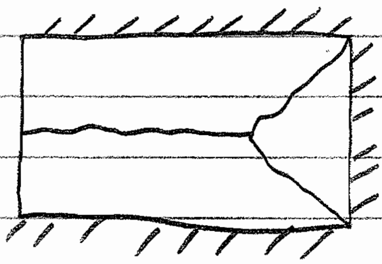
Q5 (a)

Compatible mechanisms in Fig. 5(a) (ii) (iv)

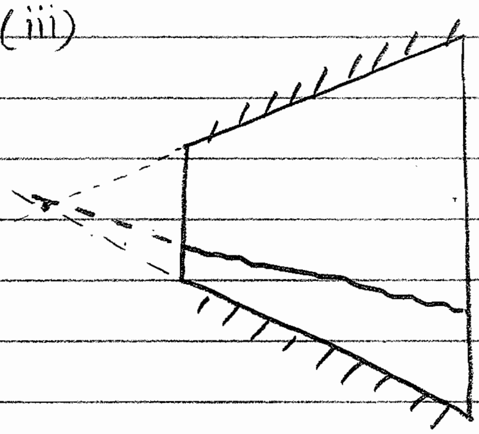
Incompatible mechanisms in Fig 5(a) (i) (iii)

Correct mechanisms for (i) & (iii)

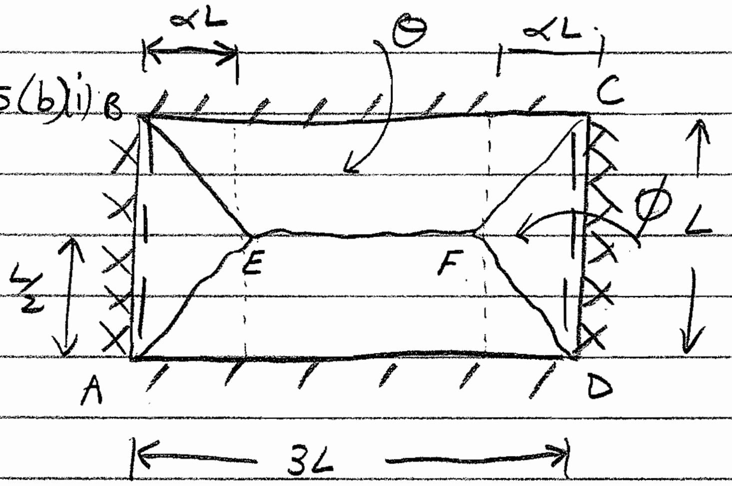
(i)



(iii)



Q5(b)(i)



Assume displacement $\delta = 1$ at E & F

$\theta = \frac{1}{L/2} = 2/L$

$\phi = \frac{1}{\alpha L}$

WORK DONE BY LOAD w. WD = Load Intensity × Volume.

Central region E-F (Vol. of triangular prism = end area × length)

$$WD_{EF} = w.\ \tfrac{1}{2} L \times L \times 1 \times (3L - 2\alpha L) = \frac{w}{2} L^2 (3 - 2\alpha)$$

End regions BA-E & F-CD. (Vol. of pyramid $= \frac{1}{3}$ vol. of cube)

$$WD_{ENDS} = w.\ \tfrac{1}{3}.2\alpha L.L.1 = \frac{2\alpha w L^2}{3}$$

$$\therefore WD_{TOTAL} = \frac{wL^2}{2}(3 - 2\alpha) + \frac{2\alpha w L^2}{3} = \frac{wL^2}{6}(9 - 6\alpha + 4\alpha)$$

$$= \frac{wL^2(9 - 2\alpha)}{6}$$

Q5(b)(i) cont.

<u>Energy dissipated in yieldlines</u> (use projection method)

$$\underset{TOTAL}{ED} = \underbrace{m.L.\phi.2}_{(BEA+CFD)} + \underbrace{m.3L.\theta.2}_{(BEFC+AEFD)} + \underbrace{m.L.\phi.2}_{(AB+CD)}$$

$$= 4mL\phi + 6mL\theta$$

$$= 4m\cancel{L}.\frac{1}{\alpha\cancel{L}} + 6m\cancel{L}.\frac{2}{\cancel{L}} = \frac{4m}{\alpha} + 12m. = 4m\left(\frac{1}{\alpha}+3\right)$$

For collapse WD = ED.

$$wL^2\left(\frac{9-2\alpha}{6}\right) = 4m\left(\frac{1}{\alpha}+3\right)$$

$$\therefore w = \frac{24m}{L^2}\frac{\left(\frac{1}{\alpha}+3\right)}{(9-2\alpha)} = \frac{24m(3\alpha+1)}{\alpha L^2(9-2\alpha)} \quad \text{as required.}$$

Q5(b)(ii) For optimum α. find $\frac{dw}{d\alpha} = 0$.

$$\frac{dw}{d\alpha} = \frac{24m}{L^2}\frac{d}{d\alpha}\frac{(3\alpha+1)}{\alpha(9-2\alpha)} = \frac{24m}{L^2}\frac{d}{d\alpha}\left[(3\alpha+1)(9\alpha-2\alpha^2)^{-1}\right]$$

$$= \frac{24m}{L^2}\left[3(9\alpha-2\alpha^2)^{-1} + (3\alpha+1)(-1)(9\alpha-2\alpha^2)^{-2}(9-4\alpha)\right]$$

$$= \frac{24m}{L^2}\left[\frac{3}{\alpha(9-2\alpha)} - \frac{(3\alpha+1)(9-4\alpha)}{\alpha^2(9-2\alpha)^2}\right]$$

$$= \frac{24m}{L^2}\left[\frac{3\alpha(9-2\alpha) - (3\alpha+1)(9-4\alpha)}{\alpha^2(9-2\alpha)^2}\right]$$

$$= \frac{24m}{L^2}\left[\frac{\cancel{27\alpha} - 6\alpha^2 - \cancel{27\alpha} + 12\alpha^2 - 9 + 4\alpha}{\alpha^2(9-2\alpha)}\right]$$

5/

Q 5(b)(ii) cont.

$$\frac{dw}{d\alpha} = \frac{24m}{L^2}\left[\frac{6\alpha^2+4\alpha-9}{\alpha^2(9-2\alpha)}\right]$$

$= 0$ when $\quad 6\alpha^2+4\alpha-9=0.$

$$\therefore \alpha = \frac{-4 \pm \sqrt{16+4.6.9}}{2.6} \qquad \text{using } \frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

$$= \frac{-4\pm\sqrt{232}}{12}. \qquad \text{+ve root only}$$

$$= 0.936$$

$\therefore \alpha = 0.94$ for minimum w.

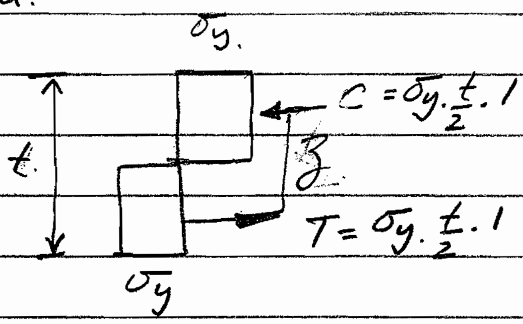
$$\therefore w = \frac{24m}{\alpha L^2}\left(\frac{3\alpha+1}{9-2\alpha}\right) = \frac{24m}{0.94L^2}\left(\frac{3\times0.94+1}{9-2\times0.94}\right)$$

$w = 13.7\frac{m}{L^2}$ is the least upper bound estimate of w.

Q5(c)(i) Valid to use steel as it is ductile and plastic analysis can be applied. Glass is brittle hence YLA is not valid.

Q5(c)(ii) For steel with $\sigma_y = 300$ MPa.

$$M_p = \sigma_y . Z_p . = \sigma_y \frac{t^2}{4}.$$

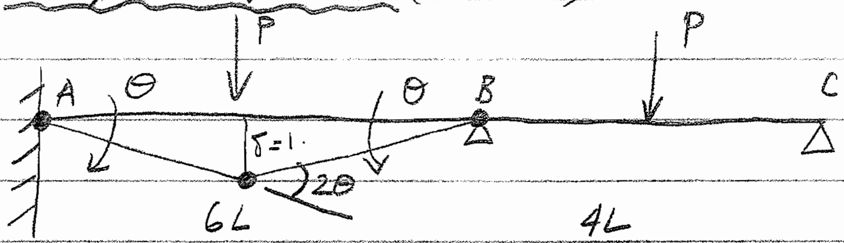


$\therefore$ for $M_p = 50$ kNm/m.

$$t = \sqrt{\frac{4M_p}{\sigma_y}} = \sqrt{\frac{4\times50\times10^3}{300\times10^6}} = 0.0258 \quad (26\text{mm})$$

$$M = T.z = \sigma_y\frac{t}{2}.\frac{t}{2} = \frac{\sigma_y t^2}{4}$$

Q 6(a) Collapse in span AB (Mode I)



Assume $\delta = 1$

Workdone $= P.1.$ $\qquad \theta = \frac{1}{3L}.$

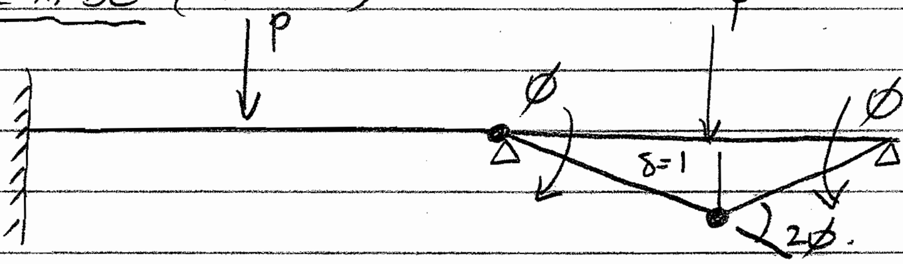
Energy dissipated $= 2M_p\theta + M_p.2\theta = 4M_p\theta$

$= \frac{4M_p}{3L}.$

For collapse WD = ED.

$\therefore \quad P = \frac{4M_p}{3L}. \qquad \left(\frac{8M_p}{6L}\right)$

Collapse in BC (Mode II)



Assume $\delta = 1.$

$\phi = \frac{1}{2L}$

$WD = P.1$

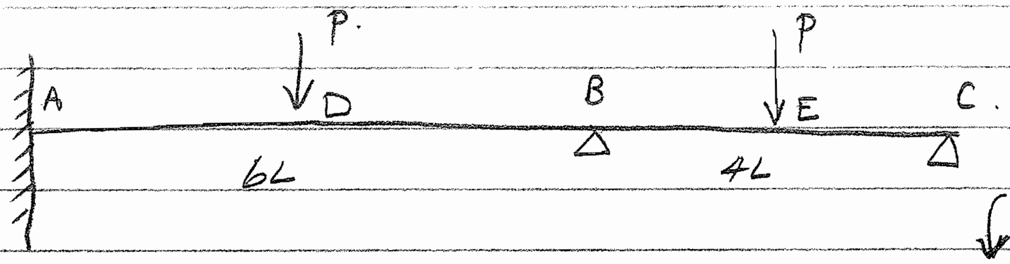
$ED = M_p.\phi + M_p.2\phi = 3M_p\phi.$

For collapse WD = ED.

$P = \frac{3M_p}{2L} \qquad \left(\frac{9M_p}{6L}\right)$

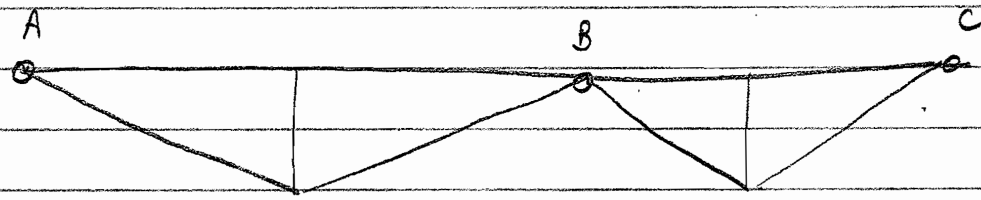
Hence maximum value (lowest upper bound) $P = \frac{4M_p}{3L}$ MODE I

Q6(b)



Particular Sol^n (assume pins at A & B)

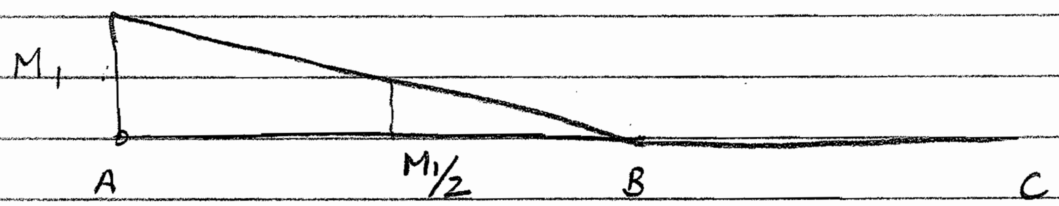
$M = \frac{WL}{4}$

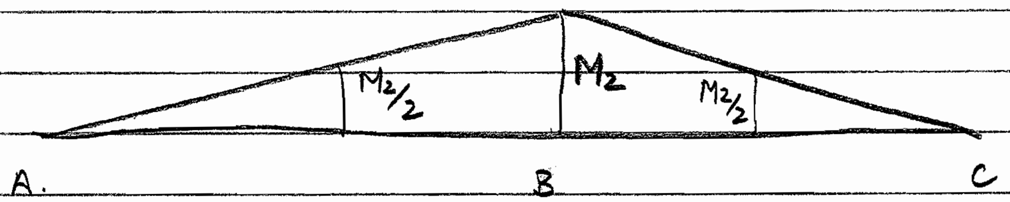


$M_D' = -P\left(\frac{6L}{4}\right)$ $\qquad$ $M_E' = -P\left(\frac{4L}{4}\right)$

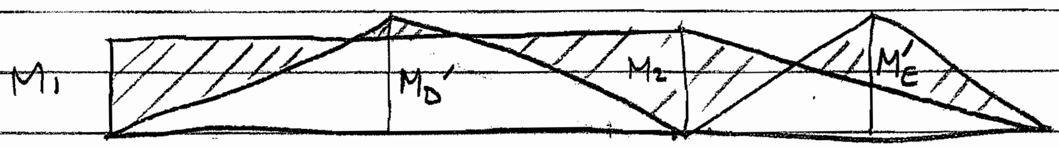
$= -\frac{3PL}{2}$ $\qquad$ $= -PL.$

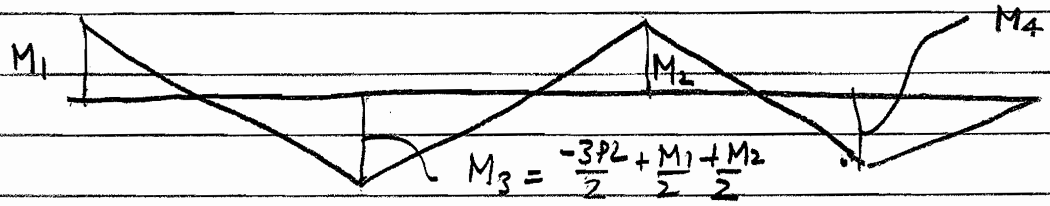
Self stress (impose moments at A & B)





General Sol^n (Particular + states of self stress)





$M_4 = -PL + \frac{M_2}{2}$

$M_3 = \frac{-3PL}{2} + \frac{M_1}{2} + \frac{M_2}{2}$

Assume $|M_1| = |M_2| = |M_3| = M_p$ (Failure in span AB)

$$M_3 = -M_p = -\frac{3PL}{2} + \frac{M_p}{2} + \frac{M_p}{2}$$

$$2M_p = \frac{3PL}{2}$$

$$P = \frac{4}{3}\frac{M_p}{L}$$

(Same as UB for span AB)
$\Rightarrow$ Unique sol^n.

Check moment M4 in span BC does not exceed Mp.

for $P = \frac{4}{3}\frac{M_p}{L}$ $\quad M_4 = -PL + \frac{M_2}{2} = -\frac{4}{3}\frac{M_p}{\cancel{L}}\cancel{L} + \frac{M_p}{2}$

$$= -\frac{8}{6}M_p + \frac{3M_p}{6} = -\frac{5}{6}M_p < M_p \text{ OK}$$

If guess $|M_4| = |M_2| = M_p$

$M_4 = -PL + \frac{M_2}{2}$ gives $-M_p = -PL + \frac{M_p}{2}$

$$\therefore P = \frac{3M_p}{2L}$$

then check M_3 for this value of P

$$M_3 = -\frac{3PL}{2} + \frac{M_1}{2} + \frac{M_2}{2} = -\frac{3}{2}\left(\frac{3M_p}{2\cancel{L}}\right)\cancel{L} + \frac{M_1}{2} + \frac{M_p}{2}$$

$$= -\frac{9}{4}M_p + \frac{M_1}{2} + \frac{M_p}{2} = -\frac{7}{4}M_p + \frac{M_1}{2}$$

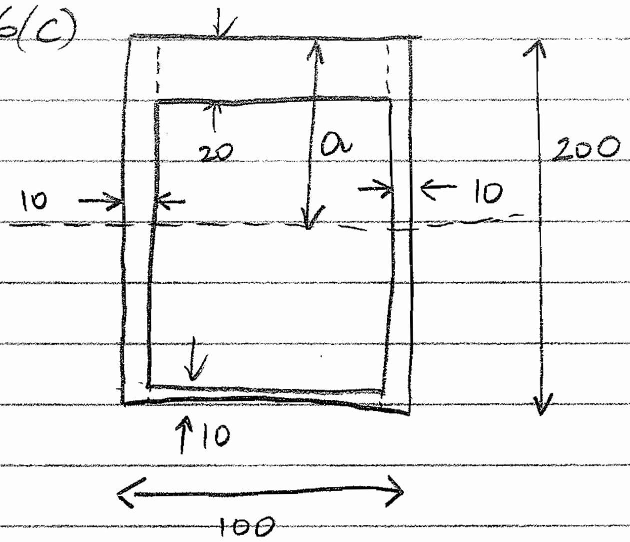
If $M_1 = 0$ $\quad M_3 = -\frac{7}{4}M_p \Rightarrow |M_3| > M_p$ not valid

If $M_1 = M_p$ $\quad M_3 = -\frac{7}{4}M_p + \frac{M_p}{2} = -\frac{7}{4}M_p + \frac{2}{4}M_p = -\frac{5}{4}M_p \Rightarrow |M_3| > M_p$

NOT VALID

$\therefore$ Maximum safe load $P = \frac{4M_p}{3L}$ (using lower bound theorem)

Q6(c)



$Z_p = \Sigma A_i y_i$

Equal area axis $80 \times \cancel{20}\,2 + 2 \times \cancel{10} \times a = 80 \times \cancel{10} + 2 \times \cancel{10} \times (200-a)$

$160 + 2a = 80 + 400 - 2a$

$4a = 320$

$a = 80 \text{ mm}$

$Z_p = 100 \times 20 \times 70 + 60 \times 10 \times 30 \times 2 + 110 \times 10 \times 55 \times 2 + 100 \times 10 \times 115$

$= 140 \times 10^3 + 36 \times 10^3 + 121 \times 10^3 + 115 \times 10^3$

$= 412 \times 10^3 \text{ mm}^3$

$M_p = \sigma_y Z_p = 300 \times 412 \times 10^3 = 123.6 \times 10^6 \text{ Nmm}$ (N/mm^2, mm^3)

$= 123.6 \text{ kNm}$

$P = \frac{4}{3}\frac{M_p}{L} = \frac{4}{3} \times \frac{123.6}{1} = 164.8 \text{ kN}$

Q6(d) Rotation would have no effect on the collapse load since final collapse is not affected by initial state of stress. (however it may effect order in which hinges form)

Q6(e) No. You cannot apply plastic theory to a brittle material such as FRP's since the theory relies on ductility of the components.