ENGINEERING TRIPOS PART IB 2006

PAPER 4 – THERMOFLUID MECHANICS

SOLUTIONS TO TRIPOS QUESTIONS

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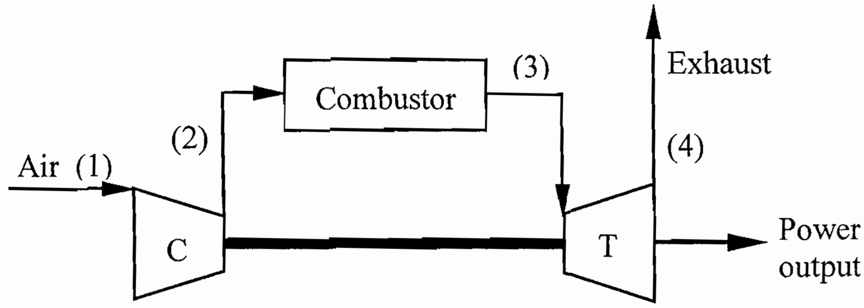
Section A

1 (a) Operation without steam injection :



For isentropic expansion in the turbine,

$$T_{4S} = T_3\left(\frac{p_4}{p_3}\right)^{(\gamma-1)/\gamma} = 1450\times\left(\frac{1}{20}\right)^{(0.35/1.35)} = 666.9\ \text{K}$$

Actual turbine outlet temperature is,

$$T_4 = T_3 - \eta_T(T_3 - T_{4S}) = 1450 - 0.85\times(1450-666.9) = 784.4\ \text{K}$$

Turbine power output is,

$$\dot{W}_T = \dot{m}_A c_p(T_3 - T_4) = 45\times1.10\times(1450-784.4) = 32.95\times10^3\ \text{kW} = 32.95\ \text{MW}$$

For isentropic compression in the compressor,

$$T_{2S} = T_1\left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = 290\times\left(\frac{20}{1}\right)^{(0.35/1.35)} = 630.5\ \text{K}$$

Actual compressor outlet temperature is,

$$T_2 = T_1 + \frac{(T_{2S}-T_1)}{\eta_C} = 290 + \frac{(630.5-290)}{0.85} = 690.6\ \text{K}$$

Compressor power requirement is,

$$\dot{W}_C = \dot{m}_A c_p(T_2 - T_1) = 45\times1.1\times(690.6-290) = 19.83\times10^3\ \text{kW} = 19.83\ \text{MW}$$

Heat input rate is,

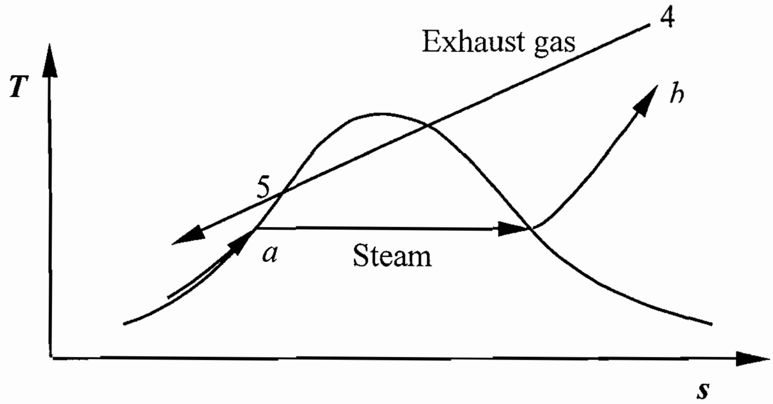
$$\dot{Q} = \dot{m}_A c_p(T_3 - T_2) = 45\times1.1\times(1450-690.6) = 37.59\times10^3\ \text{kW} = 37.59\ \text{MW}$$

Nett power output and thermal efficiency:

$$\dot{W} = \dot{W}_T - \dot{W}_C = 32.95 - 19.83 = 13.12\ \text{MW}$$

$$\eta_{Th} = \frac{\dot{W}}{\dot{Q}} = \frac{13.12}{37.59} = 0.349 \qquad [8]$$

(b) Operation with steam injection. (T-s) diagram for HRSG :



[2]

From the saturated steam tables at 20 bar, $T_a = 212.38$ °C = 485.5 K. Also, $h_a = 908.5$ kJ/kg.

From the superheated steam tables at 20 bar and 450 °C, $h_b = 3358.2$ K.

Pinch-point temperature difference is 5 °C. Hence, $T_5 = 485.5 + 5 = 490.5$ K.

To find the steam mass flowrate $\dot{m}_S$, apply the SFEE down to the pinch-point. Note that, with steam injection, the turbine outlet temperature, $T_4 = 784.4$ K, is unchanged. Hence :

$$(\dot{m}_A + \dot{m}_S)c_p(T_4 - T_5) = \dot{m}_S(h_b - h_a)$$

$$(45 + \dot{m}_S) \times 1.1 \times (784.4 - 490.5) = \dot{m}_S(3358.2 - 908.5)$$

$$\dot{m}_S = 6.84 \text{ kg/s}$$

Turbine power output with steam injection is,

$$\dot{W}_T = (\dot{m}_A + \dot{m}_S)c_p(T_3 - T_4) = (45 + 6.84) \times 1.1 \times (1450 - 784.4) = 37.96 \text{ MW}$$

The compressor power requirement is unchanged at 19.83 MW. Hence, the nett power output with steam injection (neglecting the feed pump work) is :

$$\dot{W} = \dot{W}_T - \dot{W}_C = 37.96 - 19.83 = 18.13 \text{ MW} \quad \text{(Increased from 13.12 MW)}$$ [10]

2 (a)

(i) FALSE : Entropy is an extensive, not an intensive property. [2]

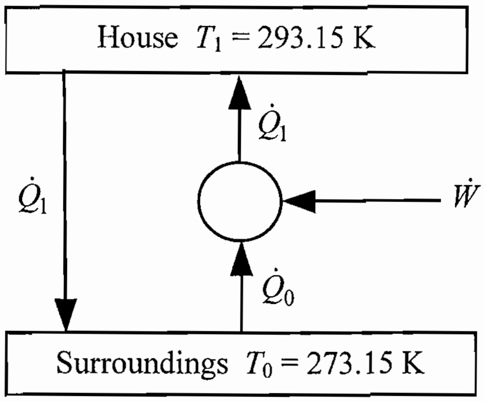
(ii) FALSE : The heat transferred to a closed system undergoing a reversible process is,

$\int_1^2 T\,dS$ which does not equal $(T_2S_2 - T_1S_1)$ unless T is constant. [2]

(iii) FALSE : The entropy of a closed system will decrease if the system is cooled. [2]

(iv) FALSE : A reversible, adiabatic process is isentropic and the work transfer has no effect on the system entropy. [2]

(b) (i) In winter, the heat pump operates as shown below :



Applying the Clausius inequality to the cyclic heat pump (noting the direction of heat flows) :

$$\frac{\dot{Q}_0}{T_0} - \frac{\dot{Q}_1}{T_1} \leq 0$$

Using the First Law then gives :

$$\frac{\dot{Q}_1 - \dot{W}}{T_0} - \frac{\dot{Q}_1}{T_1} \leq 0 \qquad \rightarrow \qquad \dot{W} \geq \dot{Q}_1\left(\frac{T_1 - T_0}{T_1}\right)$$

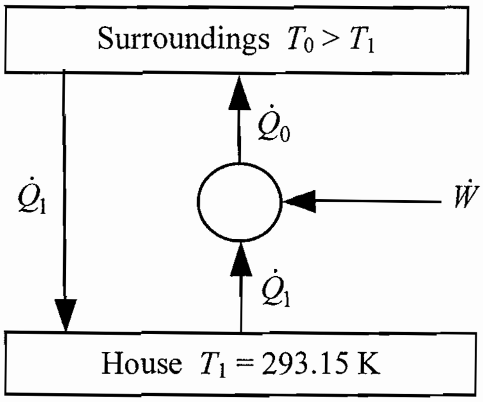
The required rate of heat transfer to the house in kW is given by,

$$\dot{Q}_1 = \frac{2400}{3600}(T_1 - T_0) = \frac{2}{3}(T_1 - T_0)$$

Therefore, the power input is,

$$\dot{W} \geq \frac{2}{3}\frac{(T_1 - T_0)^2}{T_1} = \frac{2 \times 20^2}{3 \times 293.15} = 0.91 \text{ kW}$$ [6]

(ii) In summer, the heat pump operates as shown below :



Applying the Clausius inequality to the cyclic heat pump (noting the direction of heat flows) :

$$\frac{\dot{Q}_1}{T_1} - \frac{\dot{Q}_0}{T_0} \leq 0$$

Using the First Law then gives :

$$\frac{\dot{Q}_1}{T_1} - \frac{\dot{Q}_1 + \dot{W}}{T_0} \leq 0 \qquad \rightarrow \qquad \dot{Q}_1\left(\frac{T_0 - T_1}{T_1}\right) \leq \dot{W}$$

The required rate of heat transfer from the house in kW is given by,

$$\dot{Q}_1 = \frac{2400}{3600}(T_0 - T_1) = \frac{2}{3}(T_0 - T_1)$$

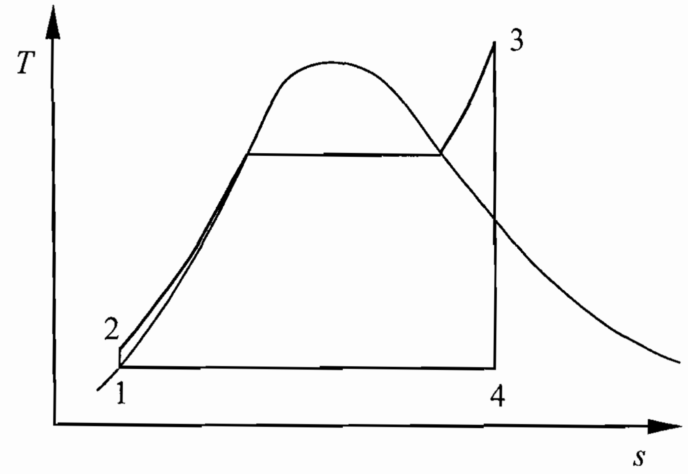
Hence, with $\dot{W} = 1.2$ kW :

$$(T_0 - T_1)^2 \leq \frac{3T_1\dot{W}}{2} = \frac{3 \times 293.15 \times 1.2}{2} = 527.7 \text{ (°C)}^2$$

Thus,

$$T_0 \leq T_1 + \sqrt{527.7} = 316.1 \text{ K} = 43.0 \text{ °C}$$ [6]

3 **(a)**



The following is based on the steam tables. Using the chart is quicker but less accurate.

(i) From tables at p_3 = 100 bar, T_3 = 550 °C : h_3 = 3502.0 kJ/kg, s_3 = 6.758 kJ/kg K.

Note that the turbine expansion is isentropic.

At condenser pressure of 0.06 bar, s_f = 0.521 kJ/kg K, s_g = 8.329 kJ/kg K. Hence :

$$6.758 = 0.521(1-x_4) + 8.329x_4 \quad \rightarrow \quad x_4 = 0.7988$$

At condenser pressure, h_f = 151.5 kJ/kg, h_g = 2566.6 kJ/kg. Hence :

$$h_4 = 151.5 \times (1.0 - 0.7988) + 2566.6 \times 0.7988 = 2080.7 \text{ kJ/kg}$$ [4]

(ii) The mass flowrate of steam is (neglecting the feed pump work),

$$\dot{m}_s = \frac{Power}{W_{net}} = \frac{Power}{h_3 - h_4} = \frac{500.0\times10^3}{3502.0 - 2080.7} = 351.8 \text{ kg/s}$$ [2]

(b) (i) Rate of heat transfer in the condenser :

$$\dot{Q} = \dot{m}_s(h_4 - h_1) = 351.8 \times (2080.7 - 151.5) = 678.69\times10^3 \text{ kW}$$

From the SFEE, the mass flowrate of cooling water is given by,

$$\dot{m}_w = \frac{\dot{Q}}{(h_{w,out} - h_{w,in})} = \frac{\dot{Q}}{c_{pw}(T_{w,out} - T_{w,in})} = \frac{678.69\times10^3}{4.18\times10} = 16237 \text{ kg/s}$$ [4]

(ii) If the total length of tubing of diameter D is L, then the rate of heat transfer is given by,

$$\dot{Q} = \pi D L \, U \, \Delta T_m$$

where U is the overall heat transfer coefficient and ΔT_m is the *Log Mean Temperature Difference*. The saturated vapour temperature at 0.06 bar is 36.16 °C and hence, from the definition of ΔT_m in the Thermofluids Data Book,

$$\Delta T_m = \frac{\Delta T_{in} - \Delta T_{out}}{\ln(\Delta T_{in}/\Delta T_{out})} = \frac{(36.16-15.0) - (36.16-25.0)}{\ln(21.16/11.16)} = 15.63 \text{ °C}$$

The total length of condenser tubing required is thus,

$$L = \frac{\dot{Q}}{\pi D \, U \, \Delta T_m} = \frac{678.69 \times 10^3}{\pi \times 0.03 \times 12.0 \times 15.63} = 38.39 \times 10^3 \text{m} = 38.39 \text{ km}$$ [5]

(iii) For steady flow, the entropy equation for a CV with multiple inlet and outlet streams is,

$$\sum_i \dot{m}_i (s_{i,out} - s_{i,in}) = \int_{Surface} \frac{d\dot{Q}}{T} + \dot{S}_{irrev}$$

The specific entropy of the steam at turbine outlet is $s_4 = s_3 = 6.758$ kJ/kg K. Hence, the contribution from the steam side is,

$$\dot{m}_s (s_1 - s_4) = 351.8 \times (0.521 - 6.758) = -2.194 \times 10^3 \text{ kW/K}$$

There is no pressure loss in the tubes and c_p for water is constant. From $Tds = dh - vdp$,

$$s_{w,out} - s_{w,in} = \int_{in}^{out} ds_w = \int_{in}^{out} \frac{dh_w}{T} = \int_{in}^{out} \frac{c_{pw} dT}{T} = c_{pw} \ln\left(\frac{T_{w,out}}{T_{w,in}}\right)$$

Hence, the contribution from the water side is,

$$\dot{m}_w (s_{w,out} - s_{w,in}) = \dot{m}_w c_{pw} \ln\left(\frac{T_{w,out}}{T_{w,in}}\right) = 16237 \times 4.18 \times \ln\left(\frac{298.15}{288.15}\right) = 2.315 \times 10^3 \text{ kW/K}$$

There is no heat loss from the condenser casing to the environment. Therefore, the nett rate of entropy creation due to irreversibility in the condenser is,

$$\dot{S}_{irrev} = 2.315 \times 10^3 - 2.194 \times 10^3 = 121 \text{ kW/K}$$ [5]

This is due to the temperature difference between the steam and the cooling water.

Section B

4 **(a)** Full dynamic similarity requires that both the Mach number and the Reynolds number are matched for the full-size aircraft and the model. Using subscript 'r' for the real aircraft and 'm' for the model.

Mach Number similarity;

$$\begin{aligned} \frac{V_r}{\sqrt{\gamma R T_r}} &= \frac{V_m}{\sqrt{\gamma R T_m}} \\ V_m &= V_r \sqrt{T_m/T_r} \end{aligned} \tag{1}$$

Reynolds number similarity;

$$\begin{aligned} \frac{\rho_r V_r L_r}{\mu_r} &= \frac{\rho_m V_m L_m}{\mu_m} \\ \frac{\rho_m}{\rho_r} &= \frac{\mu_m}{\mu_r} \frac{V_r}{V_m} \frac{L_r}{L_m} \end{aligned} \tag{2}$$

This relates to the density ratio which depends on the pressure and temperature. To get an expression for the pressure we use the perfect gas law which leads to;

$$\frac{\rho_m}{\rho_r} = \frac{p_m}{p_r} \frac{T_r}{T_m} \tag{3}$$

and substituting this into (2) and also using (1) and the expression for the dependence of the viscosity on the temperature we find

$$\frac{p_m}{p_r} \frac{T_r}{T_m} = \frac{T_m^{3/2}}{T_m + 117} \frac{T_r + 117}{T_r^{3/2}} \sqrt{\frac{T_r}{T_m}} \frac{L_r}{L_m} \tag{4}$$

Now in terms of our new symbol

$$\alpha = \frac{T_m}{T_r} \tag{5}$$

$$\frac{p_m}{p_r} = \alpha^2 \frac{1 + 117/T_r}{\alpha + 117/T_r} \frac{L_r}{L_m} \tag{6}$$

which due to a convenient choice of the value of T_r in this problem and putting in the scale-factor of 8 and the value of T_r becomes

$$\frac{p_m}{p_r} = 8\alpha^2 \frac{3/2}{\alpha + 1/2} \tag{7}$$

(b)

In this part of the problem $T_m = 288\text{K}$ and $T_r = 234\text{K}$ hence $\alpha = 1.231$ which leads to $p_m/p_r = 10.51$ and hence $p_m = 354\text{kN/m}^2$. For interest this is about 3 atmospheres which is quite achievable in readily-available wind tunnels. The velocity is $V_m = \sqrt{\alpha} V_r = 305\text{m/s}$.

(c)

Since $C_L = f(M, Re)$ then complete dynamic similarity means the lift coefficient will be the same for the real aircraft and the model.

$$\begin{aligned}
\frac{\text{Lift}_m}{\rho_m V_m^2 L_m^2} &= \frac{\text{Lift}_a}{\rho_r V_r^2 L_r^2} \\
\frac{\text{Lift}_m}{\text{Lift}_a} &= \frac{\rho_m}{\rho_r}\frac{V_m^2}{V_r^2}\frac{L_m^2}{L_r^2} \\
&= \frac{p_m}{p_r}\frac{L_m^2}{L_r^2} \\
&= 10.51/64 = 0.164 \qquad (8)
\end{aligned}$$

(d)

For the last part we make use of the equation for the pressure ratio derived above in terms of the temperature ratio α but this time we fix the pressure ratio and solve for α. This turns out not to be too hard for this problem since it reduces to a quadratic equation for α. The pressure ratio has been chosen to be exactly three to simplify the maths . Hence we get

$$\begin{aligned}
3 &= \frac{12\alpha^2}{\alpha + 1/2} \\
\alpha^2 - \alpha/4 - 1/8 &= 0
\end{aligned} \qquad (9)$$

The solution drops out nicely due to careful choice of the numbers. There is one positive solution and one negative.

$$\alpha = \frac{\frac{1}{4} \pm \sqrt{\frac{1}{16} + \frac{8}{16}}}{2} = \frac{1}{2}, -\frac{1}{4} \qquad (10)$$

Only the positive solution is sensible and we find $\alpha = 1/2$ and hence $T_m = 117$K. This is quite cold but there are large cryogenic wind-tunnels available which could achieve this. The velocity in this case is 194.5 m/s. For interest (not needed for the exam question) the basic power required to run the wind-tunnel (excluding coolers and pumps to pressurize etc.) scales with ρV^3 and for these two cases the power required in the first case is almost five times that required in the very cold case in part c) so there are advantages to cooling the working section. It is also convenient in this case that the working section is maintained at a pressure close to standard atmospheric pressure since this simplifies the structural design of the working-section and makes access from outside easier.

5 (**a**) Use mass conservation which in this case of incompressible flow of constant density is the continuity equation.

$$U_1 \frac{\pi D_1^2}{4} = U_2 \frac{\pi D_2^2}{4}$$
$$U_2 = U_1 \frac{D_1^2}{D_2^2} \tag{1}$$

(**b**) Bernoulli equation

$$\begin{aligned} p_1 + \tfrac{1}{2}\rho U_1^2 &= p_2 + \tfrac{1}{2}\rho U_2^2 \\ p_2 &= p_1 + \tfrac{1}{2}\rho(U_1^2 - U_2^2) \\ &= p_1 + \tfrac{1}{2}\rho U_1^2(1 - (D_1/D_2)^4) \end{aligned} \tag{2}$$

(**c**) Use the momentum equation. Forces on the control volume are due to the pressures on each end - we ignore friction which would result in additional tangential forces on the outside of the cylindrical control volume. The forces are then balanced by the rate of change of momentum.

$$\begin{aligned} \Sigma\text{Forces} &= \text{Rate of change of momentum} \\ (p_2 - p_3)\frac{\pi D_1^2}{4} &= -\rho \frac{\pi D_2^2}{4} U_2^2 + \rho \frac{\pi D_1^2}{4} U_1^2 \\ (p_2 - p_3) &= \rho U_1^2 (1 - (D_1/D_2)^2) \\ p_3 &= p_2 - \rho U_1^2 (1 - (D_1/D_2)^2) \\ &= p_1 + \tfrac{1}{2}\rho U_1^2 (1 - (D_1/D_2)^4 + \rho U_1^2((D_1/D_2)^2 - 1) \\ &= p_1 - \tfrac{1}{2}\rho U_1^2 \left[1 - 2\left(\frac{D_1}{D_2}\right)^2 + \left(\frac{D_1}{D_2}\right)^4\right] \\ &= p_1 - \tfrac{1}{2}\rho U_1^2 \left[\left(\frac{D_1}{D_2}\right)^2 - 1\right]^2 \end{aligned} \tag{3}$$

Difference in flux of mechanical energy

$$\begin{aligned} &= (\dot{m}/\rho)\Delta P_o \\ &= U_1 \frac{\pi D_1^2}{4}(p_1 - p_3) \\ &= \rho U_1^3 \frac{\pi D_1^2}{4}\left(\frac{D_1^2}{D_2^2} - 1\right)^2 \end{aligned} \tag{4}$$

This lost mechanical energy is converted into internal energy of the fluid so the fluid heats up very slightly.

6 **(a)** The only forces in the x-direction are due to the component of the gravitational

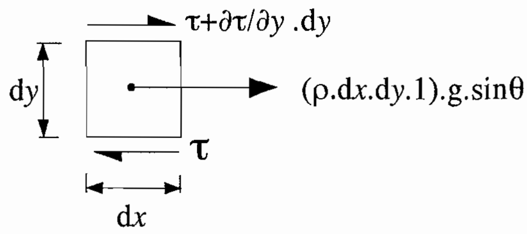


Figure 1:

force along the plate and the difference in shear stress between the top and the bottom of the element. Summing the forces in the x-direction and equating to zero since the element is in equilibrium (not accelerating),

$$(\tau + \partial\tau/\partial y.\mathrm{d}y).\mathrm{d}x.1 - \tau.\mathrm{d}x.1 + (\rho.\mathrm{d}x.\mathrm{d}y.1)g\sin\theta \;=\; 0 \tag{1}$$

Expand and simplify dividing through by $\mathrm{d}x.\mathrm{d}y$ and rearranging gives the answer.
(Note: Strictly we start with $\mathrm{d}x$ and $\mathrm{d}y$ as small finite quantities and divide through then take the limit as they go to zero. If $\mathrm{d}y$ is finite then the expansion of τ with y would involve higher order terms (and derivatives) which disappear in the limit as $\mathrm{d}y \to 0$. Hence the above is correct but not mathematically rigorous.)
The only difference between the two layers is the density and so the only difference in the equation is the change of density.

(b) Upper layer. To find the shear stress integrate the equation from part a) with respect to y after substituting $\rho = \rho_2$ for the case of the upper layer.

$$\tau_u \;=\; -\rho_2 g\sin\theta y + C \tag{2}$$

where C is a constant which may be found from the boundary condition that at $y = h_2$, $\tau = 0$ since it is a free surface and *the air above the upper film offers no resistance to the flow* and hence $C = \rho_2 g\sin\theta h_2$ which leads to the shear stress variation in the upper layer which is

$$\tau_u \;=\; \rho_2 g\sin\theta(h_2 - y) \tag{3}$$

Now at the bottom of the upper layer $y = h_1$ and so the shear stress is

$$\tau_u(y = h_1) \;=\; \rho_2 g\sin\theta(h_2 - h_1) \tag{4}$$

This does not depend on the viscosity of the fluid (or the density of the lower layer) since it is simply the force per unit area required to support the weight of the upper layer. The mass of the upper layer per unit area is $\rho_2(h_2 - h_1)$ and the component of the weight of this upper layer parallel to the plane is then just $\rho_2 g \sin\theta(h_2 - h_1)$. Changing the viscosity would lead to a change in the velocity variation in the layer which would adjust itself so that the shear stress was sufficient to support the weight of the upper layer (alternatively the thickness of the layers would change).

(c) Lower layer. To find the shear stress integrate the equation from part a) with respect to y after substituting $\rho = \rho_1$ for the case of the lower layer.

$$\tau_l = -\rho_1 g \sin\theta y + C \tag{5}$$

where C is a constant which may be found from the boundary condition (or matching condition) that at $y = h_1$, $\tau_l = \tau_u(y = h_1)$ which gives $C = \rho_2 g \sin\theta(h_2 - h_1) + \rho_1 g \sin\theta h_1$ which leads to the shear stress variation in the lower layer which is

$$\tau = \rho_1 g \sin\theta(h_1 - y) + \rho_2 g \sin\theta(h_2 - h_1) \tag{6}$$

(d) Since the fluid is Newtonian then $\tau = \mu du/dy$ and substituting into the expression for τ we can then integrate again to find the velocity.

$$u = \frac{\rho_1 g \sin\theta}{\mu}(h_1 y - \tfrac{1}{2}y^2) + \frac{\rho_2 g \sin\theta}{\mu}(h_2 - h_1)y + D \tag{7}$$

where D is a constant to be found from the boundary condition which for the lower layer is $u = 0$ at $y = 0$ and hence $D = 0$ so

$$u = \frac{\rho_1 g \sin\theta}{\mu}(h_1 y - \tfrac{1}{2}y^2) + \frac{\rho_2 g \sin\theta}{\mu}(h_2 - h_1)y \tag{8}$$

and finally we can find the value of the velocity at the interface by substituting $y = h_1$ giving

$$u = \frac{\rho_1 g \sin\theta}{\mu}\tfrac{1}{2}h_1^2 + \frac{\rho_2 g \sin\theta}{\mu}(h_2 - h_1)h_1 \tag{9}$$