ENGINEERING TRIPOS PART IB

Paper 5 ELECTRICAL ENGINEERING **Crib**

Q1

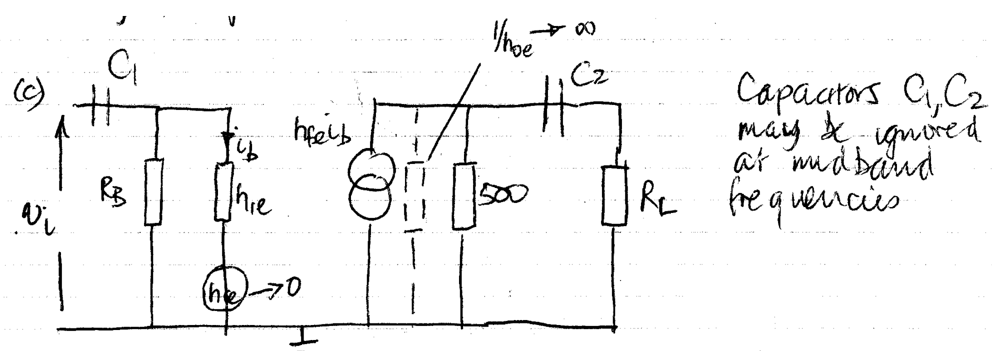
(a) Operating point. For $V_{CE} = 10$ V at $I_e = 20$ mA, we get $R_C = 10/ 20.10^{-3} = 500\ \Omega$.

For R_B. $I_B = I_C/250 = 20/250 = 80\ \mu A$.

$R_B = (20 – 0.7) / 0.08 = 241\ k\Omega$

b) Limitations. The bias point is heavily dependent on the transistor's current gain h_{FE} which may vary over a range 5:1 in typical products, and also on temperature. Improvements are 1/ connect R_B to the collector; negative feedback enhances the stability; or 2/ set up a potential divider for V_g and include an emitter resistor. This needs to be bypassed with a suitable capacitor to maintain the a.c. gain.

c)



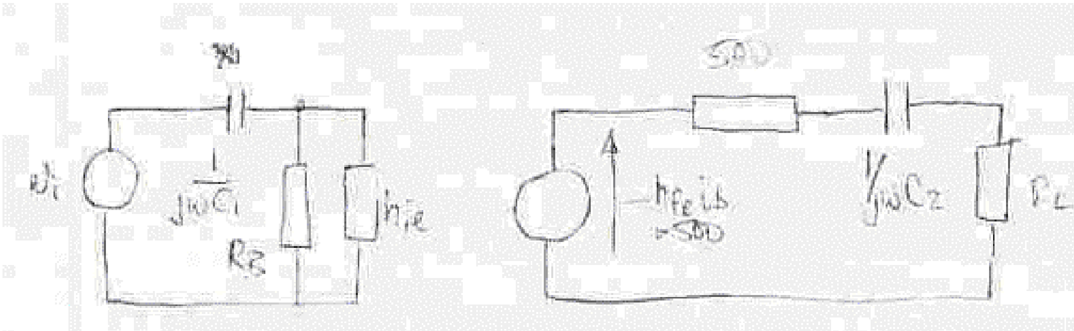
The Thévenin equivalent of the output circuit is a voltage source of $500 \times h_{fe}\, i_b$ in series with a Thévenin resistance of 500 Ω. Capacitors C_1 and C_2 may be ignored at mid-band. Hence the optimum load for maximum power transfer is $R_L = 500$ ohm.

Mid-band voltage gain is then,

$- h_{fe} . (1/h_{ie}) . (500 // 500) = 250/ 4 = 62.5$

(d)

At low frequencies, C_1 and C_2 have large impedances $\propto 1/\omega C$ and in conjunction with circuit resistances, each forms a potential divider. At low frequencies C_1 causes the base current to be reduced; C_2 limits the output current and hence the voltage developed across the load is reduced.



The output voltage v_O is developed across the terminals of R_L, as before.

Examiner's note: a common error was to take v_O *across the series combination of* C_2 *and* R_L

$$i_b = \frac{v_i}{h_{ie} + 1/j\omega C_1}$$

$$v_0 = \frac{R_L}{R_L + R_C + 1/j\omega C_2} . h_{fe} . R_c i_b$$

Hence

$$G = -\frac{h_{fe} R_C R_L}{(h_{ie} + 1/j\omega C_1)(R_L + R_C + 1/j\omega C_2)}$$

Check: as $\omega \to$ infinity, G $\to$ $-\frac{h_{fe}}{h_{ie}}(R_C // R_L)$ which is the mid-band gain.

At 100 Hz, $\omega = 628$

G = −7.68 + 15.91j

|G| = − 17.67

Q2

a/ Let v_+ and v_- be the voltages at the + and − inputs of the op-amp. If $v_{out} = G(v_+ - v_-) + G'(v_+ + v_-)$, then CMRR = G/G'.
This is important because two signals v_1, v_2 can always be broken down into components
$v_1 = v_c + v_d/2$, and $v_2 = v_c - v_d/2$, where v_c is the average or *common-mode* component and v_d is the difference, or *differential* component. In precision instrumentation it's often necessary to extract a weak signal in the presence of strong interference, often in the form of a common-mode signal. If the wanted signal can be developed as a differential signal, strong discrimination can be obtained using a differential amplifier with high CMRR.

b/ If v_+ and v_- are the voltages at the + and − inputs of the op-amp, then

$v_+ = R_3/(R_3 + R_4)\, v_q$, $v_- - v_0 = R_2/(R_1 + R_2)\, v_p$. For $G \to$ infinity, then $v_+ = v_-$
Thus $0 = R_2/(R_1 + R_2)(v_p - v_q) + R_1/(R_1 + R_2) v_0$ and $v_0 = R_2/R_1 (v_q - v_p)$

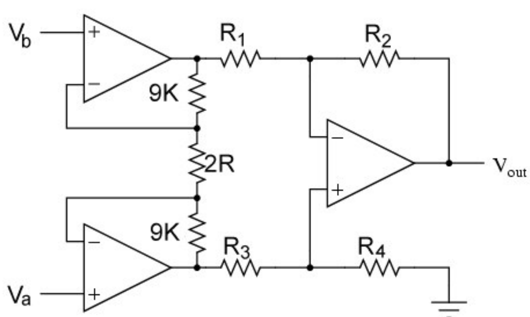
c/ In Fig. 3, $v_a = v_q + v_c$, $v_b = v_p + v_c$.

Hence, $V_0 = R_2/R_1 (v_q - v_p) = R_2/R_1 (v_a + v_c - v_b - v_c) = R_2/R_1 (v_a - v_b)$

d/ Non-idealities are finite gain, finite bandwidth, finite input resistances and finite output resistance, and distortions. All of these limit the accuracy of the assumption that $v_1 = v_2$. Finite gain is the biggest effect, and this would limit the cancellation.

e/ Differential gain required = $\dfrac{5}{50\times10^{-3} + (-50\times10^{-3})} = 50$.
This is equivalent to R_2/R_1, so $R_4 = R_2 = 500\ \text{K}\Omega$.

f/ The voltage gain G1 of Fig. 4 is given by G1 = 1 + 9000/R.



Connect two instances of Fig. 4 at each input of Fig. 3, as shown. The resultant circuit is the widely used Instrumentation Amplifier. Because of the symmetry of the circuit there is no need for the two resistors R to be grounded. This reduces the common mode gain. The extra stages:

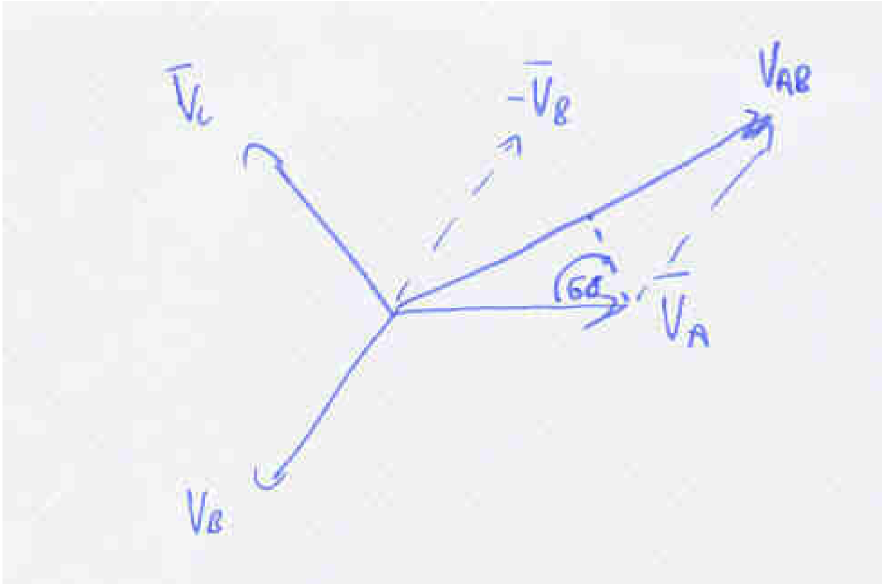
- provide additional gain
- ensure inputs *a* & *b* drive high impedance
- enhance the CMRR.

Overall gain can be set by adjusting only one resistor, R.

With R2=500 kΩ, R1 = 10 kΩ, the differential gain of Fig. 3 is 50. To achieve overall differential gain of 1000, the input amplifiers must contribute additional gain of 20. Hence choose R such that 1 + 9000/R = 20, so R = 473.7 Ω, or resistor 2R = 947 Ω.

Examiner's note: several candidates connected an instance of Fig. 4 at the output of Fig. 3. This would achieve more gain but would not improve the CMRR or input impedance.

Q3



$V_L = \sqrt{3}\ V_{ph}$. angle 60 $\qquad$ $I_L = I_{ph}$

b/ Star – $V_{ph} = V_L / 3^{1/2} = 240$ V

$I_{ph} = V_{ph} / \| 40 + 30j \| = 240/50 = 4.8$ A

$P_1 = 3\ I^2\ R =$ **2765 W**
$Q_1 = 3I^2\ X =$ **2074 VA**

Delta
$V_{ph} = V_L = 415$V

$I_{ph} = V_{ph} /| 60 + 80j | = 415/100 = 4.15$ A

$P_2 = 3\ I^2\ R =$ **3100 W**
$Q_2 = 3\ I^2\ X =$ **4133** VA

$P_{tot} =$ **5865 W**, $Q_{tot} =$ **6207 VA** $\qquad$ hence **S = 8.55 kVA** **pfactor** = P/S = **0.687**

c)
$I_L = S/(\sqrt{3}.V_L) = 8550 /(1.732x\ 415) = 11.87$ A

Line Losses are $P' = 3\ I^2\ r = 3\ (11.87)^2\ 0.8 = 338$ W
$Q' = 3\ I^2\ x = 1352$ VA

Hence new P = 338 + 5865 = 6203 W, new Q = 7559 VA
new S = 9,778 W

V(supply) = $S/\sqrt{3}\ I_L = 9778 /1.732 \times 11.87 =$ **475 V**

d) For pf =0, then P_{tot} remains same, but Q_{tot} = 0, so S_{tot} = P_{tot} = 5865 W.

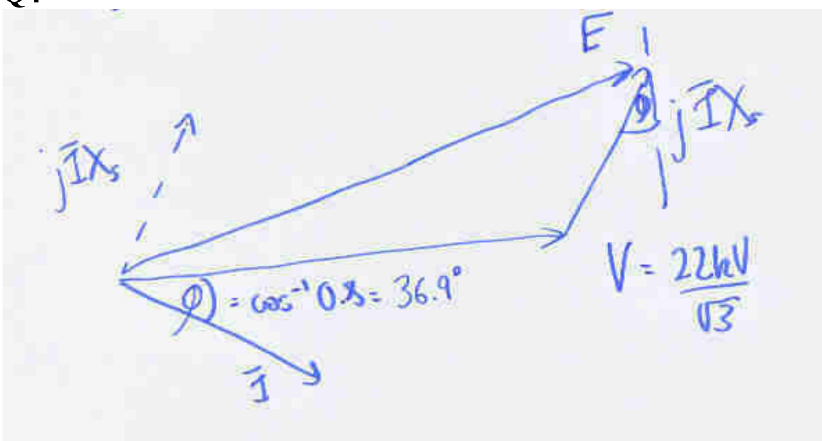
I_L = S/ $3^{1/2}$ V_L = 5865 / 1.732 x 415 = **8.15 A**

New losses P' = 159.5, Q' = 638 VA

Hence P'' = 6025 W, Q'' = 638 VA, S'' = 6059 W

New V = S''/$3^{1/2}$ I_L = 6059/ 1.732 x 8.15 = **429.2 V**

Q4



a) V_L = 22 kV, V_{ph} = 22/ 1.732 = 12.70 kV

I_{ph} = I_L = P /(1.732 x V_L cos (phi) = 320 /1.732 x 22 x 0.8 = **10.50 kA**

IX = 1.5 x 1.2 = 12.6 kV

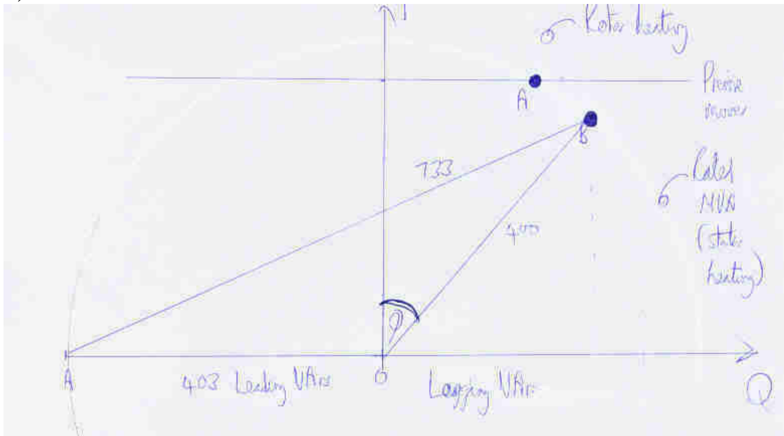
$E^2 = V_{ph}^2 + (IX)^2$ + 2 V (IX) cos (90 – phi) = $12.7^2 + 12.6^2$ + 2x12.6x 12.7 x 0.6 = 512.1

E_{ph} = 22.63 kV, **E_L = 39.2 kV**

b) H = IX cos phi = 12.6 x 0.8 = 10.08 kV

H' = (240/320) x 10.08 = 7.56 kV

$E'^2 = 12.7^2 + 7.56^2$ E'_{ph} = 14.78 kV **E_L = 25.6 kV** c)



power limit set by 350/400 = 0.875   cos phi1 = **0.875**

excitation limit set by

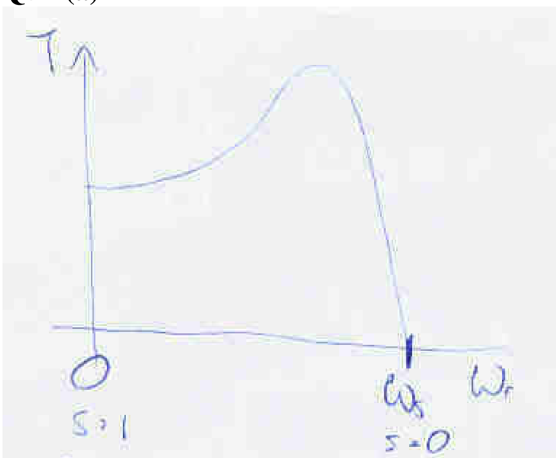
E_{ph} = 40 / 1.732 = 23.1 kV

3VI = 400, $3V^2/X$ = 403,   3VE/X = 733

so $733^2 = 403^2 + 400^2 + 2 \times 403 \times 400 \cos(\text{phi2} + 90)$

cos (phi2) **= 0.746**

Q5 (a)



(b) no load test 1500 rpm, $s = 0$ R_2'/s = infinity

assume R_i X_m are much larger than X_1 and R_1.

$3V/R_i = 600$ W, **R_i = 861 ohm.**

$I^2 = I_X^2 + I_R^2$ $16/3 = (415/X)^2 + (415/861)^2$ **X = 184 ohm**

Locked rotor test 0 rpm, s = 1.

X1 = X2'

$3 I^2 (R1 + R2) = 333$, R1 = 2, so **R2' = 3.2 ohm**

$$\frac{30^2}{(2X1)^2 + (2 + 3.2)^2} = \left(\frac{8}{\sqrt{3}}\right)^2$$ so 2X1 = 64/3 **X1 = X2 = 1.95 ohm**

c)
speed = 1480 rpm,
s = 1500 – 1480/1500 = **0.0133**

$$Z1 = R1 + jX1 + \frac{jXm(jX2' + R2'/s)}{j(Xm + X2') + R2'/s}$$

Z1 = 2 + j1 + (j100 (2/0.0133 + j1)/(j101 + 2/0.0133 + j1) = **47.9 + j70.1 ohm**

I_L = 1.732 x 415 / |47.9 + j70.1) = 1.732 x 4.89 = **8.5 A**

Q6

The characteristic impedance of a line Z_0 is the ratio of volts to current of a unidirectional wave on the transmission line.

C = 80 pF/m, L = 0.2 uH/m so $Z_0 = (L/C)^{1/2} = (2.10^{-7}/ 80 .10^{-12})^{½}$ = **50 ohm**

Phase velocity = $1/(LC)^{1/2} = 1/(2.10^{-7} \text{ x } 80.\ 10^{-12})^{1/2}$ = **2.5 x 10^8 m/s.**

For l = 20 m, wavelength should be a least 16 times longer, or 320 m.

So f = 2.5 10^8 /320 = **7.81 10^5 Hz**.

b) initially, line appears to have an infinite load.

Incident wave has

$$V = \frac{Z_0}{Z_0 + Z_S} V_S = \frac{50}{50+150}.16 = 4V$$

c) incident power = 4^2/ 50 = **0.32 W**

i) Z_L = infinity. reflection coef = $\frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\infty - 50}{\infty + 50} = 1$

Hence reflected power = 0.32 W

ii) Z_L = 0. refl coef = $\frac{0-50}{0+50} = -1$

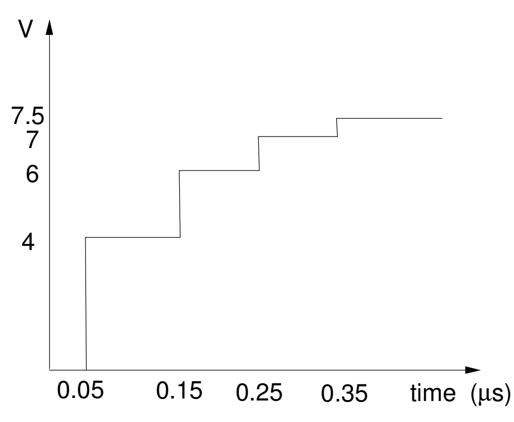
hence reflected power = 0.32 W, also

iii) Z_L = 50 Ω r = $\frac{50-50}{50+50} = 0$ so reflected power = 0 W

For Z_L = 150, r = $\frac{150-50}{150+50}$ = 0.5

Z_S = 150 ohm, also, where r = 0.5

So wave loses 50% of volts magnitude at reflections at each end.
Propagation time = 20/ 2.10^8 = 0.1 us



Q7

a) Intensity = gain . power / surface area = 2000 x 25/ 4π $[2.5\ 10^7]^2 = 6.37.10^{-12}$ W/m^2

Peak power at antenna = power density x area = 2 x = $12.7\ .10^{-12}$ W

All power goes to receiver for perfect matching

$I^2R = 12.7\ .10^{-12}$ $\quad$ $I = 5.0x\ 10^{-7}$ A

b)

$H = u_y\ H_0 \exp(jwt - jkz)$

The Maxwell eqn for free space is $\dfrac{\partial D}{\partial t} = \dfrac{\partial H_y}{\partial z}$

So $dD/dt = -jk\ H_0$ giving

$D = -(jk/jw)\ H_0$

And

$E = D/\varepsilon_0 = (k/w)/\varepsilon_0\ H = 1/(c\ \varepsilon_0)\ H = \sqrt{\mu\varepsilon}/\varepsilon H = \sqrt{\dfrac{\mu}{\varepsilon}H} = Z_0 H$

Or $E_z = Z_0.\ H_y$