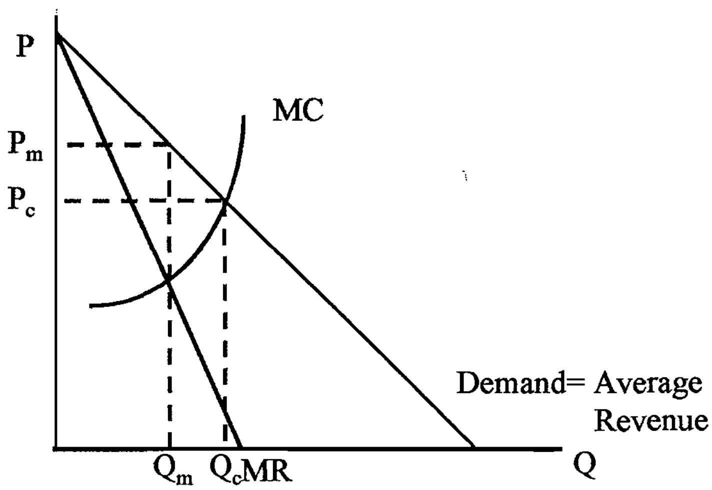
[Version 1 2006]

Engineering IB
Paper 8

SECTION A *Business Economics [CRIBS]*
Answer not more than one question from this section

1 (a) Explain the concept of monopoly. [6]

A monopoly is a firm that is the sole supplier of a certain product (almost always by virtue of being protected by some or other barrier to entry certain barriers to entry). Sources of monopoly power include horizontal integration, vertical integration, the creation of a statutory monopolies and franchises and licences. As shown below, a market characterised by monopoly has a higher price and less goods consumed compared to a perfectly competitive market.

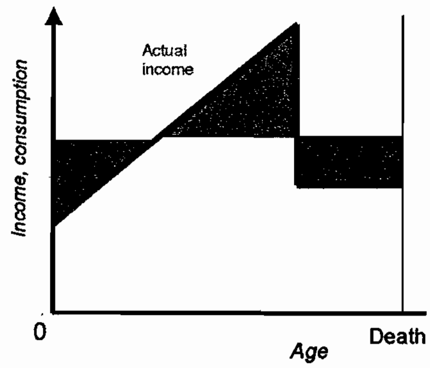


(b) What are the economic arguments against monopolies? [4]

Monopolist can take the market demand curve as its own demand curve and can enjoy some power over the setting of price or output. The standard case against a monopoly is that these businesses can earn abnormal profits at the expense of economic efficiency. The monopolist is extracting a price from consumers that is above the cost of resources used in making the product. Consumers' needs and wants are not being satisfied, as the product is being under-consumed.

(c) Outline the Life Cycle hypothesis of consumption. [6]

The Life-Cycle Hypothesis considers consumption also being as a constant proportion of long-run or normal income. The LCH emphasises the age of the consumer, and proposes that he/she attempts to smooth consumption over a lifetime in which income fluctuates widely. As shown below the hypothesis suggests, under normal conditions, dissaving in youth and old age.



(d) Assuming that the Life Cycle hypothesis is correct, how would an individual's consumption be influenced if he or she won £1million on the lottery? [4]

Although this windfall is a significant increase in actual income it is a much smaller increase in lifetime or permanent income. He or she should spread their consumption over their lifetime reflecting the estimated increase in normal income. The latter should factor in the rate off interest and the returns on other assets (including consumer durables).

2 (a) Explain the concept of profit maximisation? [4]

Profits are maximised when the next unit produced and sold, adds as much to total revenue as it does to total cost. Profit maximisation occurs when marginal revenue = marginal cost (MR=MC). If MR exceeds MC profit can be increased by increasing production, if MC exceeds MR profit can be increased by cutting back on production.

(b) Under what circumstances would firms *not* seek to maximise profits? [6]

Profit maximisation requires knowledge of cost and revenue conditions in the market so that MR and MC can be found – this may not be possible under conditions of uncertainty. Also profit maximisation assumes that owners control the management of the business. Where these roles are separated then the mangers of firms may have other objectives such as sales maximisation. Sales maximisation focuses on behaviour of manager-controlled businesses where salaries and other benefits more closely correlated with sales.

Traditional economic theory assumes there is a single goal but behavioural economists argue differently. Any corporation is an organization with various groups (employees, managers, shareholders, customers) and each group may have different objectives/goals The dominant group at any moment in time can give greater emphasis to their own objectives but maximising behaviour may be replaced by satisficing (satisficing = satisfy + suffice) – i.e. setting minimum acceptable levels of achievement for conflicting objectives

(c) Explain the Accelerator Theory of investment [6]

Accelerator model suggests that total capital investment in an economy varies directly with the rate of change of output i.e. investment is largely income-induced. The basic accelerator model assumes:

Given technological conditions
Given relative prices of capital and labour
A fixed size of capital stock needed to produce a given level of output

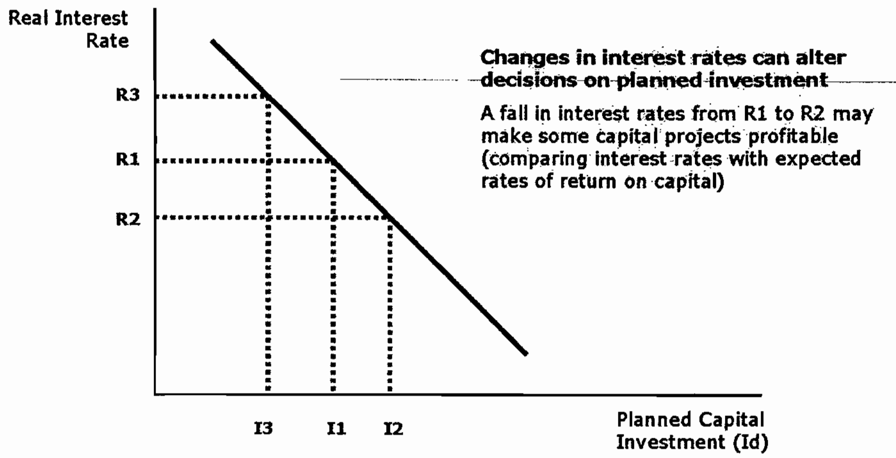
If the capital stock K is at full capacity and the capital/output ratio v is constant, then net investment In may be expressed in the following way:

$$In = v(Yt - (Yt - 1))$$

If the level of output changes, then the desired size of capital stock will also change. Net capital investment is the amount by which the required capital stock changes. It follows that the amount of investment depends on the size of the change in output. When the rate of growth of demand is strong, the size of the capital stock needs to be increased - boosting demand for capital goods.

(d) What impact would a reduction in the rate of interest have on the volume of investment? [4]

The Marginal Efficiency of Capital model suggests that a reduction in the rate of interest will normally increase the volume of investment (see below).



Better candidates may note the size of the effect will depend on interest elasticity of investment. Additionally the volume of investment will be influenced by other factors such as expected increase in demand, uncertainty, profitability, public policies, the efficiency of the financial system and technological shocks.

ENGINEERING TRIPOS PART IB
PAPER8 / SECTION B
2006
SPG MADABHUSHI

1.

(a) (i) Soil piping is subsurface erosion of soil by seeping waters from one side of the cut-off wall to the other side. The water pressure due to seepage becomes larger than the original effective stress, which leads to zero effective stress.

(ii) Liquefaction occurs in loose saturated soils. Prior to an earthquake, the water pressure is relatively low and the effective stress is positive, keeping the loose structure stable. However, shearing of the soil by earthquake shaking causes the loose structure to collapse. Because of the rapid cyclic shearing during the earthquake, the water cannot drain from the collapsing soil and the water pressure to increase to the point where the effective stress becomes zero and soil particles can readily move with respect to each other.

(b) Shield tunnelling involves using a special tunnelling machine (shield) to excavate the soil and installing pre-made segments for lining. The segments are usually made out of pre-cast reinforced concrete and are made in factory under controlled conditions. Hence, it creates a robust tunnel as long as the tunnel machine operates smoothly. The shape of the tunnel is governed by the shape of the shield and hence the method lacks the flexibility in shape.

New Austrian Tunnelling Method (NATM) involves using sprayed concrete and light reinforcement mesh/steel fibres to create a temporary tunnel lining. It can be used to create a tunnel of different shapes. Excavation sequences and face areas excavated can be continuously varied depending on the ground conditions and real time monitoring data. Potential risks come from poor workmanship. For example, inadequate sprayed concrete thickness and joint connections can lead to tunnel collapse.

(c) Piles are often used because adequate bearing capacity can not be found at shallow enough depths to support the structural loads. It is important to understand that piles get support from both end bearing and skin friction. The proportion of carrying capacity generated by either end bearing or skin friction depends on the soil conditions.

(i) when the tunnel is constructed under the pile, the capacity of end bearing may reduce and the building load will be transferred to the skin friction. This leads to pile settlement and the building may suffer by the differential settlements.

(ii) when the tunnel is constructed at the side of the pile, there will be horizontal ground movements and the pile may suffer by bending and hence lose its integrity. The variation of skin friction may also change because of vertical ground movements.

(d) Instrumentation is essential to monitor the ground and building behaviour during the construction of the tunnel. The piles should be instrumented when they are constructed. For example, inclinometers can be installed inside the pile to monitor the horizontal movement of the pile during tunnel excavation. Strain gauges can be attached to the reinforcements so that the change in axial stress distribution of the piles can be monitored during tunnel excavation. Ground surface settlements and building movements should be monitored. Ideally, extensometers should be installed to monitor the subsurface settlements of the ground.

2. (a) Diaphragm walls provide a water tight barrier to keep the excavation dry and are constructed to have a minimum subsidence behind the wall. They are formed from reinforced concrete and are constructed as normal cast-in-place walls with support which can become part of the main structure. The slurry trench method is commonly used. This involves the excavation of alternating panels along the proposed wall using bentonite slurry to prevent the sides of the excavation collapsing. The installation starts with the construction of shallow concrete or steel guide walls. The excavation is then made using special equipment, such as the thin-grab clamshell. Bentonite slurry is then pumped into the trench to provide temporary support and a prefabricated reinforcing cage is lowered in. The bentonite slurry is then replaced by concrete and the sequence proceeds onto the next panel.

(b)

Total vertical stress at the ground surface $\sigma v = 30$ kPa.

Total vertical stress at the water table (2m depth) $\sigma v = 30 + 2 \times 16 = 62$ kPa

Total vertical stress at the sand-clay interface (8 m depth) $\sigma v = 30 + 2 \times 16 + 6 \times 18 = 170$ kPa

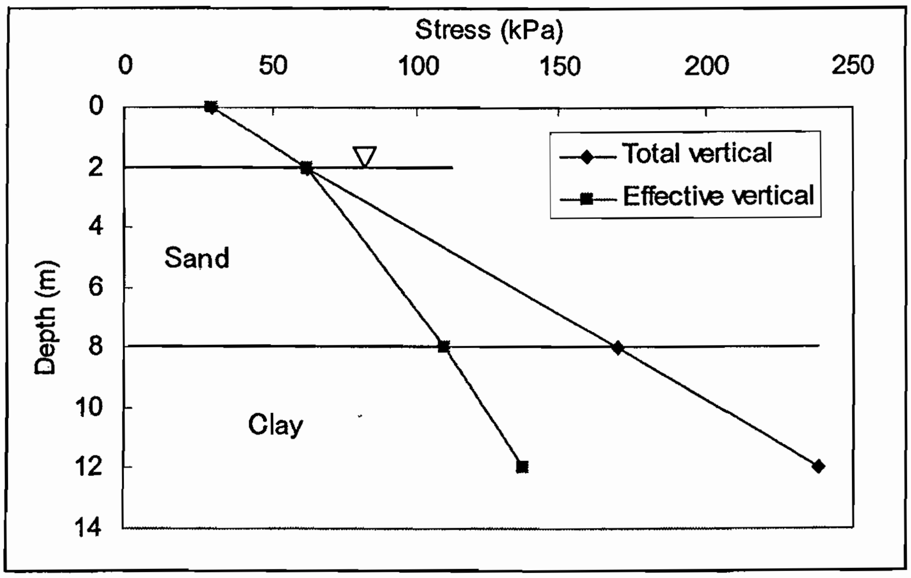
Total vertical stress at 12 m $\sigma v = 30 + 2 \times 16 + 6 \times 18 + 4 \times 17 = 238$ kPa

Effective vertical stress at the ground surface $\sigma' v = \sigma v - u = 30 - 0 = 30$ kPa

Effective vertical stress at the water table (2m depth) $\sigma' v = \sigma v - u = 62 - 0 = 62$ kPa

Effective vertical stress at the sand-clay interface (8 m depth) $\sigma' v = \sigma v - u = 170 - 6 \times 10 = 110$ kPa

Effective vertical stress at 12 m $\sigma' v = \sigma v - u = 238 - 10 \times 10 = 138$ kPa.



(ii)

Effective horizontal stress at the ground surface $\sigma'_h = K_0\sigma'_v = 0.6 \times 30 = 18$ kPa

Effective horizontal stress at the water table (2m depth) $\sigma'_h = K_0\sigma'_v = 0.6 \times 62 = 37.2$ kPa

Effective horizontal stress at the sand-clay interface in the sand (8 m depth) $\sigma'_h = K_0\sigma'_v = 0.6 \times 110 = 66$ kPa

Effective horizontal stress at the sand-clay interface in the clay (8 m depth) $\sigma'_h = K_0\sigma'_v = 1.0 \times 110 = 110$ kPa

Effective vertical stress at 12 m $\sigma'_h = K_0\sigma'_v = 1.0 \times 138 = 138$ kPa.

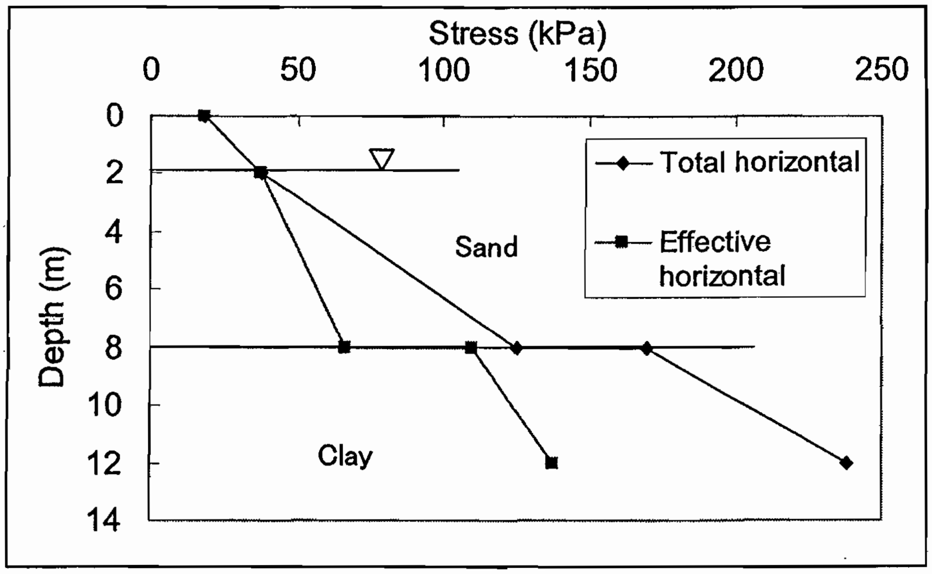
Total horizontal stress at the ground surface $\sigma_h = \sigma'_h + u = 18 + 0 = 18$ kPa

Total horizontal stress at the water table (2m depth) $\sigma_h = \sigma'_h + u = 37.2 + 0 = 37.2$ kPa

Total horizontal stress at the sand-clay interface in the sand (8 m depth) $\sigma_h = \sigma'_h + u = 66 + 6 \times 10 = 126$ kPa

Total horizontal stress at the sand-clay interface in the clay (8 m depth) $\sigma_h = \sigma'_h + u = 110 + 6 \times 10 = 170$ kPa

Total horizontal stress at 12 m $\sigma_h = \sigma'_h + u = 138 + 10 \times 10 = 238$ kPa



(iii) The active side in sand

$Ka = (1-\sin\phi)/(1+\sin\phi) = (1-\sin 35°)/(1+\sin 35°) = 0.27$

Pressure at the ground surface $\sigma_h = Ka\sigma'_v + u = 0.27 \times 30 + 0 = 8.1$ kPa

Pressure at the water table (2m depth) $\sigma_h = Ka\sigma'_v + u = 0.27 \times 62 + 0 = 16.74$ kPa

Pressure at the sand-clay interface in the sand (8 m depth) $\sigma_h = Ka\sigma'_v + u = 0.27 \times 110 + 6 \times 10 = 89.7$ kPa

The active side in clay

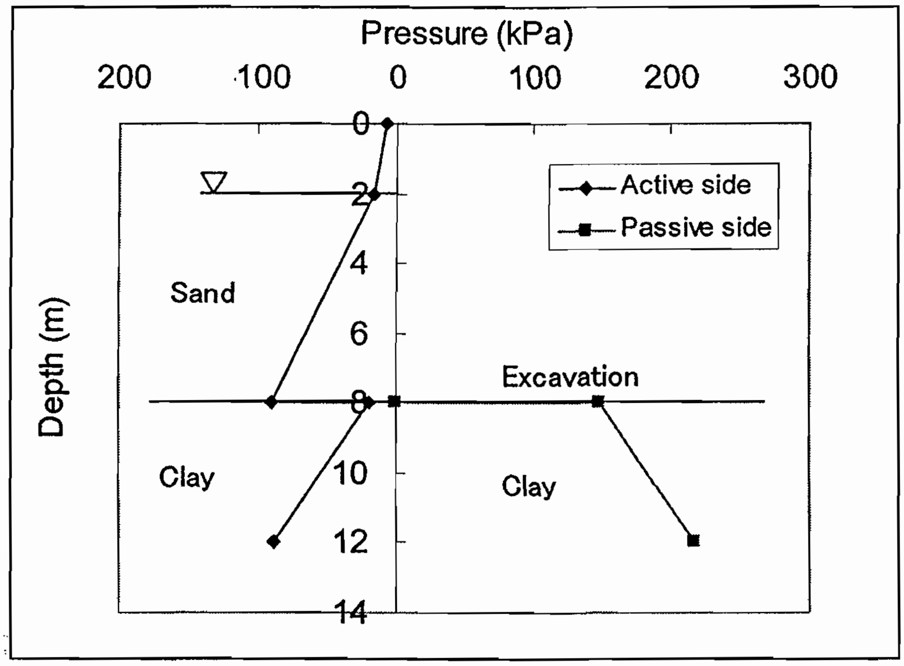
Pressure at the sand-clay interface in the clay (8 m depth) $\sigma_h = \sigma_v - 2cu = 170 - 2 \times 75 = 20$ kPa

Pressure at 12 m $\sigma_h = \sigma_v - 2cu = 238 - 2 \times 75 = 88$ kPa

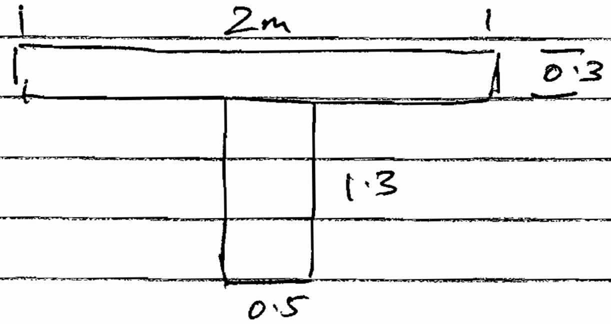
The passive side in clay

Pressure at the excavation level (8 m depth) $\sigma_h = \sigma_v + 2cu = 0 + 2 \times 75 = 150$ kPa

Pressure at 12 m $\sigma_h = \sigma_v + 2cu = 17 \times 4 + 2 \times 75 = 218$ kPa



Q5 Elective Solution

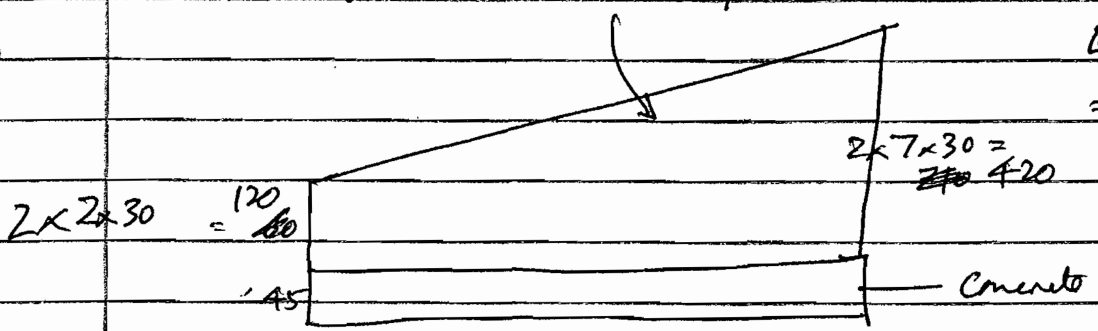


Area of concrete = $1.25\ m^2$

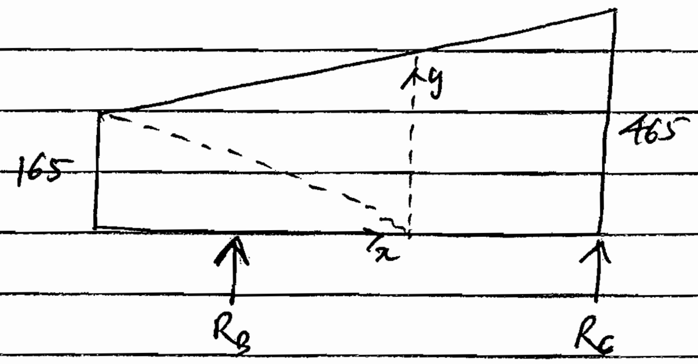
$\therefore$ Wt = 30 kN/m run

$\times 1.5$ factor = 45 kN/m

Loading due to soil / beam.



Effective density $= 20\ kN/m^3 \times 1.5$ f $= 30\ kN/m^3$



$y = 165 + 20x$

Find reactions.

Moments about C

$$R_B \cdot 11 = 165 \cdot \frac{15}{\cancel{2}} \cdot \frac{\cancel{2}15}{3} + 465 \cdot \frac{15}{2} \cdot \frac{15}{3} = 29812$$

$$\therefore R_B = 2710\ kN$$

Total load $\quad \frac{(165 + 465)}{2} \cdot 15 = 2710 + R_C$

$$\Rightarrow R_C = 2015\ kN$$

Shear force at B.

$$(y)_B = 165 + 20 \cdot 4 = 245 \text{ KN/m}$$

$$\therefore \text{Shear force} = \left(\frac{165+245}{2}\right) \times 4 = 820 \text{ KN}$$

Where is shear force zero?

$$y = 165 + 20x$$

$$(S.F)_x = \left(\frac{165 + (165+20x)}{2}\right) \cdot x - 2710 = 0$$

$$10x^2 + 165x - 2710 = 0$$

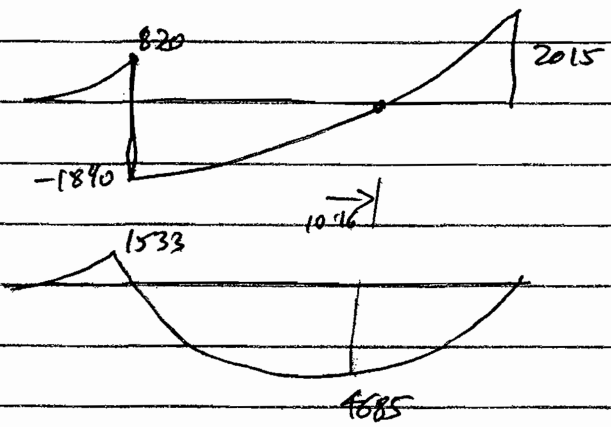
$$\Rightarrow x = \frac{-165 \pm \sqrt{27225 + 108400}}{20} = \underline{\underline{10 \cdot 16 \text{ m}}}$$

At this point moment $(M)_x$ $\qquad y = 368$

$$(M)_x = 165 \cdot \frac{(10 \cdot 16)^2}{\cancel{2}} \cdot \frac{\cancel{2}}{3} + 368 \cdot \frac{(10 \cdot 16)^2}{2} \cdot \frac{1}{3} - 2710 \cdot \cancel{6} \cdot 16 = -4685$$

(sagging)

(d) ~~Check~~ .



$M = 0.15 f_{cu} b d^2$. Take $d = 1240$ mm

$b = 500$

$= 5766$ kNm $f_{cu} = 50$

$\therefore$ Moment can be carried as singly reinforced.

To find area of steel (sagging)

Assume $x = 0.5$ (neutral axis depth)

$\therefore 4685 . 10^6 = 0.87 . 460 . A_s \; 1240 \left(1 - \frac{0.5}{2}\right)$

$\Rightarrow A_s = 12590$ mm^2

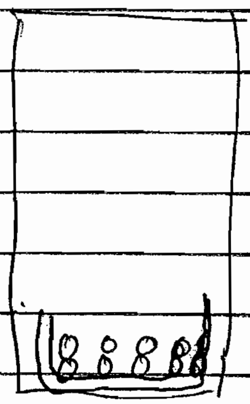
$x = 2.175 \left(\frac{460}{50}\right)\left(\frac{12590}{500 . 1240}\right) = 0.41$

recalculate $A_s = 11875$ mm^2 (nearly 2% - quite high)

$x = 0.383$ (close enough.)

Use 40 mm bars - need 10 bars 1256 mm dia

Need to be paired



In hogging $M = 1533$ kNm.

Guess $x = 0.2$

$\therefore 1533 . 10^6 = 0.87 . 460 . 1240 \left(1 - \frac{0.2}{2}\right) A_s$

$\Rightarrow A_s = 3432 \text{ mm}^2$

$x = 0.11$

$\Rightarrow A_s = 3270 \text{ mm}^2$ (0.5% OK)

$x = 0.105$ OK.

5x 32 mm bars $= 4020 \text{ mm}^2$

or $\cancel{3}$x 40 mm $= 3768 \text{ mm}^2$

Shear links. Maximum shear force = 2014 kN.

$$v_c = 0.68 . \left(\frac{100 A_s}{bd}\right)^{0.33} \left(\frac{400}{d}\right)^{0.25}$$

Take $\frac{A_s}{bd} = 0.5\%$. | Conservative assumption.

$\therefore \left(\frac{100 A_s}{bd}\right)^{0.33} = 0.8$

$$\left(\frac{400}{1240}\right)^{0.25} = 0.42$$

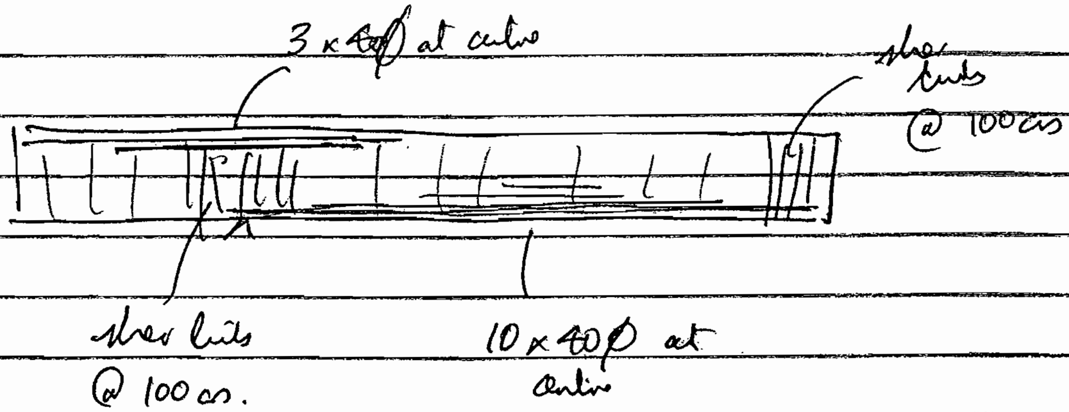
$\therefore v_c = 0.68 . 0.8 . 0.42 = 0.23 \text{ N/mm}^2$.

$v_s = \frac{2014 . 10^3}{500 . 1240} - 0.23 = \cancel{3.25}\ 3.02 \text{ N/mm}^2$

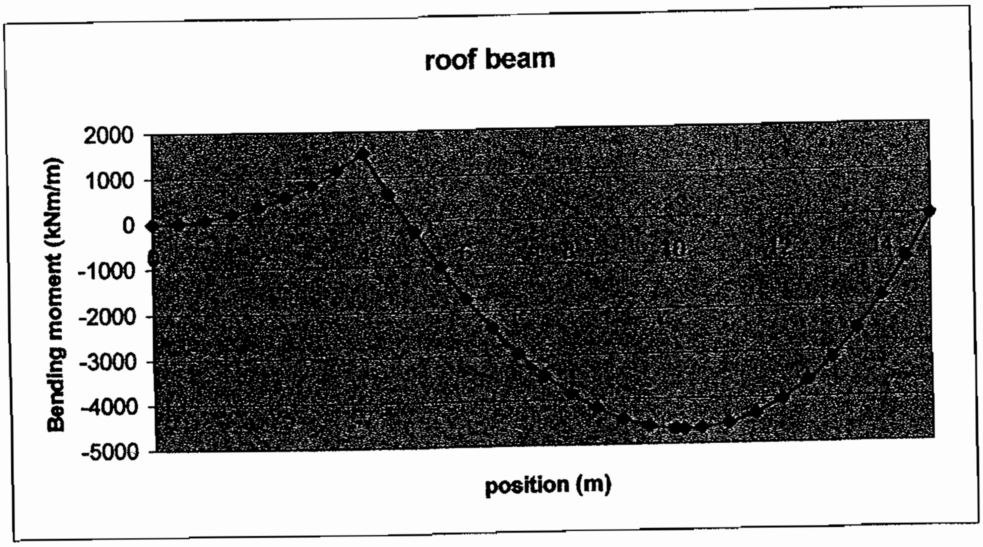
Use A_{sq} = 16 mm bars $\therefore \cancel{3.25}\ 3.02 = \frac{0.87 . \cancel{4}\ 460 . 2.201}{580 . s}$

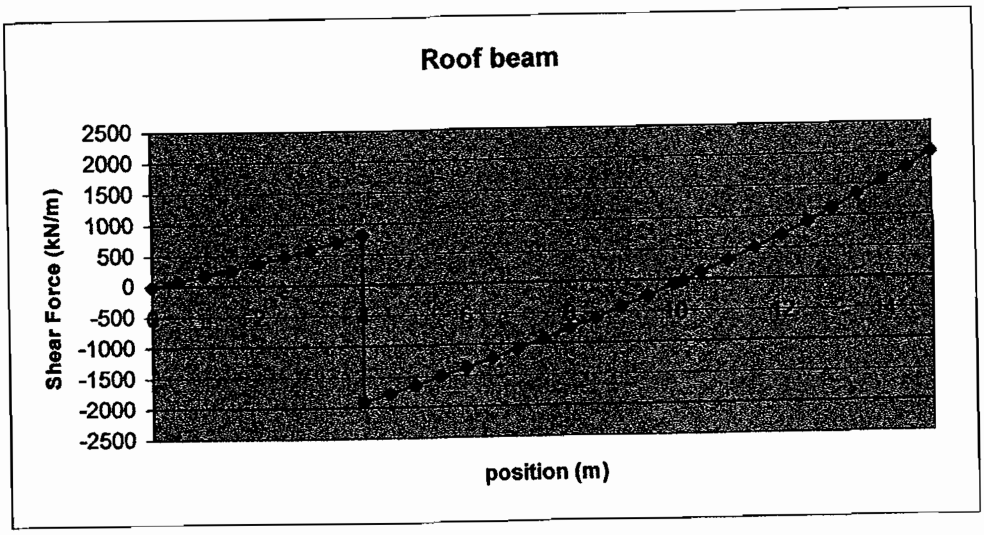
$\Rightarrow s = 106$ mm.

~~Vs needed~~

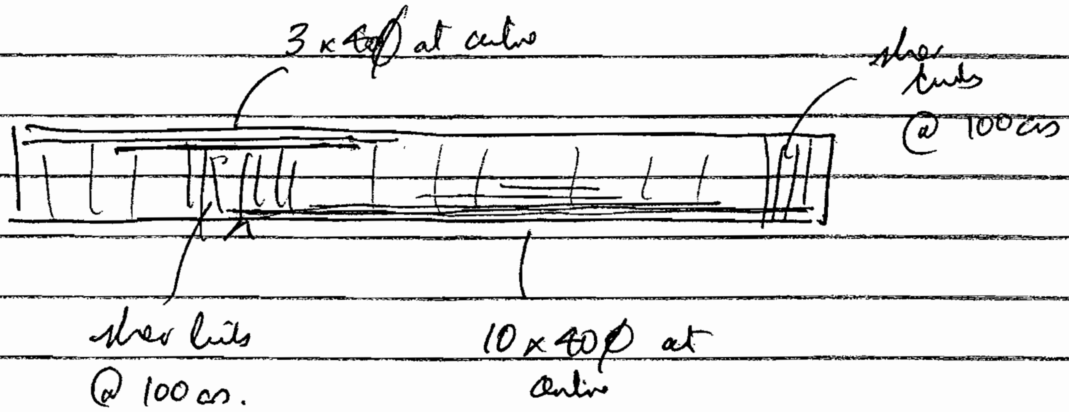


Shear links spacing reduced away from supports,





~~Us needed~~



Shear link spacing reduced away from supports.

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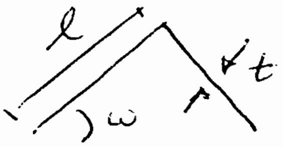
Q1

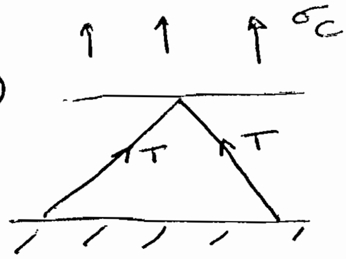
(a) Relative density of a cellular material is the ratio of density of the cellular solid treated as a homogeneous continuum to the density of the solid from which it is made.



$$\bar{\rho} = \frac{2tl}{l\sin\omega \, 2l\cos\omega}$$

$$= \frac{2t}{l\sin 2\omega}$$

(b) (i)



$$\sigma_c \, 2l\cos\omega = 2T\sin\omega$$

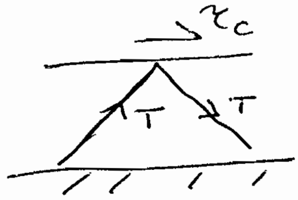
$$T = \sigma_Y t$$

$$\sigma_c = \frac{\sigma_Y t}{l} \frac{\sin\omega}{\cos\omega}$$

$$= \sigma_Y \frac{\bar{\rho}}{2} 2\sin^2\omega$$

$$= \bar{\rho}\, \sigma_Y \sin^2\omega$$

(ii)

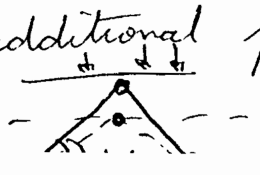


$$\tau_c \, 2l\cos\omega = 2T\cos\omega$$

$$\tau_c = \sigma_Y \frac{t}{l}$$

$$= \sigma_Y \frac{\bar{\rho}}{2} \sin 2\omega$$

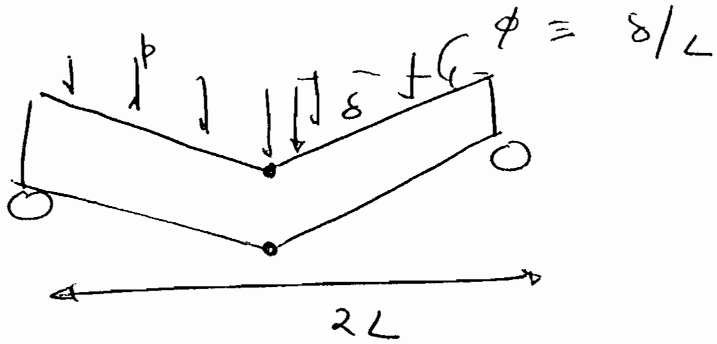
(c) A slight increase in the collapse load due to additional plastic dissipation at hinges



Q4

(a) High stiffness & strength per unit mass due to the increase in the 2nd & first moment of area of the sandwich cross-section. Also, possibility of multi-functional applications eg. heat exchangers, ~~to~~ vibration damping

(b) (i)



~~PRESS~~ In face yield max. bending moment at mid-span

$$\frac{pL^2}{2} = M_p = \sigma_Y h(h+c) + \frac{\sigma_c c^2}{4}$$

$$p = \frac{2\sigma_Y h(h+c)}{L^2} + \frac{\sigma_c}{2}\left(\frac{c}{L}\right)^2$$

(ii) $$(p\,2L)\frac{\delta}{2} = 2\tau_c\, c\, L\, \gamma + 2\frac{\sigma_Y h^2}{4}\left(\frac{\delta}{L}\right)\times 2$$

when $\gamma = \dfrac{\delta}{L}$

$$p = 2\tau_c\left(\frac{c}{L}\right) + \sigma_Y\left(\frac{h}{L}\right)^2$$

(iii) In face yield $\quad P_{FY} \approx \dfrac{2\sigma_Y hc}{L^2}$

$$P_{cs} \approx 2\tau_c\left(\frac{c}{L}\right)$$

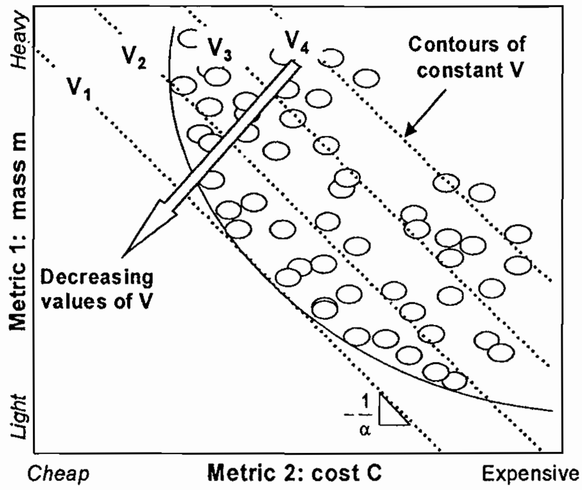
$$P_{FY} = P_{cs}$$

$$\Rightarrow \quad \cancel{k}\,\frac{\sigma_Y h \cancel{b}}{L^2} = \cancel{k}\,\tau_c \left(\frac{\cancel{d}}{L}\right)$$

$$\frac{\sigma_Y}{\tau_c} = \left(\frac{L}{h}\right)$$

Thus when $\dfrac{\sigma_Y}{\tau_c} > \dfrac{L}{h}$ core shear is the collapse mode, Q3

(a) When a problem has two objectives – minimising both mass m and cost C of a component, for instance – a conflict arises: the cheapest solution is not the lightest and vice versa. The best combination is sought by constructing a trade-off plot (see sketch below) using mass as one axis, and cost as the other. The lower envelope of the points on this plot defines the trade-off surface, and the solutions that offer the best compromise lie on this surface.



(b) We define the value function

$$V = C + \alpha m$$

where α is a constant (a weight factor) that defines the acceptable additional cost to reduce the mass by 1 unit. The best solutions are found where the line defined by this equation is tangential to the trade-off surface. (Note that a low value of V is more desirable than a high one.)

(c) For problems involving two or more competing objectives, the relative weighting of one objective with respect to each of the others must be established. These weight factors, α are found in two principal ways:

1) By full-life analysis – for instance, by calculating the cost of the fuel saved by reducing the weight of a vehicle by 1 unit;

2) By asking experts (or potential customers) for their best estimates, using a structured interviewing technique.

(d) The main contributions to the cost of manufacturing a component are:

- The cost of the material, C_m^* \$ per unit
- The capital cost of the equipment used to make the component (generally a *non-dedicated* cost), C_c (\$)
- The cost of dies, jigs and other special tooling (a *dedicated* cost), C_t^* (\$)

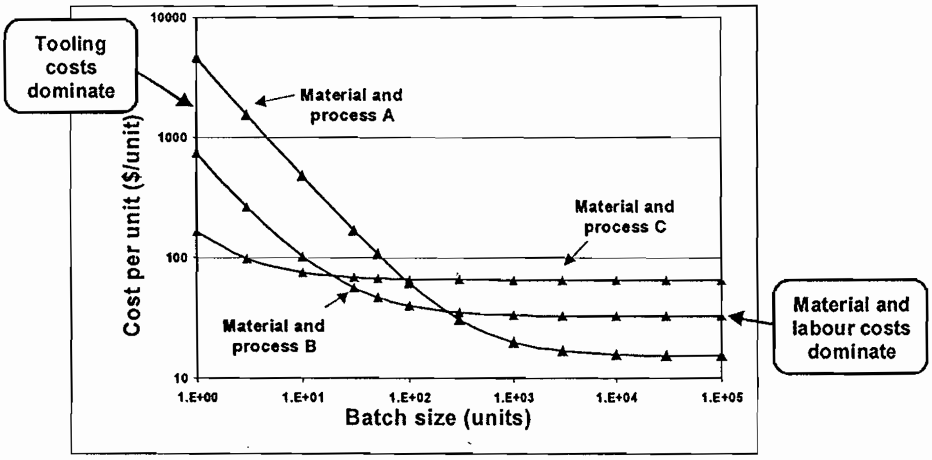
- Time-dependent costs (labour, overheads, administration, etc.), $\dot{C}_{oh}$ (\$/hour)
- The capital write-off time t_{wo} (hours)
    - The load factor (the fraction of time for which the equipment is productive), L.

We convert capital outlay into a cost per unit time by dividing C_c by $t_{wo}.L$, and sum the terms:

$$C = C_m^* + \frac{C_t^*}{n} + \frac{1}{\dot{n}}\left(\frac{C_c}{t_{wo}L} + \dot{C}_{oh}\right) = C_m^* + \frac{C_t^*}{n} + \frac{\dot{C}_{oh}^*}{\dot{n}}$$

where n is the batch size, $\dot{n}$ is the rate of production (units per hour), and the * signifies that the term contains all contributions to that item of cost.

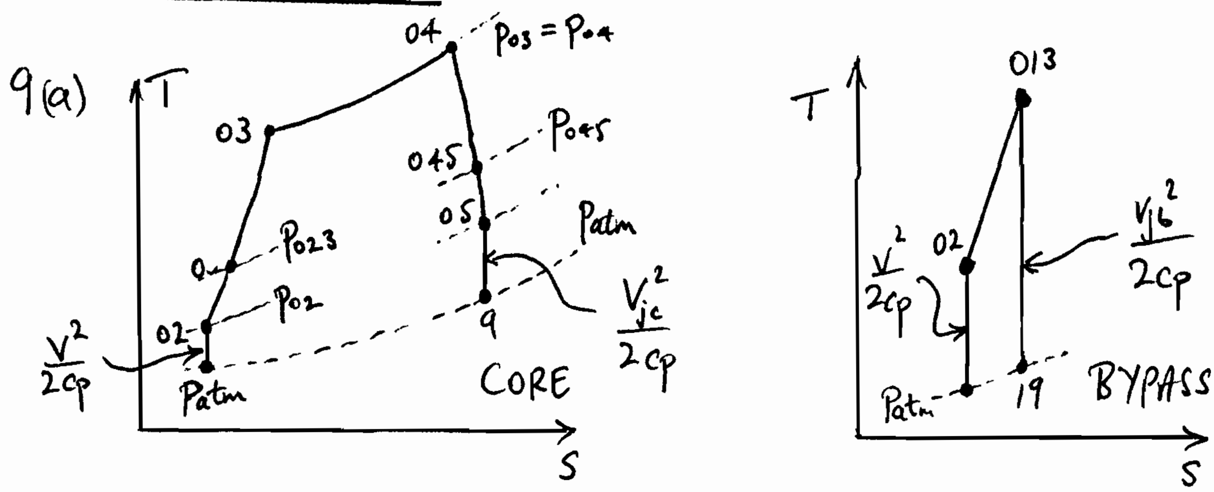
Cost varies with batch size as shown in the sketch:



The curves differ because of the differing capital, die, and time-dependent costs, and the rate of production. This leads to cross-overs, as shown, making one process economic at low batch size and another at high. Die costs often dominate at low batch sizes (Die-casting) while labour costs dominate at large batch sizes. Thus we will use EDM for small batches and die-casting for large batches.

Section D

9(a)



N.B. Different scales

(b) Good propulsive efficiency requires large $\dot{m}$, small $V_j - V$

i.e. large bypass ratio ⇒ large fan diameter

This has to be balanced against

(i) increasing fan diameter ⇒ heavier engine

(ii) increasing fan diameter ⇒ more nacelle drag

(iii) problems getting engine under wing, engine transportation, etc.

(c) (i) Take-off

Engine is producing high thrust at high ambient air temperature. Air density high ⇒ aero loads highest. Speeds and turbine temperatures highest for this op point

(ii) Top of Climb

Aircraft is at high altitude but still needs excess thrust to climb. Cycle temperature ratio is highest (but N.B not turbine temperature). $\frac{N}{\sqrt{T_0}}$ is highest i.e. internal engine Mach numbers are highest.

(iii) Cruise

Need to have best s.f.c. will spend most of flight here.

(d) LP turbine will have most stages

(i) it is powering the fan, which passes all of the mass flow of the engine & is driven only by the core mass flow

(ii) speed of LP is lower (it turns at same rotation rate as fan but with a much smaller tip radius than the fan).

<u>Examiner's note</u>

This question was, in general, well answered.

Biggest error was to mis-read the question and concentrate on airframe challenges.

The terms "aerodynamic challenge" and "thermodynamic challenge" are somewhat subjective — candidates who claimed top of climb as "thermodynamic challenge" and cruise as "aerodynamic challenge" were not penalised provided their reasoning was sound.

10. (a) $\dfrac{dW}{dt} = -\dot{m}_f\, g$ $\qquad \dot{m}_f$ = fuel flow rate

$$s.f.c. = \frac{\dot{m}_f}{\text{Thrust}} \Rightarrow \frac{dW}{dt} = -sfc\, g \text{ Thrust}$$

But $\text{Thrust} = \text{Drag} = \dfrac{\text{Drag}}{\text{Lift}}\text{Lift} = \dfrac{D}{L}W$

and $\dfrac{d}{dt} = \dfrac{ds}{dt}\dfrac{d}{ds} = V\dfrac{d}{ds}$

$$\therefore \quad V\frac{dW}{ds} = -\frac{sfc\, g}{L/D}W \Rightarrow \ln\frac{W_{end}}{W_{start}} = -\frac{g\, sfc}{VL/D}s$$

assuming sfc, $\dfrac{VL}{D}$ remain constant

Hence $s = -\dfrac{V\,L/D}{g\,sfc}\ln\dfrac{W_{end}}{W_{start}}$

[Further assumption is that take-off & landing do not contribute much to overall consumption].

(b) At 11,000 m $\dfrac{T}{T_s} = .7523 \Rightarrow \dfrac{a}{a_{sl}} = \sqrt{.7523}$ & $a_{sl} = 340$

$$\therefore \quad a = 295 \text{ ms}^{-1}$$

$\dfrac{V}{a}$ = Mach Number $= .85 \Rightarrow V = 251 \text{ ms}^{-1}$

Minimum fuel $\Rightarrow$ all fuel (apart from reserves) used

$$\therefore \ln\frac{W_{empty} + W_{payload} + W_{fuel}}{W_{empty} + W_{payload}} = \frac{9.81 \times 1.95 \times 10^{-5} \times 8000 \times 10^{3}}{251 \times 18}$$

$$= 0.339$$

$\quad \dfrac{W_{start}}{W_{end}}$ (″)

$$\Rightarrow \frac{W_{start}}{W_{end}} = 1.403 \Rightarrow W_{fuel} = \underline{50.4 \text{ tonnes}}$$

$$\text{and} \quad W_{MAX\,TO} = \underline{175.4 \text{ tonnes}}$$

(c) For 4000 km $\ln \frac{W_{start}}{W_{end}} = .169 \Rightarrow W_{fuel} = 23.1$ tonnes

$\therefore$ 8000 km in 2 legs, minimum fuel = <u>46.2 tonnes</u>

(d) Breguet as for part (c) $\frac{W_{start}}{W_{end}} = e^{.169} = 1.184$

ie. [W_e = empty wt] $\frac{W_e + 40 + W_f}{W_e + 40} = 1.184$ And $\frac{W_e}{W_e + 40 + W_f} = .5$

$$\therefore \quad W_e + 40 + W_f = 1.184(W_e + 40) = 2 W_e$$

$$\Rightarrow \quad W_e = 58.0 \text{ tonnes} \quad \& \quad W_f = 18.0 \text{ tonnes}$$

$\therefore$ For aircraft designed for 4000 km, fuel for 8000 km <u>= 36.0 tonnes</u>

(e) Against:— would need extra crew; journey time increased $\Rightarrow$ reduce aircraft utilisation & less passenger appeal; extra take-off & landing shortens airframe/engine life

For:— less fuel consumption, don't need specialist long haul aircraft

Other:— If convenient hub at 4000 km, may fit airline route plan better

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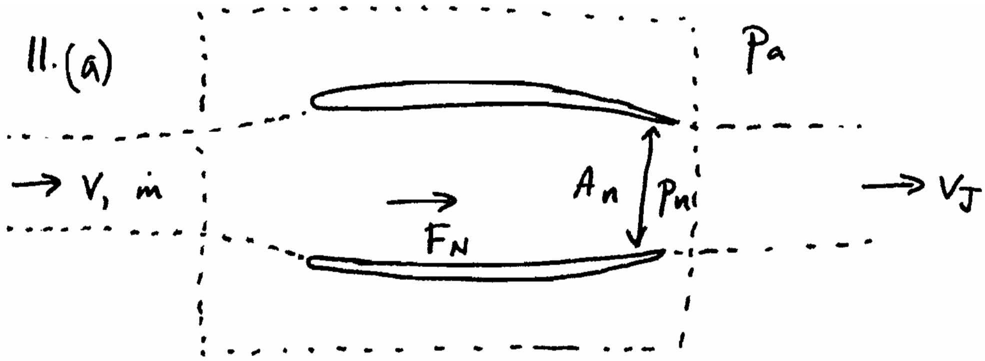
<u>Examiner's Note:</u>

Part (a) should have been a gift, but only about half could do it. For part (e) 3 of the points listed is enough for full marks.

11.(a)



Net thrust $F_N + (p_a - p_n)A_n = \dot m(V_J - V)$

$\dot m V$ = initial momentum in fluid which passes through engine

= ram drag

F_G = gross thrust = $F_N + \dot m V$

(b) "02" means stagnation conditions at compressor entry

p_a, A_n, F_G = atmospheric pressure, nozzle area, gross thrust

Assumptions

(i) fuel flow rate neglected (i.e. $\dot m_f \ll \dot m_{air}$)

(ii) nozzle is choked but with no shocks present

[This means $p_n = p_a$ and p_a determines engine exit conditions]

(iii) If being very accurate, also no loss of P_0 in intake & engine perf indep't of Re

Under these assumptions, a single engine parameter (like $N/\sqrt{T_{02}}$) determines the non-dim op point & the two parameters listed are simply a function of this.

(c) At 11,000 m $\quad T = 217\text{ K} \quad P = 22.7\text{ kPa}$

Aircraft $M = 0.8 \Rightarrow T_{02} = 217\left(1 + \frac{\gamma - 1}{2}0.8^2\right) = 244.8\text{K}$

$P_{02} = 22.7\left(1 + \frac{\gamma-1}{2}.8^2\right)^{\frac{\gamma}{\gamma-1}} = 34.6\text{ kPa}$

Hence $V = \cdot 8 \times a = .8\sqrt{\gamma RT} = 236.3 \text{ ms}^{-1}$

Drag = net thrust $\Rightarrow$ $F_G = 40\text{ kN} + 100 \times 236.2 = 63.6\text{ kN}$

per engine

$\therefore$ At altitude

$$\frac{\dot m_{air}\sqrt{c_p T_{02}}}{p_{02} A_n} = \frac{100 \times \sqrt{1005 \times 245}}{34.6 \times 10^3 \times 0.7} = 2.048$$

$$\frac{F_G + p_a A_n}{p_{02} A_n} = \frac{63.6 \times 10^3 + 22.7 \times 10^3 \times 0.7}{34.6 \times 10^3 \times 0.7} = 3.283$$

For the same non-dim^{l}. engine o/p pt, at sea level these values are the same, but $p_{02} = p_a = 101.3\text{ kPa}$

$T_{02} = T_a = 288\text{ K}$

$\frac{1}{2}$ size engine $\Rightarrow$ $A_n = \frac{0.7}{4} = .175$

$$\therefore \dot m_{air} = \frac{2.048 \times 101.3 \times 10^3 \times 0.175}{\sqrt{1005 \times 288}} = \underline{67.5 \text{ kg s}^{-1}}$$

$$F_G = F_N = 3.283 \times 101.3 \times 10^3 \times .175 - 101.3 \times 10^3 \times .175$$

$$= \underline{40.5\text{ kN}}$$

(d) Expect $\frac{\text{Blade speed}}{\sqrt{\gamma R T_{02}}}$ = same $\Rightarrow$ $\frac{\Omega_m r_m}{\sqrt{\gamma R\, 288}} = \frac{\Omega_{FS}\, r_{FS}}{\sqrt{\gamma R\, 245}}$

$$\Rightarrow \frac{\Omega_m}{\Omega_{FS}} = \frac{1}{1/2}\sqrt{\frac{288}{245}} \Rightarrow \underline{\Omega_m = 21{,}700 \text{ rpm}}$$

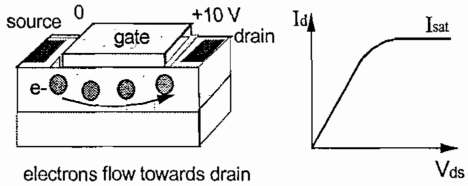
Examiner's Note

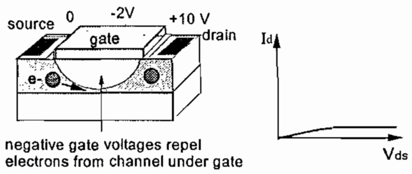
Well answered. The vast majority of candidates knew precisely what to do. A distressing number made one or other of the following howlers (i) $M = 0.8$ for static test bed. (static = stationary!) (ii) Incompressible Bernoulli used for p_{02} (iii) Confused static & stagnation quantities (iv) mis-remembered p_0, T_0 formulae ← IN DATABOOK.

ENGINEERING TRIPOS PART IB
PAPER8 / SECTION E
2006
SPG MADABHUSHI

Solutions to Section E Electrical Option

12(a)





MOSFET = oxide barrier, MESFET = Schottky Barrier

Higher reverse leakage in SB junction, normally on for depletion mode etc

(b)
accumulation (for n-channel) – Vg increases existing electron conc,
depletion – Vg depletes existing electron conc, switches off (standard type taught in this course, as above)
inversion – switches channel over to other carrier type. Normally “off” device which is very useful for various applications

Need to show Id vs Vg curves

(c)
First Part book work.
$V_T = 0.018$ V. (a bit low but for dimensions and conditions in question this is correct)

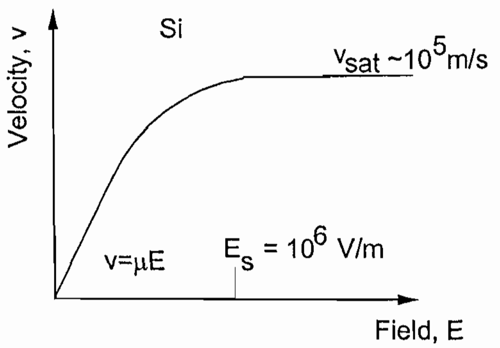
(d)

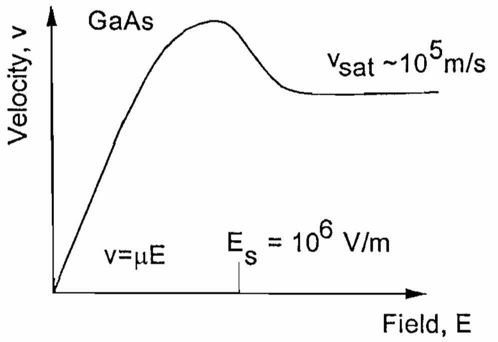
E = 9×10^5 V/m in Si. 2.8 x 10^6 V/m in the SiO2. The Si would breakdown first.

13(a)
Doping varies the carrier density in s/c whereas in a metal, each atom ‘donates’ its conduction electrons, fixed number of carriers, so conductivity is ~ fixed.

(b)
Native oxide for Silicon vs bad native oxide for In As.
Band gap in In As much lower than in Si so off current problems.
Mobility much higher for II- V material but much less well developed etc etc.

(c)





phonon scattering causes limiting velocity

(d)

$v = \mu E$, $t = L/v$, so $t = L/\mu E$

$F = ma = mv/\tau$, $F = eE$, $v = \mu E$, so $\mu = e\tau/m$.

Energy = $\frac{1}{2}mv^2$, find m from above, take v = critical velocity and hence find Energy.

$\tau = 9.1\text{x}10^{-31}\ 0.12 / 1.6\text{x}10^{-19} = 6.8\text{x}10^{-13}$ s

$E = \frac{1}{2}\ 9.1\text{x}10^{-31} \times (1.2 \times 10^5)^2 = 6.55 \times 10^{-21}$ J = 0.041 eV.

$\lambda = v\tau = 8.2 \times 10^{-8}$ m

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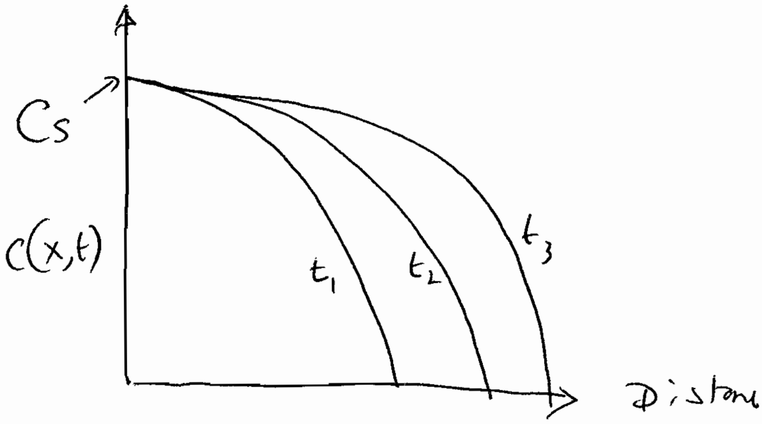
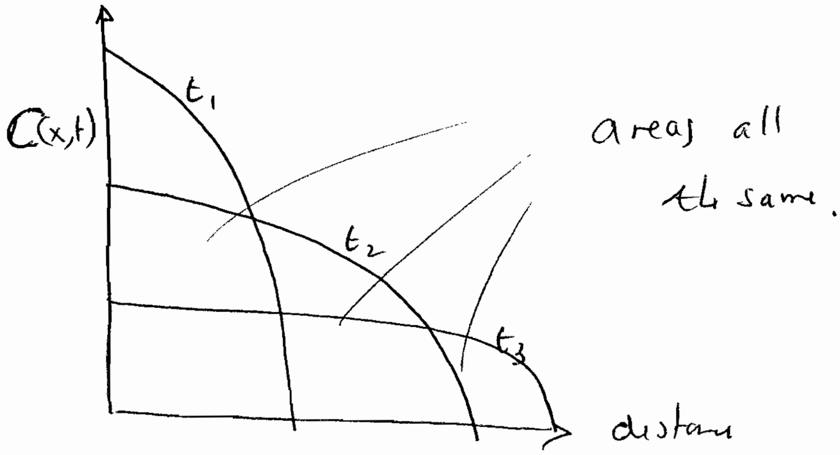
(a) Temperature, time, diffusion Coefficient, initial concentration, type of dopant etc

(b)



Constant source concentration maintained at surface by dopants from cylinders- error function distribution

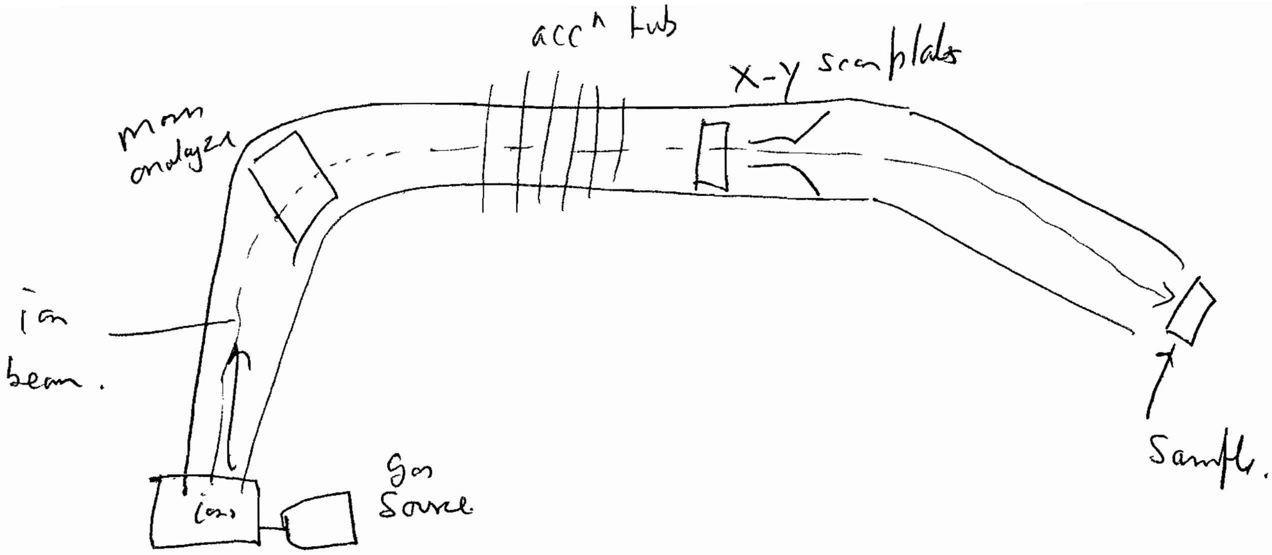


Limited Source (drive – in) amount of dopant is limited- Area under each curve in above is the same.

Etc etc

(c)

need to mention e.g. source of ions, mass filtering, scanning process, implantation depth, potential damage to substrate, so maybe annealing at low (ish) temperature needed. Good for shallow distributions, good control of dopant level, wee defined profiles etc etc



(d) second of above processes in section (b) where we have a limited source of dopants->> drive – in

Assume junction formed when concn of dopant added falls to conc in bulk

Surface concentration is 4 x 10 15 cm2

Time = 60x 60 secs

Temp = 1050C = 1323 K --- from graph D ~ 10^{-13} cm^2s^{-1}

Limited source so use

$C(x,t) = S/(\pi Dt)^{1/2} \exp[-x^2/4Dt]$

Substrate doping is 10^{17} cm^{-3} and surface concentration is 4 x 10^{15} cm^{-2}

Leads to

x ~ 1 micron (exact value depends upon value of D read from graph provided)

Engineering Tripos Part IB
Crib for Paper 8
Information Engineering

June 18, 2006

1. (a) Lowpass filters attenuate high frequencies and are used for blurring the image and for noise reduction. Highpass filters attenuate low frequencies and are used for edge enhancement and edge detection.

   (b) Let $x = \sqrt{3/2}\,t$ so that $x^2 = 3t^2/2$. In the fourier domain we have $\omega_x = \omega_t/\sqrt{3/2}$. Substituting, we have

$$H(\omega_x) = \exp\left(-\frac{3\omega_x^2}{4}\right)$$

   This is a Gaussian centered around 0. Since $h(x)$ integrates to 1, $\int h(x)dx = 1$, we know that $H(0) = 1$; the sketch should show this. This is a Gaussian low-pass filter, so it attenuates high frequencies.

   (c) $h(x)$ and $h(y)$ are both as above. The convolved image $I'(x, y)$ satisfies the following:

$$\begin{aligned} I'(u,v) &= \int \left[\int I(x,y)h(u-x)dx\right] h(v-y)dy \\ &= \int\int I(x,y)h(u-x)h(v-y)dxdy \\ &= \int\int I(x,y)g(u-x,v-y)dxdy \end{aligned}$$

   So

$$g(x,y) = h(x)h(y) = \frac{1}{3\pi}\exp\left\{-\frac{(x^2+y^2)}{3}\right\}$$

   The above can also be shown by multiplying $H(\omega_x)$ and $H(\omega_y)$ and then invese fourier transforming.
   This point spread function g is isotropic (circularly symmetric / rotationally invariant) since $x^2 + y^2 = r^2$ defines a circle in the $x - y$ plane or radius r, and does not depend on the angle (θ) when converted into polar coordinates. So g only depends on the distance of (x, y) from the origin.
   The computational advantage is that if each 1D filter has N taps, it takes order N operations to do each convolution, a total of $2N$ operations. On the other hand, naively convolving with the 2D point

spread function g takes order N^2 operations. This is a substantial speed-up.

(d) Edge adaptive filters are used to stop blurring/denoising at edges. A simple blurring filter can be obtained by convolving images with $(1/4, 1/2, 1/4)$ repeatedly, for example, but this would blur away edges in the image. We can write such a filter as $(\alpha_1, (1-\alpha_1-\alpha_2), \alpha_2)$ where $\alpha_1 = \alpha_2 = 1/4$. By varying α_1 and α_2 from 0 to 1/4 it is possible to create an edge-adaptive smoothing filter. Setting α to be negative can create sharpening. The basic idea is to make α_1 and α_2 depend on the edginess image so edges are not blurred.
A simple edge-adaptive filter would work as follows:

- Run edge detector in x and y directions separately (possibly also separately in each color channel), creating an "edginess" image. A simple edge detector can be obtained by first differencing the image, i.e. convolving with a $(-a, 1+2a, -a)$ filter.
- Convolve the image with a smoothing filter modulated by the edginess image, e.g. $(\alpha_1, 1-\alpha_1-\alpha_2, \alpha_2)$ where α is related to the edginess image at the location being enhanced.

2. (a) i. RBG is red, green, blue. YUV is a linear transform of RGB, where Y is luminance, U and V are chrominance color difference signals. Y = 0.3 R +0.6 G + 0.1 B, U=0.5 (B-Y); V= 0.625 (R-Y) [these equations differ from some available online]. HSV is hue, saturation, value, a nonlinear tranform of RGB. $V = max(R, G, B)$, $S = \frac{V-min(R,G,B)}{V}$, and H is a polar representation of direction in UV color space. S is how far away from gray (radius).

ii. **Lighting intensity correction:** need to scale all three components of RGB. In HSV we only need to change V. **Colour cast correction** (e.g. to improve photos taken in poor artificial light): essentially you need to move around in U,V space. It's complicated for RGB, keep Y constant and move in U,V space. For HSV, you can shift around the hue circle, and change saturation, without changing V.

iii. **Histogram equalisation:** to do lighting intensity correction you can take for take V (or Y) and compute the histogram over pixels in the image. You can then rescale the V axis nonlinearly so that the histogram is more uniform. The equation for histogram equalisation is:

$$y_k = \sum_{i=0}^{255} \frac{255 n_k}{N}$$

where $N = \sum_{i=0}^{255} n_k$ and value k is mapped to value y_k. In RBG you would rescale all R, G, and B identically based on the V histogram. To do colour cast correction, you need to nonlinearly rescale the hue H so the histogram is more uniform, can also be done in R G B space by equalising each independently.

(b) i. An image texture refers to approximately repeating patterns appearing in the image. Examples are grass, or pebbles. The texture sub-elements are often called *textons.*

ii. Texture can be characterised by convolving an image with a family of filters at different orientations and scales. Examples of kinds of filters include "blob" filters, "edge" filters and "bar" filters created from Gaussians, difference of Gaussians, and laplacians of Gaussians at different orientations and scales. A "difference of Gaussians filter" (a sketch would be acceptable) in one dimension, has the form $h(x;\sigma) - ah(x;b\sigma)$ where $h(x;\sigma)$ is a zero mean Gaussian with standard deviation σ, $b > 1$, and $0 < a < 1$. In 2D you can orient the filter by using 2D Gaussians. [Another choice of filters is Gabor filters.] The corresponding descriptor would be a vector of filter outputs (or perhaps squared filter outputs) for a prespecified set of orientations and scales. There might for example be able 40 filter outputs.

3. (a) A local interest point is usually a corner or an image feature with distinguishable structure in two dimensions. They correspond to peaks in the local autocorrelation function of the image. Corners can be detected efficiently using the eigenvalue approach outlined below. Edges are also sometimes interest points.

(b) The rate of change of intensity I in the direction $\mathbf{n}$ is found by taking the scalar product of ∇I and $\hat{\mathbf{n}}$:

$$I_n \equiv \nabla I(x,y).\hat{\mathbf{n}} \;\Rightarrow\; I_n^2 = \frac{\mathbf{n}^T\,\nabla I\,\nabla I^T\,\mathbf{n}}{\mathbf{n}^T\mathbf{n}} = \frac{\mathbf{n}^T \begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix} \mathbf{n}}{\mathbf{n}^T\mathbf{n}}$$

where $I_x \equiv \partial I/\partial x$, etc. Next we smooth I_n^2 by convolution with a Gaussian kernel:

$$C_n(x,y) = G_\sigma(x,y) * I_n^2 = \frac{\mathbf{n}^T \begin{bmatrix} \langle I_x^2\rangle & \langle I_xI_y\rangle \\ \langle I_xI_y\rangle & \langle I_y^2\rangle \end{bmatrix} \mathbf{n}}{\mathbf{n}^T\mathbf{n}}$$

where $\langle\,\rangle$ is the smoothed value. The smoothed change in intensity in direction $\mathbf{n}$ is therefore given by

$$C_n(x,y) = \frac{\mathbf{n}^T\mathrm{A}\mathbf{n}}{\mathbf{n}^T\mathbf{n}}$$

where A is the 2×2 matrix

$$\begin{bmatrix} \langle I_x^2\rangle & \langle I_xI_y\rangle \\ \langle I_xI_y\rangle & \langle I_y^2\rangle \end{bmatrix}$$

Elementary eigenvector theory tells us that

$$\lambda_1 \leq C_n(x,y) \leq \lambda_2$$

where λ_1 and λ_2 are the eigenvalues of A. So, if we try every possible orientation $\mathbf{n}$, the maximum change in intensity we will find is λ_2, and the minimum value is λ_1.

We can classify image structure at each pixel by looking at the eigenvectors of A:

No structure: (smooth variation) $\lambda_1 \approx \lambda_2 \approx 0$

1D structure: (edge) $\lambda_1 \approx 0$ (direction of edge), λ_2 large (normal to edge)

2D structure: (corner) λ_1 and λ_2 both large

It is necessary to calculate A at every pixel and mark corners where the quantity $\lambda_1\lambda_2 - \kappa(\lambda_1 + \lambda_2)^2$ exceeds some threshold ($\kappa \approx 0.04$ makes the detector a little "edge-phobic"). Note that $\det \mathrm{A} = \lambda_1\lambda_2$ and $\operatorname{trace} \mathrm{A} = \lambda_1 + \lambda_2$, so the required eigenvalue properties can be obtained directly from the elements of A.

(c) An example of a suitable descriptor of an interest point is the SIFT (Scale Invariant Feature Transform) descriptor. Imagine a keypoint (e.g. a local elliptical region around the point). Compute the image gradient magnitudes and orientations around the keypoint location, using the scale of the keypoint to select the level of Gaussian blur of the image. Second, rotate the coordinates of the descriptor and gradient orientations into some canonical orientation. Third, use a Gaussian weighting function of width half the width of the desciptor to assign weights to each point in the descriptor. Fourth, create a 4×4 grid of regions inside the keypoint, and in each region compute the histogram of the orientations of the gradients, e.g. with 8 bins in each histogram. This defines a $4 \times 4 \times 8 = 128$ integer valued feature vector. It might be desirable to smooth these histograms. Fifth, normalize this vector to unit length. This descriptor is invariant to affine changes in illumination.

[Another simple descriptor is a normalized image patch.]

(d) The nearest neighbour is defined as the keypoint with minimum Euclidean distance to the invariant descriptor, f_i. A simple implementation of nearest neighbour (NN) search: (1) compute for each feature, f_i, the distance to all keypoint features in the database. (2) For each feature f_i find the database feature with the smallest Euclidean distance to f_i. This algorithm is not fast.

Fast exact algorithms for NN are not known for high dimensional data (e.g. 128 dimensions). A fast algorithms for finding nearest neighbors is based on building a tree data structure such as a **k-d tree**. The space (e.g. 128-D) is partitioned recursively creating a binary tree. At each node in the tree a decision based on one of the features is used to divide up the points into the left and right branches of the tree (e.g. $x < 5$ or $x \geq 5$). When trying to find data nearest neighbors, one descends the tree from the root taking left and right branches depending on the decision at that node. The left reached either contains the nearest neighbor or the tree needs to be ascended up a few levels to find the nearest neighbor in one of the adjacent branches. This might not work well in high dimensions, but at least it will find an approximate nearest neighbor.

[A picture of a tree and partitions of the space would be useful].

PAR

ENGINEERING TRIPOS PART IB
PAPER8 / SECTION G
2006
SPG MADABHUSHI

18. Structures at the Molecular Scale

(a) A strand of the double-helical DNA molecule consists of a string of four different kinds of base — A, C, G, T — which carry a coded message specifying all the details of construction and operation of an organism (whether it be a bacterium, plant or animal, etc.). Each successive 3 letters specifies uniquely one of 20 different amino acids (there is a name redundancy, as there are 64 three-letter "words").

The cellular machinery translates the DNA code into a string of amino acids on a (covalently bonded) poly-peptide chain.

The amino acids have distinct chemical characteristics. Thus some are hydrophobic ("oily", water-hating) while the hydrophilic (water-liking) ones may carry positive or negative electric charge. In consequence the chain folds up into a unique conformation, with the structural pattern determined by such factors as the hydrophobic amino acids' desire to be buried in the centre of the globular proteins. (α-helices are a particular "motif" in which the amino-acid sequence has a seven-repeat, with hydrophobic amino acids forming a "stripe" on one side of the "rod", enabling it to form "coiled-coils" with neighbouring α-helices. In general, the folded form of the polypeptide chain of a protein is determined by the *weakest* bonds, such as hydrogen bonds and hydrophobic effects — much weaker than the covalent bonds of the (flexible) polypeptide backbone.) [4]

(b) Some proteins are *hard & tough* (claws, finger-nails); some are *stringy* (hair, tendons); some are *elastic* (skin, tissue); some are transparent (eye lenses); some *combine with calcium* (bones and teeth).

Some proteins build *pumps* and *motors*; some act as *electrical switches* in membranes and nerve cells. Others play a key role in *muscle contraction.*

Enzymes are a class of proteins which catalyse reactions, sometimes by recognising and holding together other proteins and DNA.

Receptors, eg. in the bacterial membrane, recognise individual external molecules of nutrients ("food") and send a signal through the membrane to the interior of the cell — which may set in train a process leading to a reversal of the sense of rotation of a flagellar motor. [4]

(c) The slender flagellar filaments — the bacterium's "propellers", each rotated by a motor embedded in the cell wall — are built from a single protein called flagellin. The flagellin molecules act as building-blocks to make a sort of tubular "tower": the blocks lie on a single spiral path, with approximately 11 blocks every 2 turns. (No need for this to be an integral number: small differences from an integer result in overall *twist* in the building-pattern — which is required if the filaments are to have their overall helical form.)

If all the blocks are in identical environments with their neighbours the "tower" will (obviously) be *straight.* But flagellar filaments are actually helical; and this means that (in addition to twist) they must have *curvature.* How can we build a curved tower out of identical building blocks?

The key to the situation is that the flagellin molecules can switch between two stable forms which have slightly different 3D geometry. Call these forms A and B.

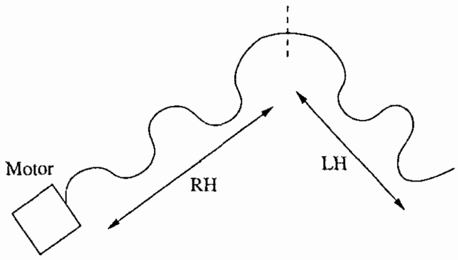
If *all* the blocks were in form A we could find that the blocks would not fit together properly ("close") to form a tube. Neither would they fit together if all were in form B. However, if a certain proportion were A and the rest B, then (with a little elastic distortion of the blocks) the tubular building pattern could occur. If all As were on one side and all Bs on the other; and if As and Bs had slightly different lengths, the resulting "tower" would be curved.

(The building takes place by a process of *self-assembly.* The form of the tower/tube is specified by the details of the geometry (etc.) of the building-block. The nature of the bi-stable switch is not yet fully understood in detail.) [4]

(d) These bacteria swim in straight lines of "smooth swimming" (in which the half-dozen or so corkscrew-like flagella associate to form a single bundle) punctuated by brief "tumbles" (in which the flagellar bundle flies apart). After a tumble, the cell goes off in smooth swimming in a random direction.

If this direction is towards a higher concentration of nutrients (the cell measures a temporal gradient) a control mechanism prolongs the run of smooth swimming; if not, not. In this way the cell homes in on supplies of nutrients in the surrounding water.

At the beginning of a "tumble" some (or all) of the motors go into reverse rotation. This turns the corkscrew-like filaments in reverse. But the filaments are not *rigid* corkscrews. Instead, when driven in reverse the torque makes them switch from a LH to a RH helix. (This is a consequence of the structure described in (c): the torque distorts the structure and its building blocks elastically, and the proportions A/B change, resulting in a different member of a family of discrete helical forms.)



The proximal (near the motor) part becomes Right-handed; and it joins the distal (far) portion smoothly as shown. The dog-leg shape thrashes around in "tumbles". (In particular the growth of the RH portion enables the filament to screw itself, backwards, out of the (smooth-swimming) bundle of flagella.) [4]

(e) The detailed specification of any living organism — whether bacterium, fungus, worm, plant, animal (or virus, which has no autonomous life) — is specified precisely by its DNA. In the course of transcription (in the process of making protein) and replication (duplicating before cell-division) small copying errors may occur (although there is machinery for correcting errors); and this will result in different amino-acid sequences and different proteins.

In this way changes can occur from generation to generation. Radiation may also cause mutations. This is one of the features of *evolution*, as described by first by Darwin. A small change in an organism may later give it an improved chance of survival if the environment changes (eg. a less hairy woolly mammoth may be better able to survive higher temperatures when a crustal plate moves from pole to tropics; and so become the precursor to present-day elephants.)

Now that it is possible, in a more or less routine fashion, to determine the DNA sequence of any arbitrary organism, it is possible to compare the DNA of different species, and to map the branching of the (single) evolutionary tree, which begins with a "primordial ancestor" and branches to produce the rich variety of life currently on earth.

Other points:

* Human genome is about 10^9 letters long.
* Life on earth has evolved over about 10^9 years.
* A change of one DNA letter in the flagellin protein can produce a different helical form of the flagellum — a significant change in the organism.
* A change of one DNA letter does not necessarily change the protein sequence (since the code is redundant). So DNA sequences are better then amino acid sequences for resolving branching in the "tree of life".
* Viruses evolve very rapidly — on a timescale of months. [4]

Assessor's comments: This question required the students to write short notes on topics related to DNA, protein structure and bacterial flagella. It was a popular question and generally well answered. Candidates showed particularly good knowledge of the swimming and navigation of bacteria and many were able to describe the way triplets of bases in DNA encode the structure of proteins. The precise three-dimensional structure of flagellar filaments was less well understood, although a small number of students explained this tricky concept with great clarity.

19. Human Sensorimotor Control

(a) (i) Supervised learning is a technique for approximating an input-output function from training data. For each input vector the desired output vector is supplied. The task of the network is to predict the value of the output for each input after having seen a number of training examples The error between actual output and desired output is used to update the weights. Update rules include the delta rule or backpropagation.

Reinforcement learning is a technique for creating an input-output function in which the desired output is not provided for each input. Instead a scalar value is given which represents reward and the task of the network is to try to maximise reward accumulated over time (often with a discounting factor so that earlier rewards are more valuable than later rewards). Reinforcement learning is often used in a situation in which a learner is in a given state and must take an action, but the consequences of the action have complex relationship with future rewards, such as finding your way out of a maze. [3]

(ii) Supervised learning could be used to train a chess machine by providing training data with inputs of board layout and the desired output obtained by either asking experts what move they would make or by examining past games played by experts. Reinforcement learning could be used by the machine playing against an expert and being provided with a reward signal if it wins the game and a punishment signal if it loses. In fact training is a lot faster with reinforcement learning if you have the machine play a copy of itself. [2]

(b) Visual inputs are ambiguous as, for example, many different three dimensional structures cast the same two dimensional retinal image, and uncertain, due to noise in the visual processing. Bayes rule underlies many perceptual processes which generate a percept from sensory inputs. The interpretation of a visual image will therefore depend on our prior beliefs about the state of the world and on the likelihood of a visual image given different states of the world. These can be used to compute the posterior:

$$\mathrm{P(state\ of\ world \mid sensory\ input)} \propto \mathrm{P(sensory\ input \mid state\ of\ world)} \times \mathrm{P(state\ of\ world)}$$

Illusions can arise when the prior biases the percept so that perception is not veridical but is in fact optimal given the uncertainty in the stimulus. Two examples of priors are the "light form above" prior and the motion illusion prior over object speeds which is approximately Gaussian and centred on zero (most objects in the world do not move and slower objects are more common than faster). These bias the interpretation of ambiguous or noisy stimuli.

When judging speed we have a prior over speeds which is likely to be approximately Gaussian and centred on zero. That is most objects in the world do not move and slower objects are more common than faster. Visual information can also be used to judge speed but as this becomes noisy or uninformative, such as when driving in fog, we tend to rely more on the prior, which biases our percept to the peak of the prior (zero speed) so that we perceive speed as lower than it really is. [5]

(c) The path of the hand tends to take a roughly straight-line hand path (whereas the joint angle path is complex) and the hand speed is symmetric and bell-shaped for fast movements.

The idea taken from optimal-control is that we can associate a cost for every possible way of achieving a task (such as a point to point movement) and the optimal movement is the one with the lowest cost. Movement planning then amounts to selecting the movement with the lowest cost. For arm movements, the observed smoothness (straight hand path with bell-shaped speed profiles) has led to the idea that the cost function is the mean-squared jerk (the temporal derivative of the acceleration) of the hand or the mean squared rate of change of the joint torques.

An alternative cost is the variability in movement endpoints around the target (mean squared error). This variability is assumed to arise from signal-dependent noise, in which the standard deviation of the noise in the motor command signal is proportional to the magnitude of the motor command (constant coefficient of variation). Given this noise there is in general one unique way to move to minimise variability at the end of the movement and this is in good agreement with empirical data. [5]

(d) (i) There are two distinct strategies people employ when learning novel dynamic tasks, such as moving while an external force acts on their hand. First, learning the forces required to compensate for an externally imposed perturbation (with an inverse model) they can directly counteract the perturbing influence. Alternatively, by co-contracting muscles, they can increase the stiffness of their arm and thereby reduce the displacement caused by an external force. When reaching in a predictable force field people tend to employ a low-stiffness strategy and learn to represent the compensatory forces. Early in the process of learning the stiffness of the arm reduces systematically as these compensatory responses are learned.

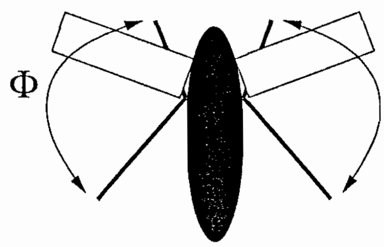
(ii) In several situations, however, it is not possible to reliably predict the forces the hand will experience, and therefore compensation is difficult. For example, when drilling into a wall with a power drill, the aim is to maintain the drill bit perpendicular to the wall while applying an orthogonal force. This situation is inherently unstable, in that any deviations from orthogonality lead to forces that destabilise the posture. In this situation the stiffness of the hand can be increased in all directions, thereby stabilising the system. Recently it has been shown that stiffness can be increased in a task specific manner to match directions of instability in the task.

(iii) Two mechanisms can be used to distinguish between stiffness (co-contraction) and compensation. First if the perturbation is unexpectedly turned off the change in movement path (after-effect) can be assessed. If there is no change this suggests that a stiffness control strategy is being used, whereas a large after-effect suggests a compensation strategy. Alternatively the electromyogram (EMG) from the muscle can be recorded and co-contraction assessed by determining to what extent opposing muscles are active. [5]

Assessor's comments: This question was about types of learning, understanding optical illusions in a Bayesian framework and the control of movement in humans. The discussion of learning algorithms was generally weaker than the understanding of motor-control. The answers to the part of the question about optical illusions could be separated into those from candidates who had a conceptual understanding and those from candidates who preferred a more mathematical explanation. Some of the brighter students managed to present both these perspectives and relate them together.

20. Animal Flight and Sports Engineering

(a)



R = wing length

c = chord = 0.3R

$$\phi = \overline{\phi} + \frac{1}{2}\Phi\cos(2\pi f t)$$

(i) The question says that the downwash is negligible compared with the flapping velocity, so use the small angle approximations from lecture 2 for the relative velocity and vertical force:

$$\begin{aligned} U_r &\approx \text{Flapping velocity} \\ F'_{\text{vert}} &\approx L' \end{aligned}$$

From first principles (of from the lecture handout), derive the flapping velocity as

$$U(r,t) = r\frac{d\phi}{dt} = -r\pi\Phi f\sin(2\pi f t)$$

So the lift per unit span is

$$L' = \frac{1}{2}\rho c U_r^2 C_L \approx \frac{1}{2}\rho c r^2 \pi^2 \Phi^2 f^2 C_L \sin^2(2\pi f t)$$

and the mean lift per unit span is

$$\overline{L'} = \frac{1}{4}\rho c r^2 \pi^2 \Phi^2 f^2 C_L$$

Integrate the mean lift per unit span along one wing length and multiply by 2 for the other wing. Equate this to the weight since, according to the small angle approximation, the lift force is vertical.

$$\begin{aligned} mg &\approx \frac{1}{4}\rho\pi^2\Phi^2 f^2 C_L\left[2\int_0^R cr^2\,dr\right] = \frac{1}{4}\rho\pi^2\Phi^2 f^2 C_L\left[2\int_0^R (0.3R)r^2\,dr\right] \\ mg &\approx 0.05\rho\pi^2\Phi^2 f^2 C_L R^4 \end{aligned}$$

[6]

(ii) Simply substitute to get

$$(0.01\text{kg})(9.81\text{m/s}^2) = 0.05(1.2\text{kg/m}^{-3})\pi^2(2\text{rad})^2 f^2(1.8)(0.15\text{m})^4$$

$$f = 6.74\text{Hz}$$

[2]

(iii) From part (a) we have $mg \propto \Phi^2 f^2 C_L R^4$. Over 6 orders of magnitude for mass, Φ and C_L can vary only by relatively small amounts and therefore should be considered as constants. Hence the proportionality reduces to $m \propto f^2 R^4$. Given that $R \propto m^{\frac{1}{3}}$, we would expect frequency to vary as $f \propto m^{-\frac{1}{6}}$. [2]

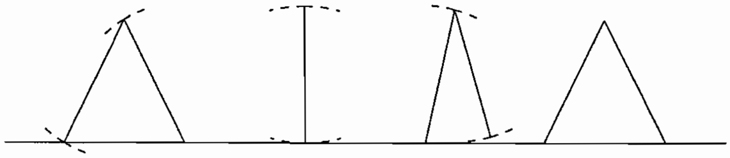
(b) (i) A *kinematic analysis* of human motion describes positions, velocities and accelerations. There is generally no reference to the causes of motion. This is one of the most basic types of analysis for sports applications.

Kinematic data can be collected in a number of ways. Three of these are outlined below (N.B. three methods will be discussed in more detail in the lecture course, but others will also be mentioned, so a discussion of these others will also be allowed).

- **Optical Motion Capture**
  This is one of the most common and easily available forms of performing kinematic analysis. Such systems are multiple-camera setups in which each camera synchronously records the image-positions of bright markers placed on the subject. If the system is calibrated (i.e. we know the relative positions of the cameras and their internal optics) we are able to triangulate to reconstruct the 3D positions of the markers. The number of cameras in typical systems ranges from 2 to 24, depending on the nature of the motion to be captured. The speed of the cameras ranges from 50Hz to 1000Hz. Markers are either active LEDs (usually with some sort of internal identification, e.g. modulation) or small passive retro-reflective markers which reflect IR pulsed by a source around the cameras. The advantage of the identified LEDs is there is no need for matching of points between cameras, the disadvantage is that, because they are wired, they can sometimes restrict movement. The faster the cameras, the easier it is to track the bright points. Optical systems will give only the position of markers – from the positions of several markers on each limb, the limb rotation can be inferred.
- **Magnetic Motion Capture**
  Magnetic systems consist of small sensors (wired) placed on the body to measure a low-frequency magnetic field generated by a transmitter source; the sensors report positional and rotational information. The sensors and source are cabled to a control unit that correlates their reported locations within the field. The more expensive systems now have a wireless connection between the control unit and the sensors which produces fewer restrictions in the type of motion that can be captured.
  In general the frame-rate obtained from magnetic systems decreases with the number of sensors used. Since each sensor requires its own (fairly thick) shielded cable, the tether used by non-wireless magnetic systems can be quite cumbersome. With AC-based systems there is often adverse reaction to metal in the environment, which makes such systems very difficult to use in arbitrary laboratories. One advantage of such magnetic systems is that there is no tracking of sensors required and fewer sensors are required as each sensor produces both position and rotation information.
- **Electro-mechanical Motion Capture**
  In these systems the subject wears a specially made suit which consists of rigid, connected links (usually metal) which are strapped onto the limbs of the body. As the body moves, potentiometers attached to the suit determine the rotation of each link. Such suits have a wide working-space as they are not fixed to any particular set of cameras or transmitter. They can also capture data at a high rate and have the additional advantage of being less expensive than other systems. The big disadvantage is that they are very cumbersome to wear and inhibit free motion – they are not often used for sports analysis.

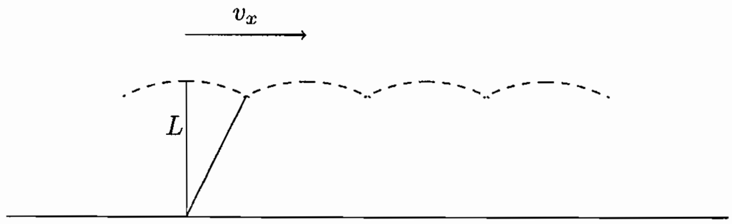
When the data has been collected from the sensor-based systems, we have the 3D positions of the sensors over time throughout the motion observed. To turn this into movement of the body's skeleton, we must either ensure that we place the sensors on the joints (very hard to do accurately) or that we extract the best estimate of the joint from the marker data (centre of rotation estimation). With the resulting data for joint position and rotation over time we can perform any sort of kinematic analysis we wish to achieve. [5]

(ii) In the *inverted pendulum* model of walking, let us model the legs as rods which are pivoted at the hip and are free to swing with their natural period. We will assume for the purposes of analysing dynamics and energetics that the centre of mass of the body is at the hips. However, we also assume, for the purposes of calculating the natural period of the leg swing, that the legs are uniform cylinders. As one leg comes into contact with the ground, that leg pivots about the point of contact while the other leg swings through – this is illustrated in the figure below where T is the natural period of the leg swing.



$t = 0$ $\qquad$ $t = T/4$ $\qquad$ $t = T/2$

We assume that the knee bends to avoid the swinging leg hitting the ground but otherwise we will not consider this knee-bend in the model (we assume that the legs are stiff).
The swinging leg makes contact with the ground when it has swung through its full range – the cycle then begins again, with each foot leaving contact with the ground as soon as the other touches. The centre of mass of the body therefore moves through a sequence of circular arcs as illustrated below.



The normal walking speed is then related to the natural period of the pendulum (in this model a faster walk would require a faster natural swing).
For a physical pendulum the period of natural swing (time taken to return to the same position travelling in the same direction) is given by

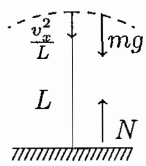
$$T = 2\pi\sqrt{\frac{I_{supp}}{MgL_{CoM}}}$$

where I_{supp} is the moment of inertia of the pendulum about the point of support, M the mass of the pendulum and L_{CoM} the distance of the CoM from the point of support. In our case we are modelling the leg as a uniform cylinder swinging about one of its ends – the moment of inertia is a standard result and is given by $\frac{1}{3}ML^2$ where L is length of leg. Thus, using this for I_{supp} in the above and replacing L_{CoM} with $\frac{1}{2}L$ gives us

$$T = 2\pi\sqrt{\frac{(1/3)ML^2}{Mg(1/2)L}} = 2\pi\sqrt{\frac{2L}{3g}}$$

as required. Note that the above is the small angle approximation and the leg swing is not exactly small angle – it nevertheless gives us an approximate value of the period.

Transition from walking to running: In the above model, as long as there is contact maintained with the ground at all times, the body centre of mass moves along a circular trajectory as above. Consider the point at which the leg in contact with the ground is vertical and consider the forces acting on this leg – see diagram below:



We have the vertical upward normal reaction from the ground, N, and the weight acting vertically downwards, mg. As the leg is moving in a circular arc the acceleration will be v_x^2/L, where v_x is the forward velocity of the body. Thus, by Newton's 2nd law we are able to write

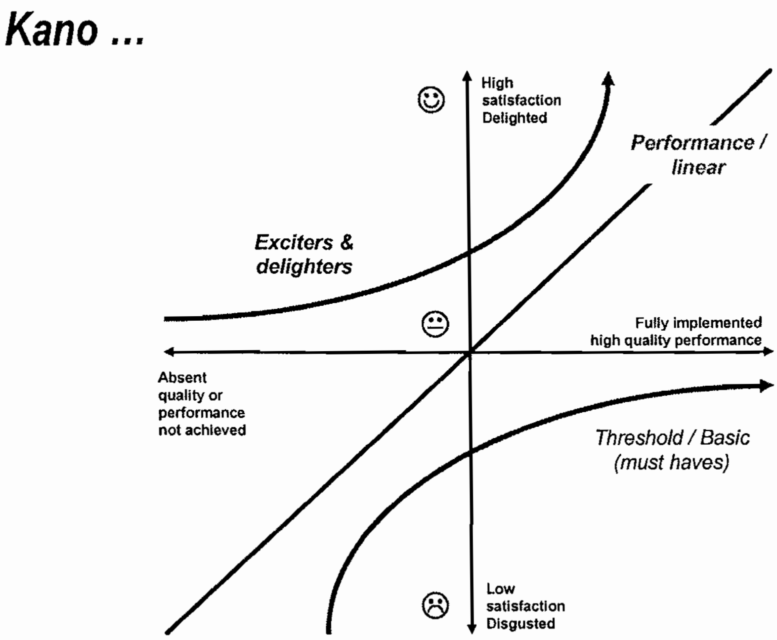
Part IB 2006 Paper 8 section H crib

Question 21

(a)

The Kano model was developed by Noriaki Kano in the 1990s. It is a good way to classify requirements based on perceptions of customer quality. Kano classifies attributes into 4 main categories:

Indifferent: attributes to which the customer pays no attention "If they are present, it is nice. If they are not present, it does not matter"



Expected attributes (Threshold / Basic): have to be present in the product in order to make it successful. Failure to offer these attributes represents secured failure for the product and customer remains neutral regardless of how well the feature is executed. Provide no differentiation (brakes).

One dimensional attributes (Performance / Linear): Characteristics that are directly correlated to customer satisfaction. Increased functionality results in increased customer satisfaction. Decreased functionality results in greater dissatisfaction. Price is related to these attributes

Attractive attributes (Exciters / Delighters): Customers get great satisfaction from a feature - and are willing to pay a premium. Satisfaction will not decrease (below neutral) if the product lacks the feature. Unspoken and unexpected by customers. Often satisfy latent or unknown needs.

A good answer will comment that these attributes change over time and that today's delighter may be expected or linear in the future.

Examples relevant to this problem:

- Indifferent: Choice of basket material, advertising space (customers indifferent, but suppliers – may be a delighter)
- Expected: baby seat, separate 'vegetable' area
- One dimensional: basket size
- Attractive: novel basket shape, compartments for different types of goods, space for bags, extra narrow to stop congestion etc.

Strengths and weaknesses:

- It is a good way to encourage a design team to consider the customer benefits of their ideas – better than a ranking of demand/wish.
- It can be difficult to classify attributes. It can be hard to classify some technical features.
- It enables a team to debate and discuss ideas.
- It is perhaps over-simplistic about how people really respond.
- As with all design tools, it can be used too rigidly, without sufficient critical judgement on behalf of the team.

(b)

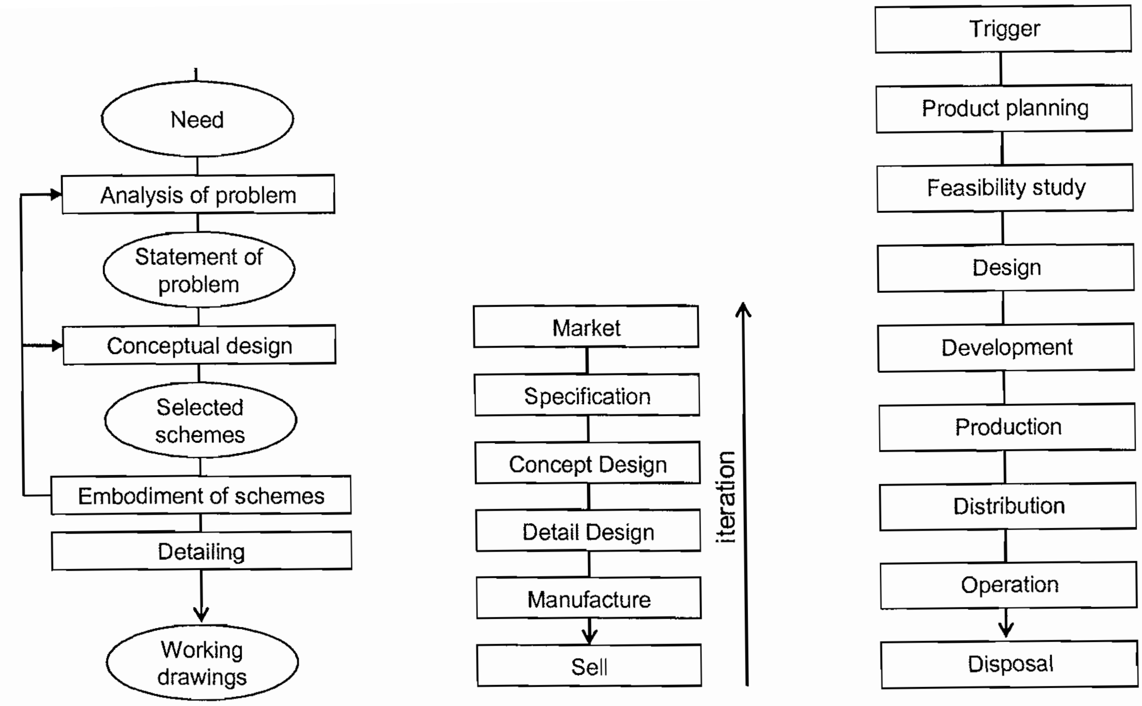
A good answer should comment on different approaches to describing the design process:

Prescriptive models vs descriptive models of the design process. Descriptive do not mandate any specific stages or actions but illustrate a typical design sequence. Prescriptive processes provide a series of steps to follow.

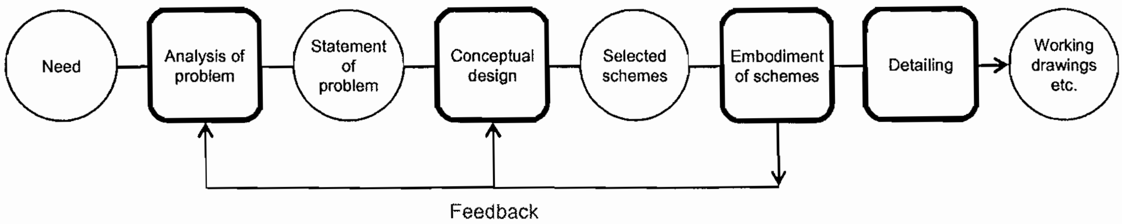
It would include a sketch of a design process, outlining a number of steps, starting from idea, progressing through to production.

A good sketch will include iterative loops and an indication of the expected outputs at the end of each stage.

Some example processes from the lectures are shown below:



Cross **Pugh, simplified** **BS7000**



A good answer would suggest specifically why the choice would be appropriate for the development of a shopping trolley – for example …

There would need to be a clear requirements understanding stage

There would need to be a lot of testing and evaluation.

(c)

A good answer should firstly give an indication of the overall design process, starting from idea through to final production. If they are sensible, this will be the same process described in part (b).

A simple process might include three stages: requirements capture, concept design, detailed implementation.

The purpose of all prototypes is to help reduce either market or technical risks.

Requirements capture: here, you would expect to use 'low fidelity' prototypes to help reduce market risks. The simplest would be a concept sketch, storyboards or a 'block model', that could be used to gather early user/customer feedback. It may also be possible to have early stage technical/functional prototypes to test the feasibility of new ideas. These prototypes by necessity should be low cost

Concept design: here, the prototypes would be used to test market and technical risks. Prototypes might range from analytical models to ealy stage concept models. If necessary, higher fidelity functional prototypes might be necessary to test a new technical feature. Visual models might enable different concepts to be tested with users.

Detailed implementation: here, you would expect to see high fidelity prototypes to enable detailed evaluation of technical performance. These would include full production prototypes. Such high fidelity models are expensive, but are necessary before the design is released to production.

Question 22

(a) Three from the following:

Business model #1
CDT invested in the further development of the technology and planned to invest in production capability to allow the company to manufacture displays based upon their technology, and then to sell these displays on to computer, communications and electronics companies. Resources = R&D labs, volume production facility, distribution, sales and marketing, after-sales support, etc.

Business model #2
CDT continues to invest in the further development of the technology does not build any factories. Instead, they find partners who wish to licence the technology who will then make the investment in production capability to make display devices themselves, and then sell them to a range of customers. Resources = R&D labs, sales and marketing, IP management

Business model #3
CDT continues to invest in the further development of the technology, find partners who wish to licence the technology who will then make display devices and sell them. In addition, CDT builds a small factory to develop and sell knowledge of how to produce displays based upon LEPs, and also produce own displays for niche market applications. Resources = R&D labs, sales and marketing, IP management, low-volume production facility, service delivery team, sales and marketing, after-sales support

Business model #4
CDT stops making and selling products for niche markets, but continues to invest in the further development of the technology, find partners who wish to licence the technology They also continue to sell knowledge of how to produce displays based upon LEPs. Resources = R&D labs – inc production processes, sales and marketing, IP management, service delivery team, sales and marketing

(b)
Equity investment = selling part of the ownership (shares) of a business in return for cash. The assumption is that whoever buys shares will wish to sell them to someone else in the future at a higher price. The only reason for doing this is if the investor can be convinced that: (a) the business is really going to grow (and quite fast) and that (b) there will be someone else willing and able to buy their share in the company at a later date

Business Angels = wealthy individuals who choose to invest some of their own money in new business ventures. The term 'smart' is used to refer to those business angels who are able to bring not only money but also expertise of particular technologies, markets and industries, based upon their own experience.

Venture capitalists (known as 'VCs') are a form of high risk, high return investment. VCs raise money from large institutions such as pension funds and then re-invest portions of this money in high growth potential businesses. They know that a number of their investments will fail, so they seek to ensure that those that do succeed do so in such a manner that not only covers the costs of the failed investments, but also generates the very high levels of returns required to please their investors and cover the VC fund's own management costs.

Venture capitalists will look for a strong plan in all of the following areas: the market, the offering (product and /or service), the management team, the business operations, the financial projections, the marketing strategy, the resources required, and the exit opportunities. Some VCs say that they really just focus on the team and core idea to see if they are both 'stellar'. Between these two, preference is often for the quality of the team. The level of ambition of the team and realistic opportunity for an investment exit are also major concerns for VCs

(c)
Revenue = money from selling something. If a company hasn't yet got a product to sell, there still may be ways of getting revenue. For example, selling expertise (consultancy) in your specialist area may be one way of bringing money in to fund the development of a product.

Grant = a 'gift' from an organisation such as the UK Department of Trade and Industry (DTI) or NESTA (National Endowment for Science, Technology and the Arts). The type of projects that are eligible for funding through this route may be very restricted.

Debt = borrowing money from a bank or specialist finance organisation. You can only borrow money if you can convince the bank that you can repay the money, plus interest, exactly when they want it. This is usually very hard for a new company to do.

Question 23

(a)

In the lectures, five such strategies were described:

- Leasing assets instead of purchasing- for instance, leasing a photocopier
- Designing the product to reduce the need for fixed assets – for instance using manual labour to bend sheet metal components rather than developing dedicated tooling
- Purchasing services to avoid indirect costs – for instance payroll and accounting services
- "Outsourcing" a non-core activity, such as production of a product, where design is the 'core-competence' of the business
- Developing a joint venture to share the risk of development – for instance as done by CDT Ltd to share risk in developing products that used CDT's novel light emitting polymers.

(b)

(i) Breakeven volume (V) satisfies $75V = 10{,}000 + 50V$, so $V = 400$

(ii) Let $Q =$ monthly level of production.
Price $P = 75$ ($Q<=300$),
$P = 75 - 0.1*(Q-300)$ ($Q>300$)

Then if $Q>300$, Profit = Revenue – Costs
= revenue for first 300 + revenue for (Q-300) - Costs
$= 300*75 + (Q-300)*0.5*(75 + (75-0.1(Q-300))) - (10000 + 50Q)$
$= 55Q-0.05Q^2-14{,}500$
(a simple sketch graph is useful to work out mean price for $Q > 300$)

This is maximised when $Q = 550$, at which point Profit = £625

(Alternatively and simpler method: maximum profit occurs when marginal cost = marginal revenue. So $75 – 0.1x = 50$; therefore $x = 250$; therefore $Q = 300 + x = 550$)

(iii)

At $Q = 550$, total costs will be £10 000 + 550 x £50 = £37 500 which represents £37500/550 = £68.18 per unit. The average selling price is (£37 500 + £625)/550 = £69.32 per unit so the average profit per unit is £1.14. By avoiding all fixed costs, the product need be only just profitable per sale to achieve the same maximum profit. Avoidance of fixed costs reduces risk, as no minimum sales volume is required to cover the initial investment. However, if sales exceed the breakeven volume, profitability will be greater with lower variable costs.