ENGINEERING TRIPOS PART IB

Monday 5 June 2006 9 to 11

Paper 1

MECHANICS

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

The answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Single-sided graph paper	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 A rotor consists of three similar uniform discs, A, B and C, mounted on a single shaft. Each disc has a radius 0.5 m and the discs are spaced axially at 1 m from each other. The rotor is supported on two bearings, P and Q, 3 m apart. Discs A and C are spaced at 0.5 m from bearings P and Q respectively as shown in Fig. 1. Disc C is perfectly mounted on the shaft with no unbalance but discs A and B are mounted with their centres of mass displaced from the shaft resulting in net unbalances of 0.01 kg-m each. The angular displacement between the centre of unbalance of disc B is measured to be 90° counterclockwise from the centre of unbalance of disc A, as viewed along the shaft from P to Q.

(a) Distinguish briefly between statically and dynamically balanced rotating shafts. [2]

(b) If the shaft runs at a speed of 1000 rad s^{-1}, calculate the forces supplied by the bearings. [8]

(c) It is proposed to achieve static and dynamic balance for this system by adding masses to the rims of discs A and C. Estimate the magnitude and angular position (relative to the offset of centre of mass of A) of the balancing masses. [10]

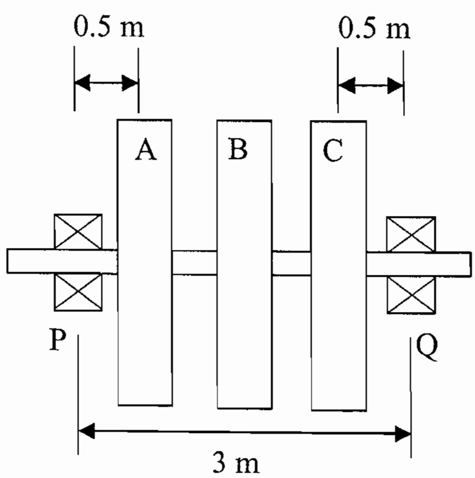

Fig. 1

2 A thin rod of mass m and length l is connected to a trolley at P by a frictionless hinge as shown in Fig. 2. The trolley is initially at rest and is then given a constant horizontal acceleration a. The rod is initially at rest with $\theta = 0$.

(a) Show that the angular acceleration $\ddot{\theta}$ is given by:

$$\ddot{\theta} = \frac{3}{2l}(a\cos\theta - g\sin\theta)$$

Hence, find an expression for the angular velocity $\dot{\theta}$ of the rod as a function of θ. [8]

(b) Write down expressions for the axial and transverse reactions on the rod at point P as functions of θ. [6]

(c) Write down an expression for the bending moment at the mid-point of the rod as a function of θ. [6]

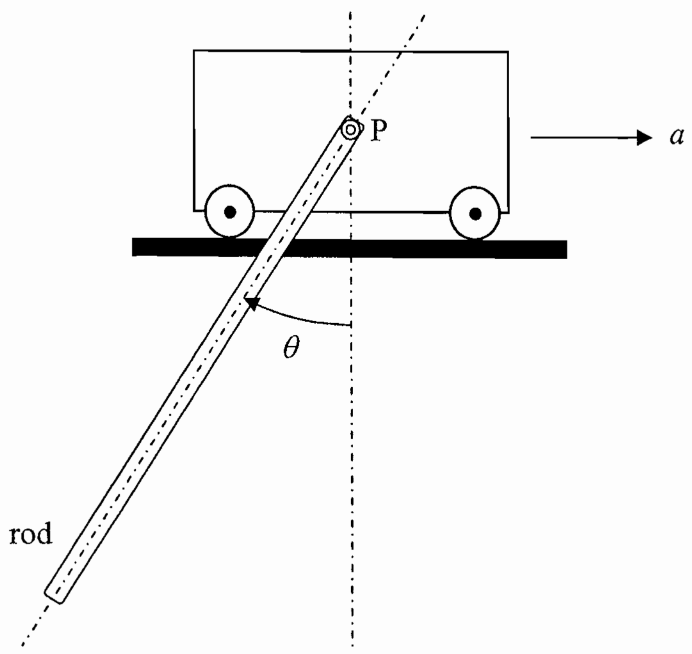

Fig. 2

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3 A thin uniform wire of mass m is bent into a closed semi-circular hoop of radius r as shown in Fig. 3. The hoop is hung from a peg at P, and its centre of mass is located at G. P lies on the axis of symmetry and the radius of the peg is small compared to r. The hoop is initially at rest. It is then set into small amplitude oscillations about P.

(a) Show that the centre of mass of the closed semi-circular hoop G is located on the axis of symmetry at a distance $\pi r/(\pi + 2)$ below P. What is the moment of inertia of the closed semi-circular hoop about P? [8]

(b) Write down the kinetic and potential energy of the hoop for small amplitude oscillations about P as a function of θ and $\dot{\theta}$. [6]

(c) Using conservation of energy, determine the natural frequency of small oscillations about P. [6]

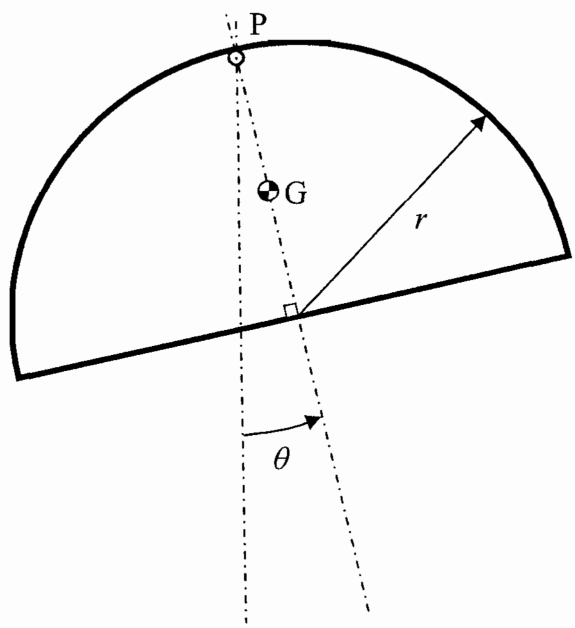

Fig. 3

SECTION B

4 (a) Find the moment of inertia of the thin rectangular lamina of mass m shown in Fig. 4 about an axis perpendicular to the plane of the lamina and through its centre of mass G. [4]

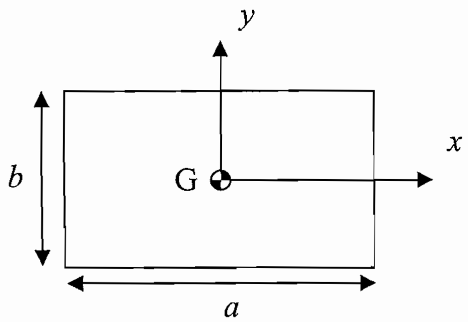

Fig. 4

(b) Fig. 5 shows a simple model of a collision between two vehicles. Vehicle P has mass m and a velocity before impact of $5u$. The larger and heavier vehicle Q has mass $5m$ and an initial velocity of u in the opposite direction. The inertia of vehicle Q may be modelled as a lamina of uniform density, whereas vehicle P should be modelled as a particle. After the impact, the vehicles remain connected together at the corner of vehicle Q (labelled O).

(i) Find the velocity of P and the angular velocity of vehicle Q immediately after the impact. [8]

(ii) Find the energy dissipated in the impact. [8]

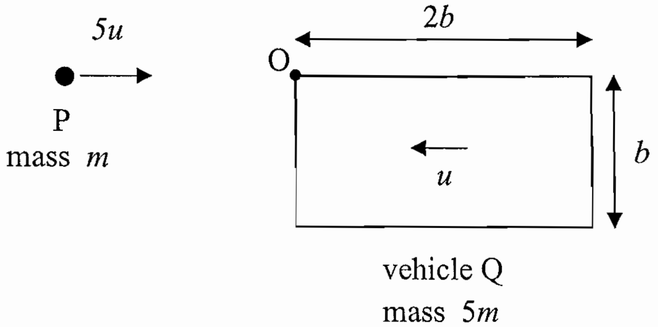

Fig. 5

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5 Figure 6 represents part of a fun fair ride in the Cartesian co-ordinate frame (**i**, **j**, **k**) with origin O. OP is a rigid shaft of length $2a$ which rotates with constant angular velocity Ω about the vertical axis **k**. Three 'cars' Q, R and T, each of mass m, are mounted on arms of length a which are rigidly connected to the shaft OP. The arms rotate with constant angular velocity ω about the axis OP. In the configuration shown Q has position vector OQ = $2a\,\mathbf{i} + a\,\mathbf{k}$.

At the instant shown, find:

(a) The absolute velocity of Q. [8]

(b) The absolute acceleration of Q. [8]

(c) The forces acting in the link arm PQ. [4]

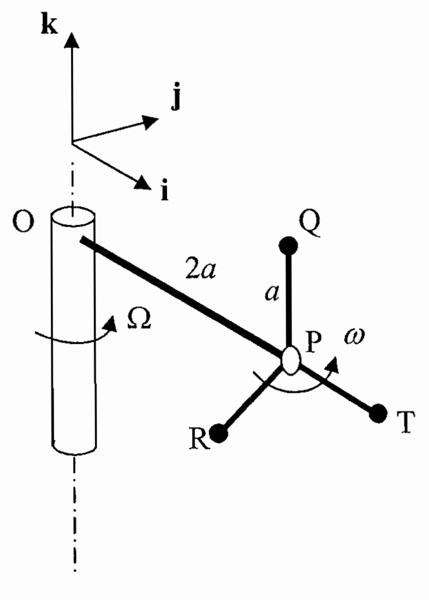

Fig. 6

6 Figure 7 shows a mechanical digger consisting of two rigid links, BCD (length 2 m) and DEF (length 3 m). The links are controlled by two hydraulic rams AC and CE. In the configuration shown the digger is stationary and the tip of the digger scoop F is moving horizontally at a constant speed of 1 m s^{-1}.

(a) Find the extension speed of each hydraulic ram at this instant. [8]

(b) Find the extension acceleration of each ram at this instant. [12]

Note: If you wish to construct velocity and acceleration diagrams, separate graph sheets are provided which you should hand in with your script. A suggested scale for a velocity diagram is 100 mm = 1 m s^{-1} and a suggested scale for an acceleration diagram is 100 mm = 0.25 m s^{-2}.

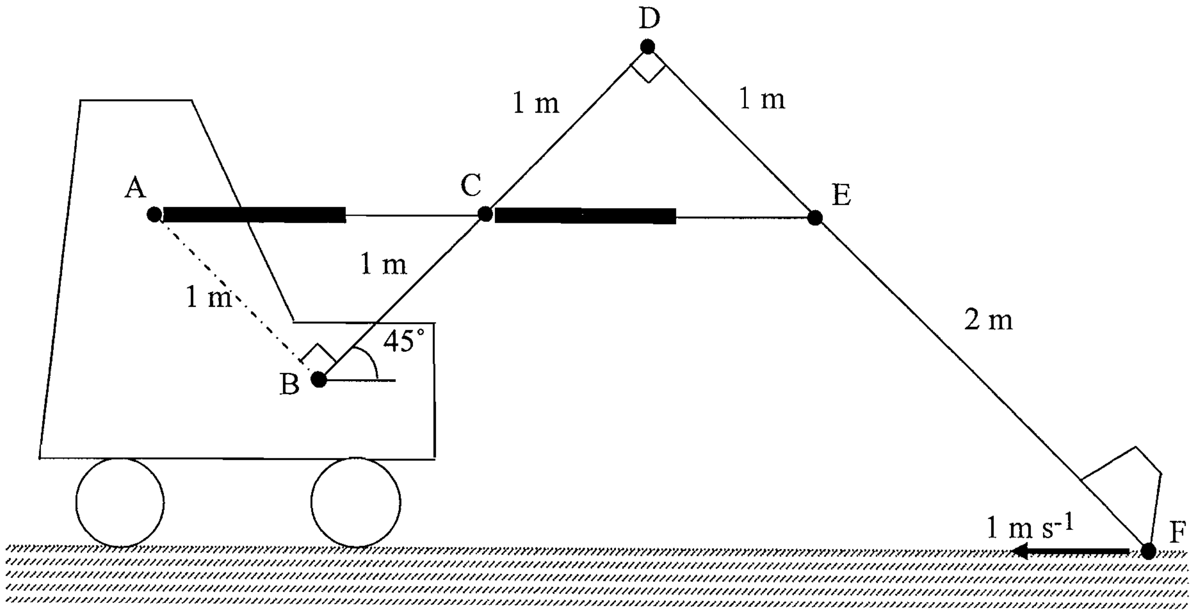

Fig. 7

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