ENGINEERING TRIPOS PART IB

Monday 5 June 2006 2 to 4

Paper 2

STRUCTURES

Answer not more than ***four*** *questions, which may be taken from either section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 Figure 1 shows a pin-jointed structure, where only one node is not fixed to a foundation.

(a) Find a state of self-stress in the structure. [3]

(b) All bars have cross-sectional area A, are made from material with a Young's Modulus E, and have negligible self-weight. The structure is initially unstressed when it is unloaded. A load W is now applied to the structure at node B as shown. By considering the symmetry of the structure find the elastic solution for the forces in the bars. [7]

(c) In a revised design, bars I and III are replaced with *cables* of the same cross-sectional area A. The cables also have a Young's Modulus E in tension, but are unable to carry compression due to slackening. A turnbuckle is added to member II that allows the bar to be extended thereby stressing the system before any external load is applied. The level of self stress is such that cable III will first slacken when a load of $2W$ is applied at B.

(i) Write down the forces in the members when this load of $2W$ has been applied, and cable III has slackened. [5]

(ii) The load $2W$ is now removed. Find the forces in the three members. [5]

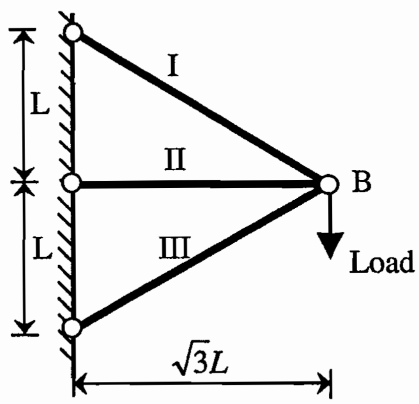


Fig. 1

2 The steel beams shown in Fig. 2(a) and (b) are to be analysed *elastically*. Both beams are UB305 × 127 × 48 and unstressed when unloaded. Point loads of W_1 and W_2 are applied as shown.

(a) The beam in Fig. 2(a) is supported on three pin joints and cannot lift off these supports.

(i) An antisymmetric load $W_1 = 10$ kN, $W_2 = -10$ kN is applied to the structure. Calculate the deflections at Q and S, and the rotation at R. [5]

(ii) A symmetric load $W_1 = 10$ kN, $W_2 = 10$ kN is applied to the structure. Find the bending moment in the beam at R. [5]

(b) The point supports at P and T in Fig. 2(a) are now replaced by fully-fixed supports, as shown in Fig. 2(b). A load $W_1 = 20$ kN, $W_2 = 0$ kN is applied. Find the bending moment in the beam at R and the rotation of the beam at R. [10]

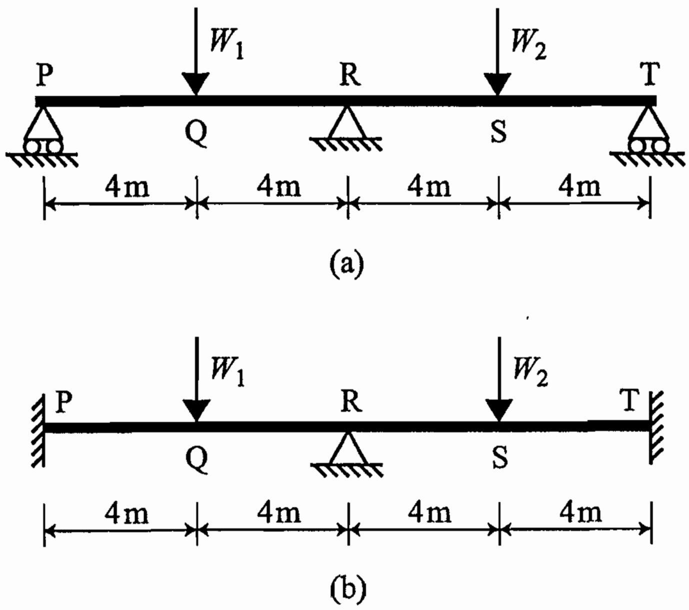


Fig. 2

3 Figure 3(a) shows a cantilever, fully restrained at its base. The beam has a symmetric rectangular hollow cross-section as shown in Fig. 3(b). A tip load of 50 kN is applied along the vertical centre-line of the beam. Two points on the beam are marked, both close to the root of the cantilever: A is at the centre of the top of the flange, B is at the top of the web.

(a) (i) Determine the shear stress at both A and B, located at the cross-section of the beam shown in Fig. 3; [6]

(ii) calculate the normal longitudinal bending stress at both A and B. [2]

(b) The loading on the beam is to be increased by a factor λ. The beam is made of steel with a uniaxial yield stress of 245 MPa. Considering only points A and B, estimate λ when first yielding occurs: (i) using the Tresca Yield Criterion; and (ii) using the Von Mises Yield Criterion. [12]

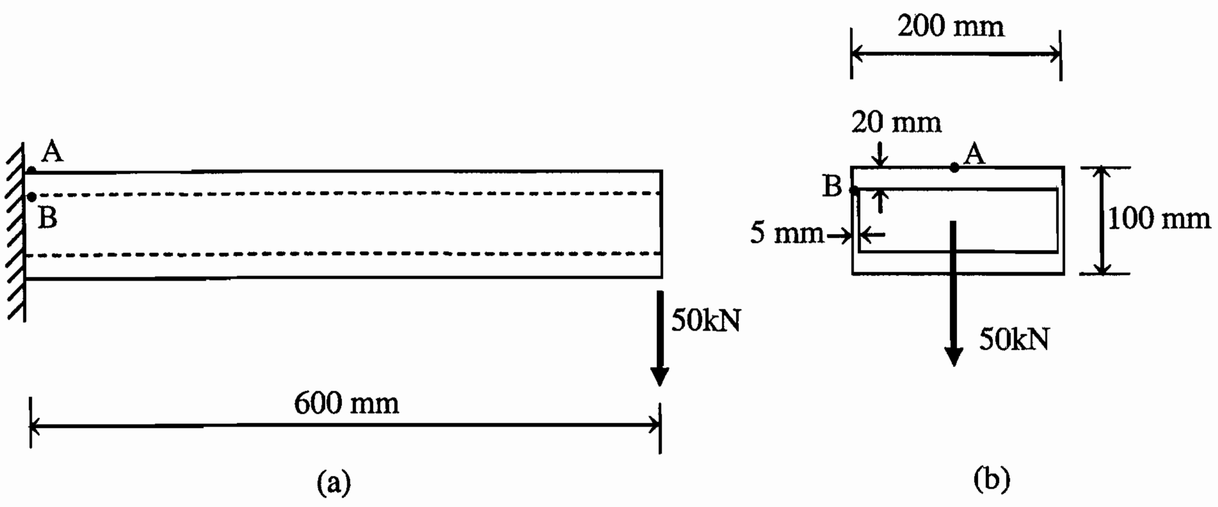


Fig. 3

SECTION B

4 Figure 4(a) shows a cranked pipe ABC made from a thin-walled hollow steel tube of uniform cross-section as shown in Fig. 4(b). The pipe is fixed rigidly to the wall at A. The angle at B is 90°. Vertical point loads of magnitude P are applied through the centre of the pipe upwards at B and downwards at C. The self-weight of the pipe is negligible.

(a) For each leg of the pipe sketch separate diagrams of bending moment, shear force and torque, marking on your graphs the magnitudes of each of these force resultants at key locations. [6]

(b) Show that for a thin-walled circular pipe the polar second moment of area J is equal to $2\pi r^3 t$ and the second moment of area I is equal to $\pi r^3 t$. [2]

(c) If $P = 2000$ N, $r = 25$ mm, $t = 2$ mm and $L = 100$ mm:

(i) determine the vertical displacement of point C due to this loading; [4]

(ii) calculate the longitudinal bending stress and the shear stress at A_1 located at the crown of the pipe immediately adjacent to the support at A; [2]

(iii) draw the Mohr's circle of stress for the point A_1 and determine the magnitude and direction of the three principal stresses at this location. [6]

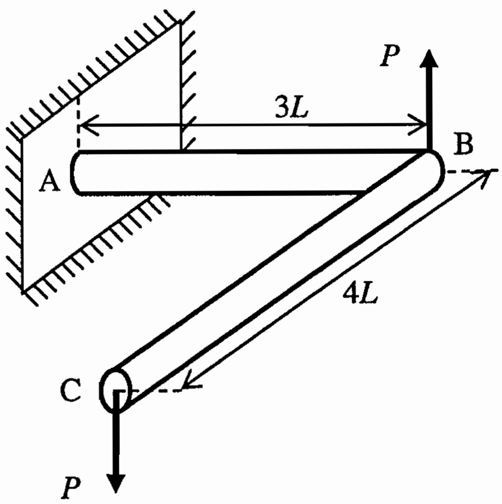


Fig. 4(a)

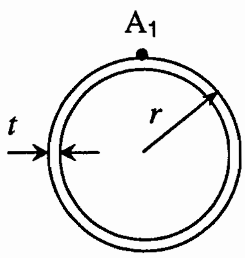


Fig. 4(b)

5 (a) Which, if any, of the four yield-line patterns shown in Fig. 5(a) are geometrically compatible? For any invalid pattern sketch an alternative compatible failure mechanism. [4]

(b) A rectangular reinforced concrete building slab ABCD shown in Fig. 5(b) is simply supported along the two longer sides and fully fixed along the shorter sides. The slab is required to carry a uniformly distributed load of w per unit area. The slab is reinforced so that its moment capacity in sagging and hogging anywhere in the slab is m per unit width. The self-weight of the slab may be ignored. For the mechanism shown in Fig. 5(b):

(i) show that the collapse load w is given by:

$$w = \frac{m}{L^2}\frac{24(3\alpha+1)}{\alpha(9-2\alpha)}$$ [5]

(ii) find the optimum value of the geometric parameter α and hence estimate the least upper bound value of w for this mechanism. [7]

(c) A designer wishes to examine the use of either a steel plate, with yield strength 300 MPa, or a solid glass plate, with tensile strength 30 MPa, as alternatives to the reinforced concrete slab in Part (b).

(i) For which, if any, of these materials would it be valid to use yield-line analysis to determine the collapse load? Justify your answer. [2]

(ii) For the alternative material(s) deemed suitable for plastic analysis, what thickness of plate would be required to support the same uniformly distributed load w calculated in Part b(ii) assuming the reinforced concrete slab has a moment capacity of $m = 50$ kNm/m. [2]

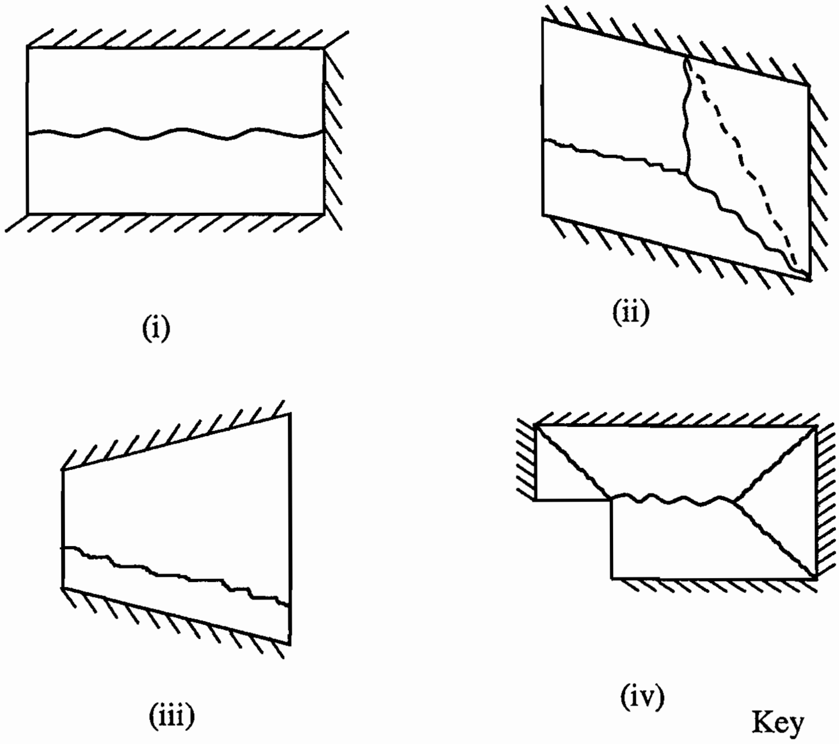


Fig. 5(a)

Simply supported
Fully fixed
Sagging
Hogging

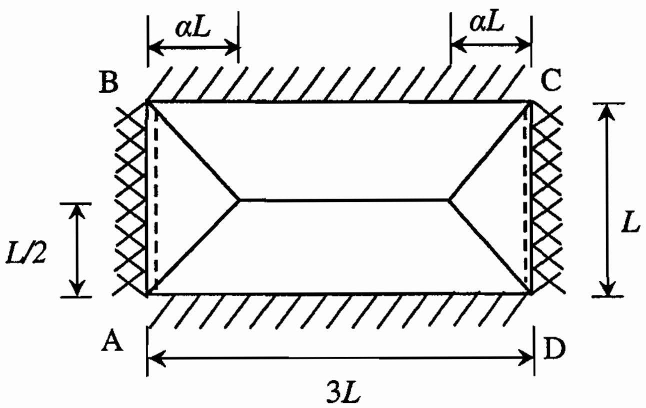


Fig. 5(b)

(TURN OVER

6 The beam ABC shown in Fig. 6(a) has cross-section as shown in Fig. 6(b) and plastic moment capacity M_p. The loading consists of a vertical point load P located at midspan of each of the spans. Self-weight of the beam may be ignored.

(a) Postulate one simple collapse mechanism for each span and hence, using the *upper bound theorem,* derive an expression for the maximum value of P, in terms of the plastic moment capacity M_p, that can be carried safely by this structure. [5]

(b) As an alternative approach, use the *lower bound theorem* to derive an expression for the maximum value of P in terms of the plastic moment capacity M_p that could be carried safely by this structure. Draw the bending moment diagram for this beam at collapse. [9]

(c) Calculate the value of the plastic moment capacity of the section and hence the load capacity P, assuming the yield stress of steel is $\sigma_y = 300$ MPa and $L = 1$m. [4]

(d) What would be the effect on the collapse load of this structure if the support at A rotates clockwise through 1° due to rotation of the foundation? The beam is constrained so that it cannot lift off the supports at B or C. [1]

(e) Would the same calculations be valid if the steel beam was replaced by one made from FRP (fibre reinforced plastic), a material with a linear stress-strain curve to failure? Explain your answer. [1]

(cont.

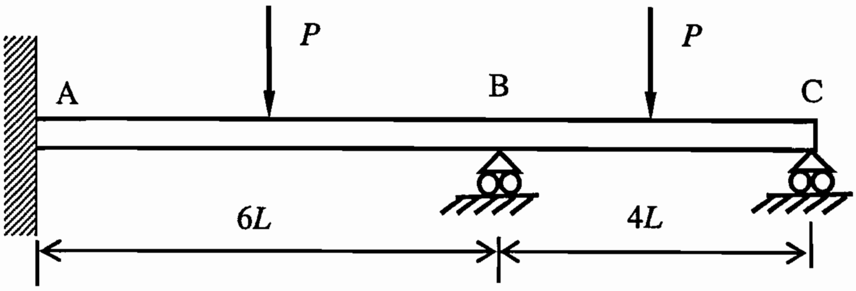
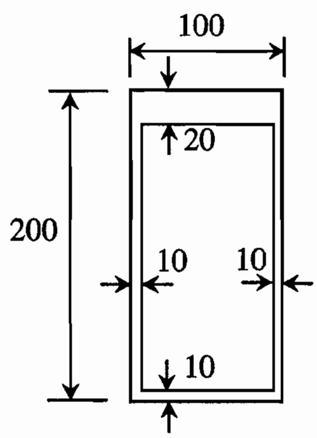


Fig. 6(a)



All dimensions in mm

(Not to scale)

Fig. 6(b)

END OF PAPER