ENGINEERING TRIPOS   PART IB

Thursday 8 June 2006   2 to 4

Paper 6

INFORMATION ENGINEERING

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Additional copy of Fig. 2
Additional copy of Fig. 4

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

Answer two questions from this section

1 (a) Consider the feedback system shown in Fig. 1. Explain how you would determine its gain and phase margins from a Bode diagram. [4]

(b) The Bode diagrams of two systems with transfer functions:

$$G_1(s) = \frac{100}{s(s+1)(s^2+3s+100)}$$

and

$$G_2(s) = \frac{10}{s(s+10)(s^2+3s+1)}$$

are given in Fig. 2. Label the graphs on the additional copy of Fig. 2 with the name of the corresponding transfer function, explaining your choice. If a controller $K(s) = 1$ is used, estimate the gain margin and phase margin of each feedback system. [8]

(c) A controller is added to the second system. The transfer function of the controller is:

$$K(s) = \frac{5s+1}{s+2}$$

Sketch the Bode diagram of $K(j\omega)G_2(j\omega)$ on the additional copy of Fig. 2, estimate the new gain and phase margins and comment on your result. [8]

The additional copy of Fig. 2 should be handed in.

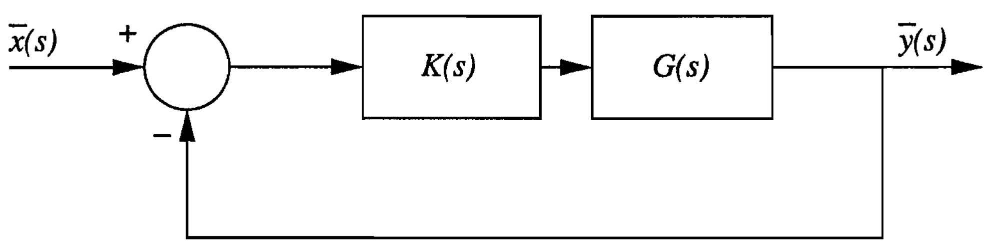


Fig. 1

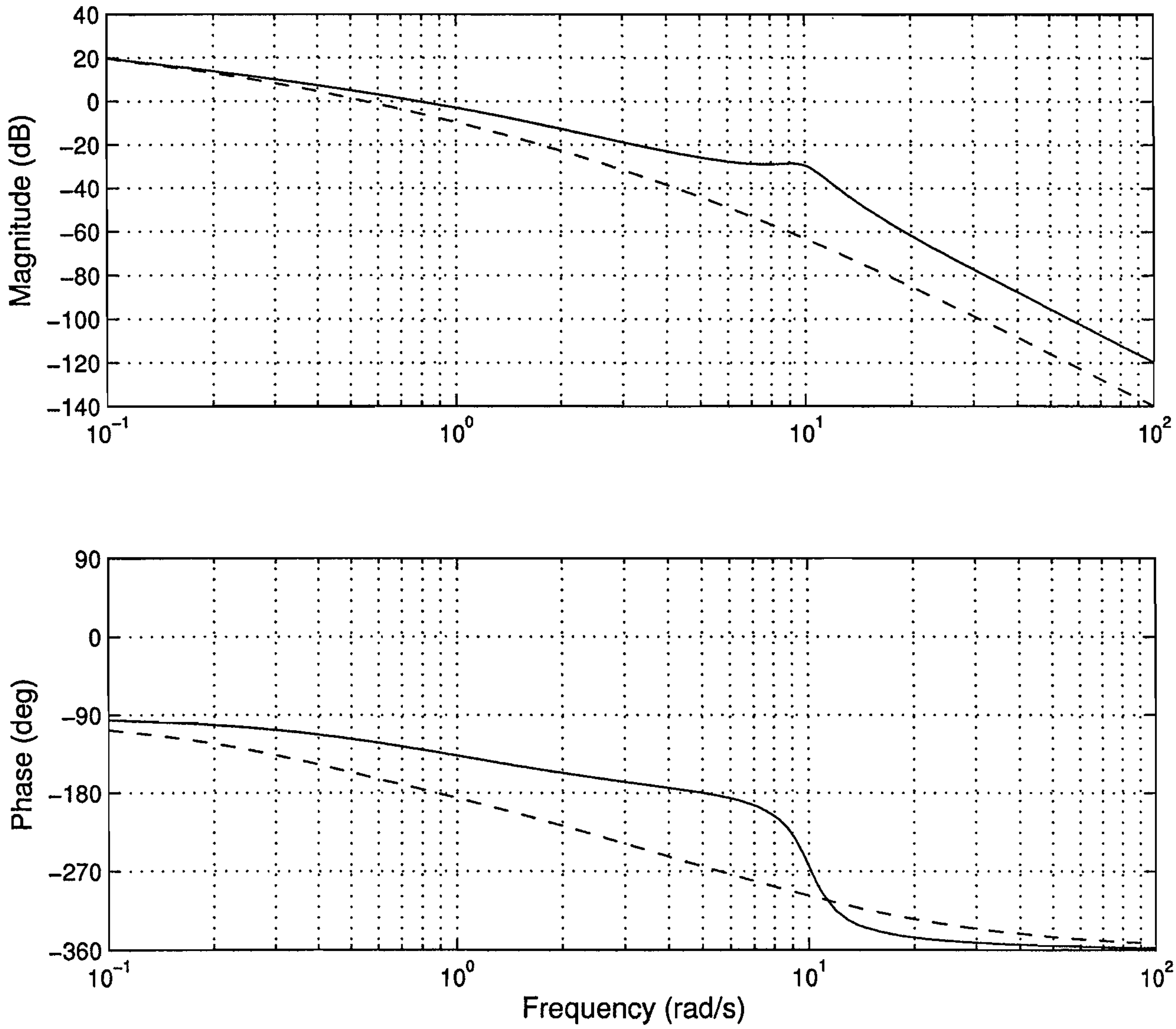


Fig. 2

(TURN OVER

2 (a) Consider the feedback system shown in Fig. 3. How would you determine experimentally the data necessary to plot its Nyquist diagram? [4]

(b) Given a system with transfer function

$$G(s) = \frac{1}{s(s+1)^2}$$

determine the behaviour of its Nyquist diagram as $\omega \to 0$ and $\omega \to \infty$. Find the frequency where the imaginary part of $G(j\omega)$ becomes zero and hence complete the Nyquist diagram of $KG(s)$ on the additional copy of Fig. 4, for $K = 1$. Calculate the gain margin and estimate the phase margin of the feedback system. For what range of K is the feedback system stable? [8]

(c) For $K = 1$ and for a frequency of 0.4 rad/s, estimate from the graph the magnitude of $K(j\omega)G(j\omega)/[1 + K(j\omega)G(j\omega)]$ and comment on your answer. [4]

(d) For what range of frequencies (if any) will the feedback attenuate the effect of any disturbances $\bar{w}(s)$, if $K = 1$? [4]

The additional copy of Fig. 4 should be handed in.

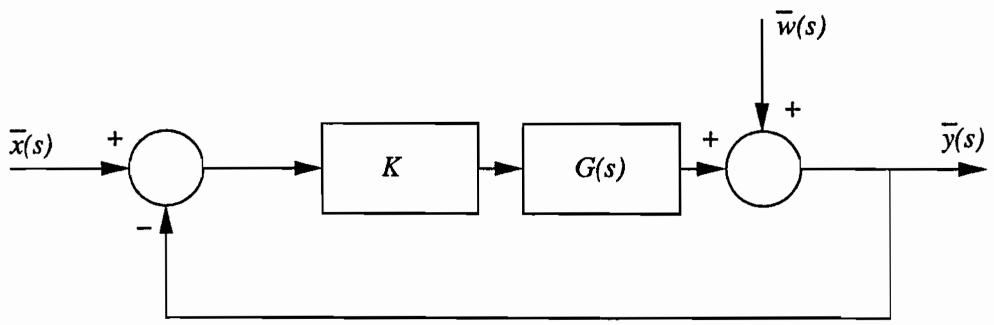


Fig. 3

(cont.

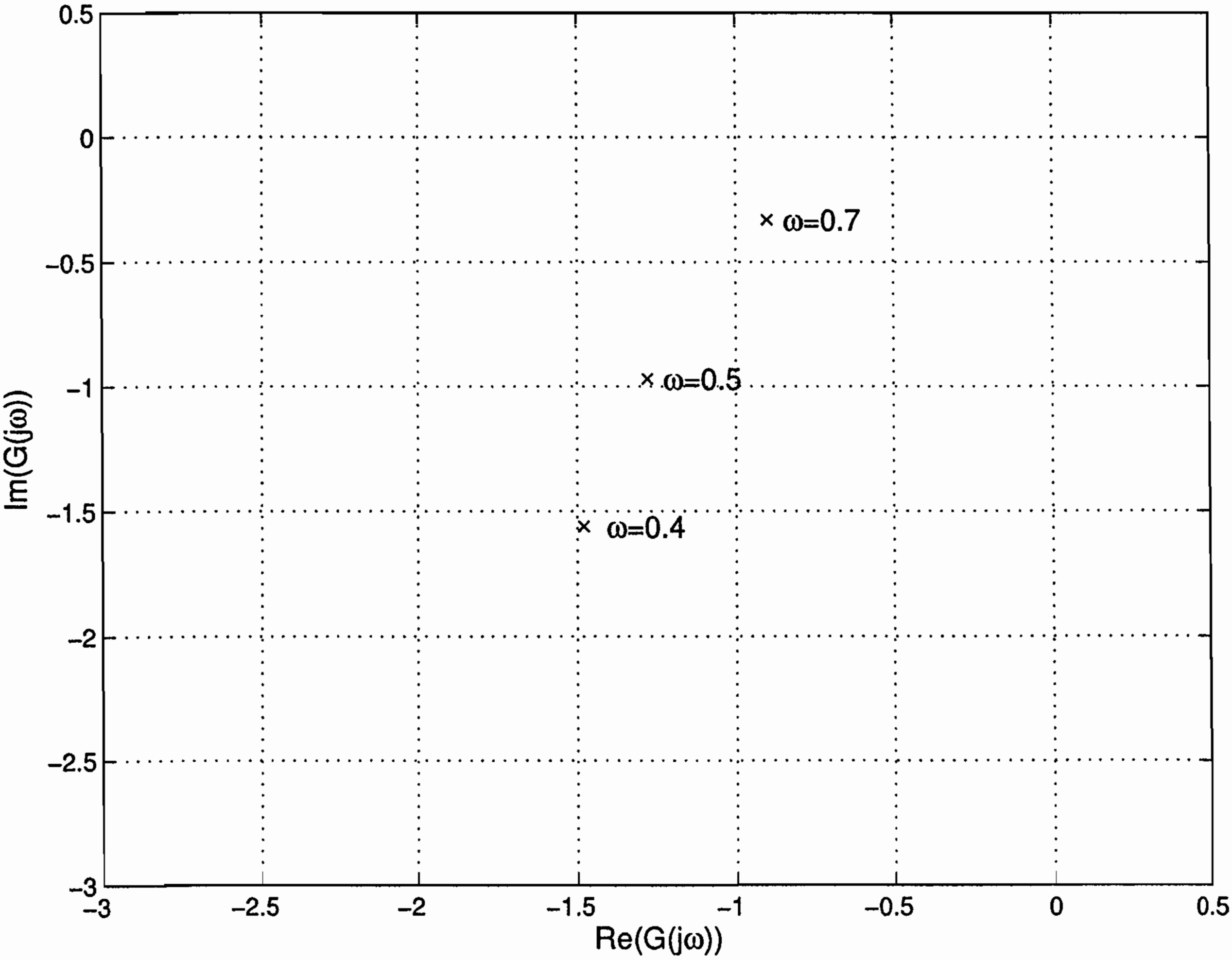


Fig. 4

(TURN OVER

3 (a) Describe what is meant by Proportional plus Derivative (PD) control. What are the advantages of this type of feedback? [2]

(b) The dynamics of a particular boat can be described by the following ordinary differential equation:

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + \alpha y = x$$

where α is a constant, x is the rudder angle and y the heading of the boat.

Find the transfer function from $\bar{x}(s)$ to $\bar{y}(s)$. Hence calculate and sketch the impulse response of this system for:

(i) α =3

(ii) α =4

(iii) α =8 [8]

(c) A controller is added to control the motion of the boat, so that that following equation holds:

$$\bar{x}(s) = K(s)(\bar{r}(s) - \bar{y}(s))$$

where $\bar{r}(s)$ is the reference signal and $K(s) = k_1 + sk_2$.

(i) Draw a block diagram for the closed-loop system and calculate the closed-loop transfer function from $\bar{r}(s)$ to $\bar{y}(s)$ in terms of α , k_1 and k_2 . [5]

(ii) Explain how the values of k_1 and k_2 influence the performance of the closed-loop system. [5]

SECTION B

Answer two questions from this section

4 (a) Explain what is meant by the frequency spectrum of a signal and explain how it can be computed for both periodic and non-periodic signals. [5]

(b) Calculate the Fourier transform of the function:

$$f_1(t) = \begin{cases} -1 & \text{when } -T < t < 0 \\ 1 & \text{when } 0 \leq t < T \\ 0 & \text{otherwise} \end{cases}$$

[5]

(c) The duality theorem states that if $F(\omega)$ is the Fourier transform of $f(t)$, then $2\pi f(-\omega)$ is the Fourier transform of $F(t)$. Use this result to find the Fourier transform of the function:

$$f_2(t) = \frac{\sin^2 at}{\pi t}$$

where a is a real constant. [5]

(d) Hence use Parseval's theorem to calculate:

$$\int_{-\infty}^{\infty} \frac{\sin^4 t}{t^2} dt$$

[5]

(TURN OVER

5 (a) Explain the reasons for modulating an analogue signal prior to transmission. Illustrate your answer with specific reference to the transmission of radio waves. [5]

(b) Suppose that a form of amplitude modulation for two signals, $g_1(t)$ and $g_2(t)$, is given by:

$$u(t) = g_1(t)\cos\omega_0 t + g_2(t)\sin\omega_0 t$$

where ω_0 is the carrier signal frequency and $u(t)$ is the modulated signal.

If

$$g_1(t) = \cos\frac{\omega_0}{4}t$$

and

$$g_2(t) = \cos\frac{\omega_0}{8}t$$

find the spectrum and the bandwidth of $u(t)$. [8]

(c) Consider now $g_1(t)$ and $g_2(t)$ as unknown functions. Show that they can be demodulated separately by multiplying $u(t)$ by $\cos(\omega_0 t + \phi)$ and passing the resulting signal through a low-pass filter. Give the values of ϕ needed to extract $g_1(t)$ and $g_2(t)$. [7]

6 (a) Define the discrete Fourier transform and state the assumptions on which it is based. [4]

(b) A signal

$$x(t) = a\cos\left(\frac{2\pi q}{NT}t\right)$$

(where q is an integer) is sampled with period T over a window of N samples. Show that the coefficients of the discrete Fourier transform X_k, for $k = 0 \,..\, N-1$, are given by

$$X_k = \begin{cases} aN/2, & k = q \text{ and } k = N - q \bmod N \\ 0, & \text{otherwise} \end{cases}$$

[5]

(c) Show that the discrete Fourier transform of the product of two sampled signals $x_n = x_{1n}x_{2n}$ has coefficients:

$$X_k = \frac{1}{N}\sum_{m=0}^{N-1} X_{1m}X_{2|k-m|}$$

where X_{1k} and X_{2k} are the coefficients of the discrete Fourier transforms of x_{1n} and x_{2n} respectively and $|k-m|$ is evaluated modulo N. [5]

(d) Hence, for $N = 8$, find the coefficients of the discrete Fourier transform of the sampled signal

$$y(t) = \frac{1}{8}\left(1 + 3\cos\frac{2\pi}{NT}t\right)\cos\frac{4\pi}{NT}t$$

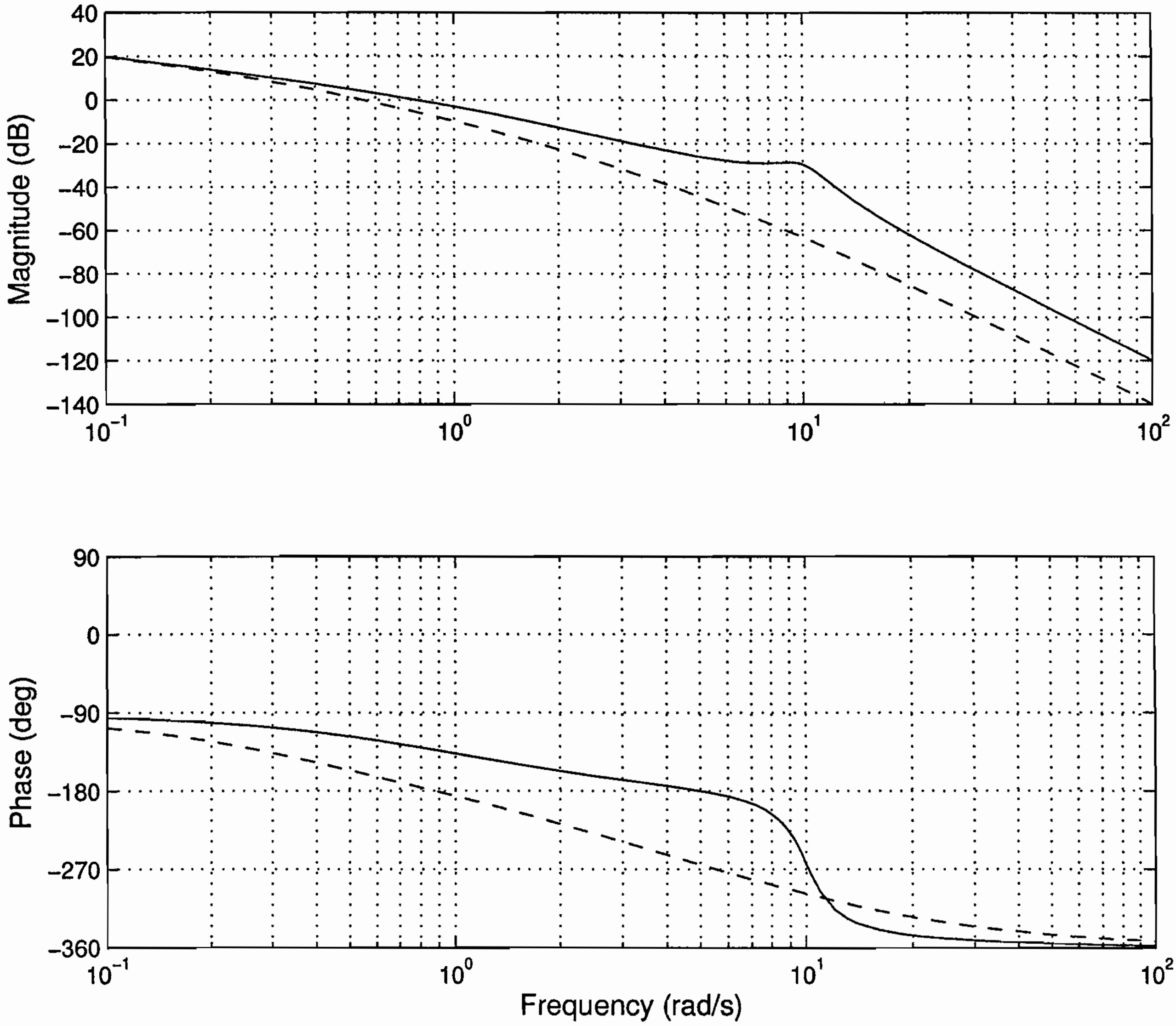
[6]

END OF PAPER

ENGINEERING TRIPOS PART IB, 9 June 2005

Candidate number:

Extra copy of Fig. 2 which should be annotated and handed in with your answer to Question 1.



ENGINEERING TRIPOS PART IB, 9 June 2005

Candidate number:

Extra copy of Fig. 4 which should be annotated and handed in with your answer to Question 2.