ENGINEERING TRIPOS PART IB

Friday 9 June 2006 2 to 4.30
2 to 3.30 Foreign Language Option

Paper 8

SELECTED TOPICS

*Answer **one** question from Section A. In addition:*

> *If you are not taking the Foreign Language option, answer **four** questions, taken from only **two** of Sections B – H. Not more than **two** questions from each section may be answered.*
> *If you are taking the Foreign Language option, answer **two** questions from **one** of Sections B – H.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each Part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Data Sheet for Section B (6 pages)
Data Sheet for Section E (2 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A *Business Economics*

Answer not more than ***one*** *question from this section*

1 (a) Explain the concept of monopoly. [6]

(b) What are the economic arguments against monopolies? [4]

(c) Outline the Life Cycle hypothesis of consumption. [6]

(d) Assuming that the Life Cycle hypothesis is correct, how would an individual's consumption be influenced if he or she won £1million on the lottery? [4]

2 (a) Explain the concept of profit maximisation. [4]

(b) Under what circumstances would firms *not* seek to maximise profits? [6]

(c) Explain the Accelerator Theory of investment. [6]

(d) What impact would a reduction in the rate of interest have on the volume of investment? [4]

SECTION B *Civil and Structural Engineering*

Answer not more than ***two*** *questions from this section*
You may refer to the data sheet as necessary at the end of the examination paper

3 (a) Explain the mechanism of each of the following soil fluidisation phenomena:

(i) soil piping when a cut-off wall is constructed;

(ii) soil liquefaction during an earthquake. [5]

(b) Outline two different tunnelling methods for London clay and discuss the advantages and the disadvantages of each. [5]

(c) A new building with a pile foundation is to be constructed. Three years after the building construction, a tunnel will be excavated under the building and there is concern over the effects of tunnelling on the piles. Discuss potential problems when:

(i) the tunnel is constructed under the piles;

(ii) the tunnel is constructed at the side of the piles. [5]

(d) Identify the instrumentation that should be specified to monitor the effects of tunnelling on the building discussed in Part (c) above. [5]

4 An 8 m deep excavation is to be made to accommodate a new building. The excavation is supported by a smooth diaphragm wall, as shown in Fig. 1. A building exists on one side of the wall and a surcharge of 30 kN m^{-2} is applied at the ground surface to represent the load applied by the building. The site investigation revealed a dense sand deposit of 8 m underlain by stiff clay. The wall penetrates a distance of 4 m into the stiff clay and is propped at a depth of 2 m below the ground surface. The water table is at 2 m below the ground surface. During construction the water table on the excavation side is to be maintained at the bottom of the excavation as shown in Fig. 1.

The unit weight of the saturated sand is $\gamma = 18$ kN m^{-3}, whereas the unit weight of the sand above the water table is $\gamma = 16$ kN m^{-3}. The critical state angle of friction of the sand is $\phi' = 35$ degrees. The unit weight of the clay is $\gamma = 17$ kN m^{-3} and the undrained shear strength is $s_u = 75$ kN m^{-2}. The coefficient of earth pressure at rest K_0 equals 0.6 for sand and 1.0 for clay.

(a) Describe why a diaphragm wall is suitable for this case and how it is constructed. [5]

(b) Estimate the initial total and effective *vertical* stress distribution of the ground under the existing building prior to excavation. Sketch the stress distribution with depth. [5]

(c) Estimate the initial total and effective *horizontal* stress distribution of the ground under the existing building prior to excavation. Sketch the stress distribution with depth. [5]

(d) Assuming that the wall moves sufficiently for active pressures to be mobilised behind it and passive pressures in front of it, calculate the pressures acting on the wall, and sketch the pressure distribution on both sides of the wall immediately after the excavation. [5]

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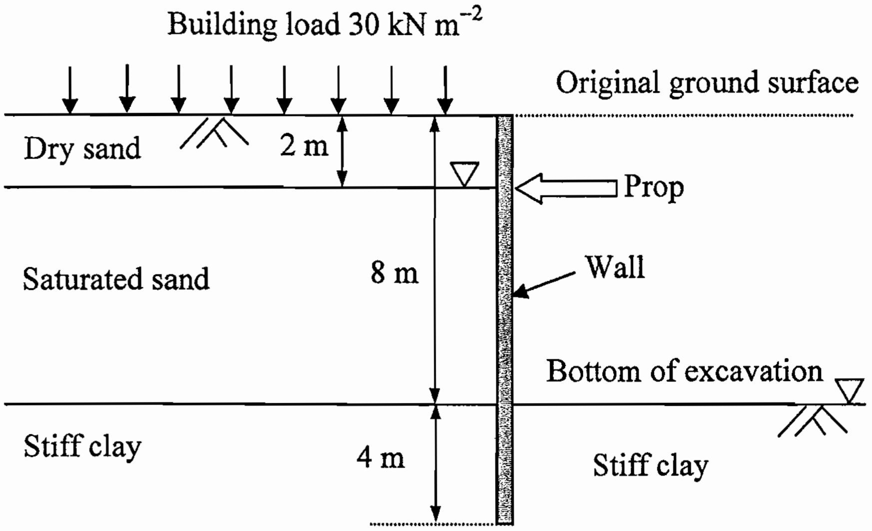

Building load 30 kN m^{-2}
Original ground surface
Dry sand
2 m
Prop
Saturated sand
8 m
Wall
Bottom of excavation
Stiff clay
4 m
Stiff clay

Fig. 1

5 The roof of an underground storage bunker in sloping ground consists of a wide slab 0.3 m deep that rests on (but does not act compositely with) reinforced concrete beams ABC at 2 m centres. These beams are 0.5 m wide by 1.3 m deep and 15 m long. They are simply-supported on the back of the wall of the bunker C and on columns B 4 m in from the entrance A, as shown in Fig. 2. The overburden consists of soil that is 2 m deep at the front, rising to 7 m deep at the back. The section through YY is shown in Fig. 3. The concrete has a unit weight of 24 kN m^{-3}, while the soil has a unit weight of 20 kN m^{-3}. The concrete has a cube strength of 50 N mm^{-2}, and the steel reinforcement has a characteristic yield strength of 460 N mm^{-2}. The beams are to be designed for a safety factor of 1.5 on the applied loads.

(a) Construct the bending moment and shear force diagrams for beam ABC, showing clearly the maximum and minimum values and their locations. [10]

(b) Check whether the beam can be designed as singly reinforced. Design the flexural reinforcement at the locations of maximum hogging and sagging bending. [5]

(c) Design the shear reinforcement required. [5]

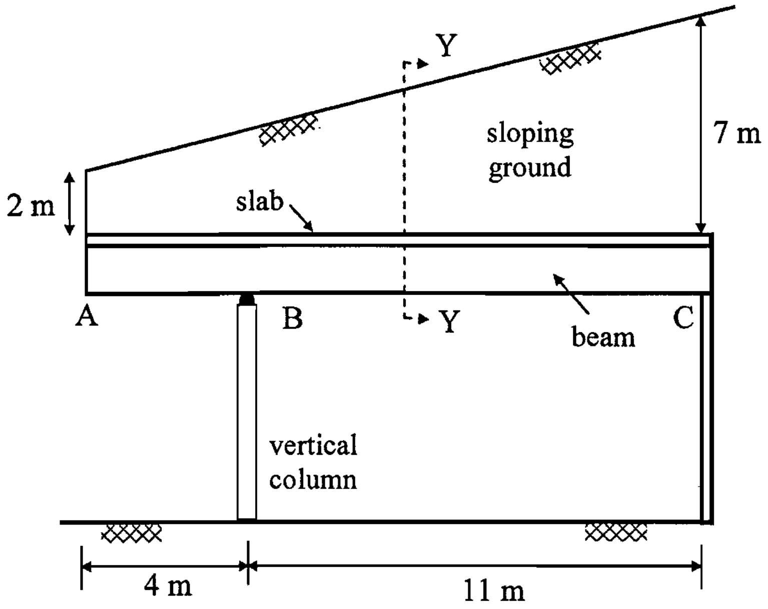
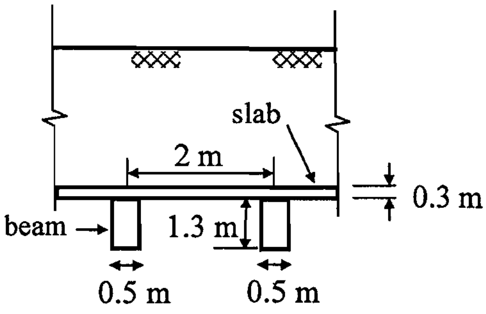


Fig. 2



section through YY

Fig. 3

SECTION C *Mechanics, Materials and Design*

Answer not more than ***two*** *questions from this section*

6 Consider a sandwich beam with rigid face-sheets and a corrugated core of height c as sketched in Fig. 4. The core comprises struts of thickness t inclined at an angle ω with respect to the face-sheets. The struts are made from a rigid ideally plastic solid with a tensile yield strength σ_Y.

(a) Define the relative density $\bar{\rho}$ of a cellular material and obtain an expression for $\bar{\rho}$ of the corrugated core sketched in Fig. 4 in terms of c and t. [4]

(b) Assuming that the struts are pin-jointed to the face-sheets calculate:

(i) The net compressive yield strength σ_c of the core in the x_2 direction. [6]

(ii) The shear strength τ_c of the core for loading in the $x_1 - x_2$ plane. [6]

(c) Qualitatively discuss how the results in Part (b) above would change if rigid joints are assumed rather than pin-joints. [4]

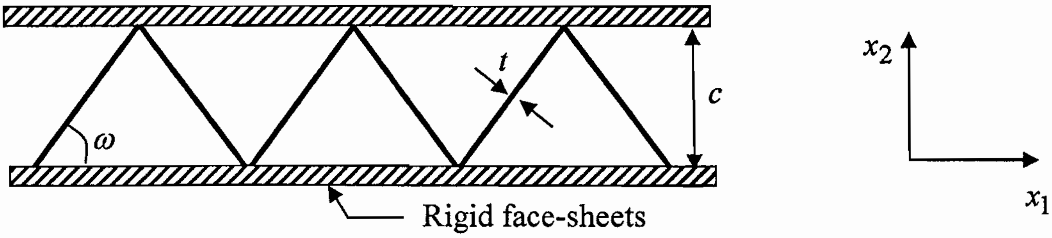


Fig. 4

7 (a) Briefly explain the advantages of sandwich construction in aerospace applications. [4]

(b) The simply-supported sandwich beam of span $2L$ sketched in Fig. 5 comprises face-sheets of thickness h, made from a rigid ideally plastic material with a tensile yield strength σ_Y, and a rigid ideally plastic foam core of height c, with a shear strength τ_c and a compressive strength σ_c. The beam is uniformly loaded with a pressure p.

(i) Obtain an expression for the collapse strength of the beam, assuming a face yield collapse mode. [6]

(ii) Propose a collapse mode assuming core shear deformation and hence calculate an upper bound to the collapse strength of the beam. [6]

(iii) Assuming that $h \ll c$ and $\sigma_c \ll \sigma_Y$, obtain a criterion for the transition from the face yield to the core shear collapse modes. [4]

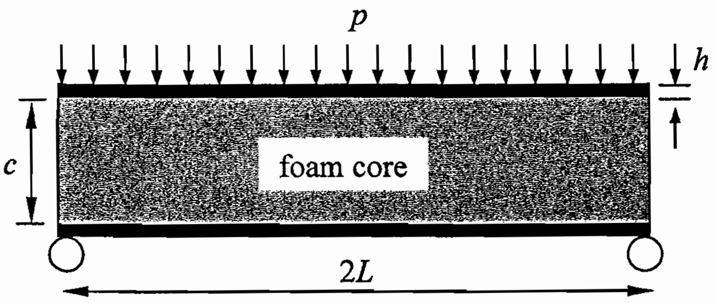


Fig. 5

8 As a chief design engineer, you are required to choose between competing materials and manufacturing methods for your products. With this context in mind, answer the following questions.

(a) What is meant by a trade-off surface? Sketch such a surface, using mass and cost as competing objectives. [6]

(b) Explain the meaning of a value function. Illustrate such a function on the diagram sketched in Part (a) above. [4]

(c) A value function requires that performance metrics such as mass be combined with cost to give a single function to be minimised. Explain using an example how mass is introduced into a cost function and how the necessary constants are determined. [6]

(d) What are the main factors driving the manufacturing cost of a component? Under what circumstances will it be more economical to use electro-discharge machining (EDM) to manufacture a component rather than die casting? [4]

SECTION D *Aerothermal Engineering*

*Answer not more than **two** questions from this section*

9 (a) Figure 6 shows a schematic diagram of a twin-spool bypass engine. Sketch $T-s$ diagrams for the core and bypass flows, labelling the locations in accordance with the figure. [4]

(b) Explain why engines of this design are used to power modern civil aircraft and describe the factors that influence the choice of bypass ratio for a given application. [4]

(c) For the aero-engine designer, which flight conditions represent:

(i) the greatest *mechanical* challenge;
(ii) the greatest *aerodynamic* challenge;
(iii) the greatest *thermodynamic* challenge ?

Describe the particular aspects of these conditions which make them most challenging. [8]

(d) Which of the turbines will have the most stages ? Explain why. [4]

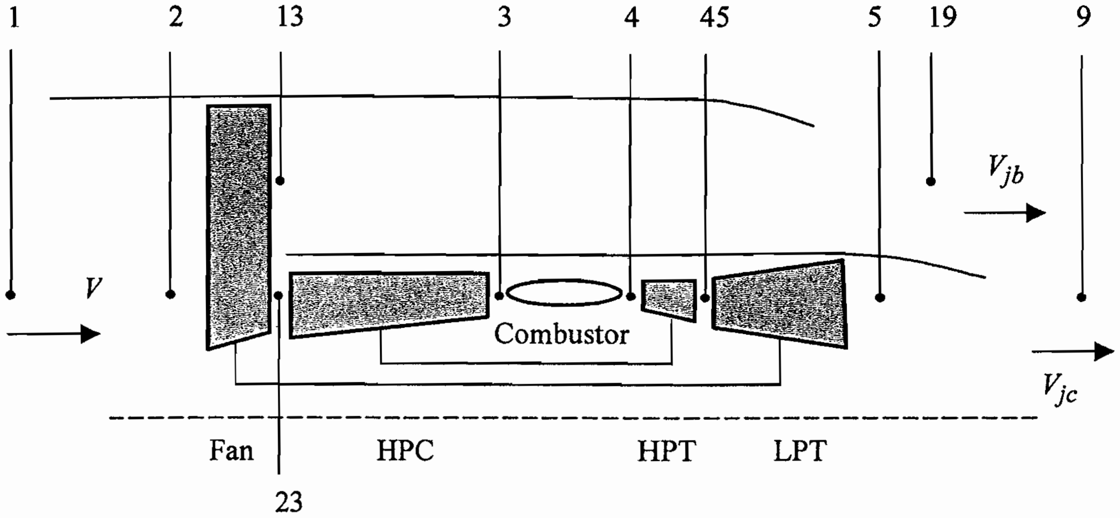


Fig. 6

10 (a) Show that the distance, s, between the start and end of an aircraft flight is given by Breguet's range formula

$$s = -\frac{V\,L/D}{g\,sfc}\ln\left(\frac{W_{end}}{W_{start}}\right)$$

where V is the aircraft speed, sfc is the thrust specific fuel consumption, L/D is the lift-drag ratio, g is the gravitational acceleration and W is the weight of the aircraft. State any assumptions made in the derivation of this formula. [4]

(b) A 250 passenger aircraft has a design range of 8000 km. The aircraft design characteristics are

Empty Weight	85 tonnes
Payload	40 tonnes
sfc	1.95×10^{-5} kg $s^{-1}N^{-1}$
L/D	18
Cruise Mach No.	0.85
Cruise Altitude	11,000 m

The payload includes the weight of the passengers, crew, cargo and the weight of emergency fuel reserves. Find the minimum weight of fuel consumed during an 8000 km flight. Hence find the maximum aircraft take-off weight. [4]

(c) Find the minimum weight of fuel consumed when flying the same distance in this aircraft in two legs, each of 4000 km, when the aircraft is re-fuelled after the first leg. [2]

(d) Estimate the minimum fuel consumed when flying two 4000 km legs for a 250 seat aircraft which has a design range of 4000 km. You may assume that this aircraft has the same *aerodynamic* characteristics as those of the 8000 km range aircraft, which are listed in the table, and that its empty weight is half of its maximum take-off weight. [6]

(e) Discuss the factors other than fuel consumption that would influence the decision of an airline to schedule 8000 km as a single leg or as two. [4]

11 (a) Explain what is meant by the term *ram drag* and how it enters into the definitions of gross thrust and net thrust. [4]

(b) For a turbojet engine operating at a fixed non-dimensional operating point, standard non-dimensional expressions for the air mass flow rate and the gross thrust produced are respectively:

$$\frac{\dot{m}_{air}\sqrt{c_p T_{02}}}{p_{02} A_n} \quad \text{and} \quad \frac{F_G + p_a A_n}{p_{02} A_n} \; .$$

Explain the meaning of the various terms in these expressions and indicate what assumptions are necessary for the engine performance to scale in this way. [4]

(c) When an aircraft powered by two turbojet engines flies at a Mach number of 0.8 at an altitude of 11,000 m, the drag on the airframe is 80 kN. The mass flow of air through each engine is 100 kg s^{-1} and they each have a nozzle area of 0.7 m^2. A half-size model of an engine is to be tested on a static sea-level test bed at the same non-dimensional engine operating condition. Estimate the mass flow through the model engine and the thrust that it is expected to produce. [8]

(d) If the turbojet engines installed in the aircraft rotate at 10,000 rpm for the flight conditions of Part (c), what rpm should be used for the ground test of the half-size model ? [4]

At 11,000 m the properties of the standard atmosphere are:

Pressure	Temperature	Density
22.7 kPa	217 K	0.365 kg m^{-3}

SECTION E *Electrical Engineering*

Answer not more than ***two*** *questions from this section*
You may refer to the data sheet as necessary at the end of the examination paper

12 (a) Explain with the aid of diagrams how the gate voltage controls the source-drain current in a FET. How does this differ for MOSFETs and MESFETs ? [3]

(b) Contrast the gate control characteristics of FETs in depletion, accumulation and inversion modes. [4]

(c) A silicon MOSFET has a channel length L in the direction of current flow and width W. The channel below the gate insulator has been uniformly doped n-type at dopant density N to a depth D so that the device is normally on for a gate-source voltage $V_{GS} = 0$ V. Silicon has a relative permittivity of 12.

Starting from Gauss' Law of electrostatics, show that the gate threshold voltage V_T required to turn the device OFF is given by

$$V_T = \frac{eND^2}{2\varepsilon_0\varepsilon_r}$$

where e is the electronic charge and ε_r is the relative permittivity of silicon.

What is V_T if $N = 1.5 \times 10^{22}$ m^{-3} and $D = 4 \times 10^{-8}$ m ? [7]

(d) What is the maximum electric field in the channel at this threshold voltage? If the MOSFET uses silicon dioxide with relative permittivity of 3.9, what is the maximum field in the oxide ? Which material is likely to breakdown first ? [6]

13 (a) How is electrical conductivity in a semiconductor different to that in a metal ? [2]

(b) Compare the advantages and disadvantages of Si and InAs for use as channel materials for metal - insulator field effect transistors. [4]

(c) Sketch the velocity versus electric field characteristic for an electron in silicon, and explain the various features. What causes the limiting velocity ? [3]

(d) Derive the expression relating the carrier transit time t to the carrier mobility μ, channel length L, and source-drain field E_d. Under what conditions does it hold ? [3]

(e) Using Newton's laws or otherwise, derive an expression relating mobility to the electron mass and the mean free time between collisions. Hence, from the limiting velocity, calculate the energy lost per collision in eV. Assume $m^* = 1$. Take the electron mobility as $0.12\ \mathrm{m^2V^{-1}s^{-1}}$, and the limiting velocity as $1.2 \times 10^5\ \mathrm{m\ s^{-1}}$.

What is the mean free path between collisions ? [8]

14 Diffusion is used to dope semiconductors during device manufacture.

(a) Write down four factors that affect the dopant concentration at a given distance below the semiconductor surface. [2]

(b) Two types of dopant profile can be achieved depending upon the initial conditions. Sketch profiles to show how dopant concentrations vary for each type, giving names for the profiles and stating the initial conditions that lead to each profile. [6]

(c) Describe, with the aid of sketches, the process of ion implantation. [4]

(cont.

(d) Following ion implantation of silicon by phosphorus leading to a surface concentration of n-type dopant of 4×10^{15} cm^{-2}, a silicon wafer is annealed in an inert atmosphere for 60 minutes at 1050° C. Assuming that the Si substrate was p-type with a dopant concentration of 10^{17} cm^{-3} :

(i) state which of the two profiles described in Part (b) approximates most closely the final profile, giving the reasons for your choice; [2]

(ii) using Fig. 7, estimate the depth below the silicon surface at which the p-n junction is formed following the anneal. State any assumptions made. [6]

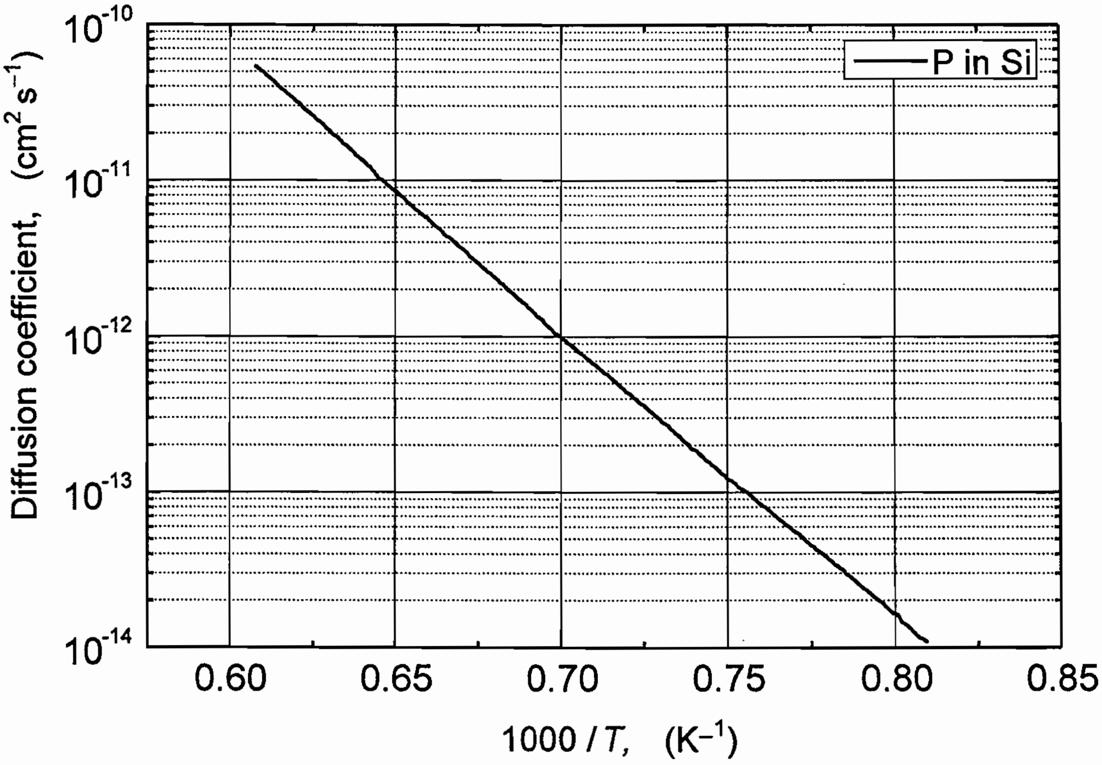


Fig. 7

SECTION F *Information Engineering*

*Answer not more than **two** questions from this section*

15 (a) Low pass and high pass filters are used for processing images. Explain the purpose of each type of filter within the context of photo-editing. [4]

(b) The impulse response of a one-dimensional sampled filter is approximated by the continuous function

$$h(x) = \frac{1}{\sqrt{3\pi}} \exp\left(\frac{-x^2}{3}\right)$$

Obtain an expression for the frequency response of this filter, sketch it, and identify what type of filter it is. [6]

Hint:

The Fourier transform of $\frac{1}{\sqrt{2\pi}} \exp\left(\frac{-t^2}{2}\right)$ is $\exp\left(\frac{-\omega^2}{2}\right)$

(c) If this filter is applied first to the rows and then to the columns of an image, $I(x,y)$, show that the result is equivalent to convolving $I(x,y)$ with a two-dimensional point-spread function $g(x,y)$ and obtain an expression for $g(x,y)$. Hence show that $g(x,y)$ is isotropic and comment on the computational advantages of separable 2-D filtering. [6]

(d) Briefly explain how *edge-adaptive* filters may be constructed and describe their key advantages over non-adaptive filters, when used for image enhancement. [4]

16 (a) (i) Three commonly used colour representations for digital images are RGB, YUV and HSV. Explain each representation and describe how the YUV and HSV representations can be derived from an image in RGB form. [4]

(ii) Two common processes for digital photograph enhancement are *lighting intensity correction* and *colour cast correction.* Discuss how each of these processes may be achieved for images both in RGB form and in HSV form. [4]

(iii) Explain how histograms may be used to automate the above two correction processes. [4]

(b) (i) What is meant by *image texture*? [2]

(ii) Describe how texture can be characterized by convolving the image with a family of filters. Give full details of a typical set of filters that are used and the corresponding descriptor. [6]

(TURN OVER

17 Local interest points and their descriptors are often used in image retrieval and matching.

(a) What is meant by a local interest point in an image? [2]

(b) Show how one type of local interest point can be detected and localised by examining the eigenvalues of the 2 × 2 matrix

$$\begin{bmatrix} \langle I_x^2 \rangle & \langle I_x I_y \rangle \\ \langle I_x I_y \rangle & \langle I_y^2 \rangle \end{bmatrix}$$

evaluated at each pixel of an image $I(x, y)$, where $\langle \ \rangle$ denotes a 2D smoothing operation and where $I_x = \frac{\partial I}{\partial x}$ and $I_y = \frac{\partial I}{\partial y}$. You should also state the algorithm for labelling a pixel as a local interest point. [8]

(c) Give details of a suitable descriptor which is approximately invariant to lighting and small *affine* transformations, and how it is computed from the neighbourhood of the interest point. [4]

(d) Describe the *nearest neighbour search* algorithm which uses these descriptors to match local interest points to interest points stored in a large database. Give details of how to do the *nearest neighbour search* efficiently. [6]

SECTION G *Biological and Medical Engineering*

Answer not more than ***two*** *questions from this section*

18 (a) Explain how the base-sequence in the DNA of a living organism prescribes the amino-acid sequence of poly-peptide chains and how this sequence determines the conformation of protein molecules. [4]

(b) Describe the wide range of materials and devices which constitute the protein family. [4]

(c) How does a bi-stable feature within the flagellin protein molecule permit identical building-blocks to construct the *curved* tubular structure of bacterial flagellar filaments? [4]

(d) Discuss the consequences of the reversal of the direction of rotation of the flagellar motors in the swimming and navigation of bacteria such as *E. Coli.* [4]

(e) Describe the role of DNA mutations in the evolution of different bacteria, insects, plants and animals. Explain how genomic sequences can be used to map the phylogenetic 'tree of life'. [4]

19 (a) In the context of neural network learning:

(i) describe the main features of supervised and reinforcement learning; [3]

(ii) describe how supervised and reinforcement learning could be used to train a machine to play chess. [2]

(b) Optical illusions are characterized as visual percepts that are deceptive or misleading. Describe how visual illusions can arise from a rational (Bayesian) system which is designed to make judgements in the face of uncertainty. Thereby explain why drivers tend to underestimate their speed when driving in foggy conditions. [5]

(c) There are an infinite number of movements which could move the hand from one point in space to another, yet such movements in humans are stereotypical. What are the stereotypical features of these arm movements? Describe the computational principles by which one movement could be selected over another. [5]

(d) Humans can adapt to a wide variety of externally imposed dynamic (force) perturbations.

(i) Describe the strategy people employ when adapting to predictable dynamic perturbations (such as reaching while holding a mass).

(ii) Describe the strategy people employ when adapting to unpredictable dynamic perturbations (such as trying to hold still the hand of an unco-operative child).

(iii) A robot is used to perturb a subject's arm and after adaptation the movements become similar to unperturbed movements. How could you experimentally assess which of the two strategies in Part (d)(i) or Part (d)(ii) the subject has used? [5]

20 (a) The weight mg of a hovering animal is supported by a pair of approximately rectangular wings, each of length R with constant chord c equal to $0.3R$. The wings flap through an angle A in the horizontal plane, and the flapping angle varies with time t as:

$$\phi = \overline{\phi} + \frac{1}{2} A \cos\left(2\pi f t\right)$$

where $\overline{\phi}$ is the mean flapping angle and f is the wingbeat frequency. The downwash, or induced velocity, is negligible compared with the flapping velocity.

(i) Assume that the lift per unit span L' is given by the usual blade element relation $L' = \frac{1}{2}\rho c U_r^2 C_L$, where ρ is the density of air, U_r is the relative velocity and C_L is the lift coefficient. Taking C_L to be constant and the mean of $\sin^2(2\pi f t)$ to be 0.5, show that a vertical force balance requires $mg \approx 0.05 \rho \pi^2 A^2 f^2 C_L R^4$. [6]

(ii) A 10 g bird has wings with length $R = 15$ cm that are flapped through an angle of 2 radians in air of density $\rho = 1.2$ kg m^{-3}. What frequency is needed for weight support, assuming a maximum lift coefficient of 1.8 ? [2]

(iii) The wings of flying animals tend to scale isometrically, such that $R \propto \mathrm{m}^{1/3}$. Given that the masses of hovering animals range over six orders of magnitude, how would you expect frequency to vary with body mass? [2]

(b) (i) Explain what is meant by a *kinematic analysis* of human motion and describe three commonly used methods of collecting kinematic data, discussing the relative merits of each method. [5]

(ii) Describe the basic ideas behind an '*inverted pendulum*' model of human walking, noting carefully the assumptions made. Explain how this model for walking can be adapted for running and can predict that the transition from walking to running occurs at a speed of approximately $(gL)^{0.5}$, where L is the leg length and g is the acceleration due to gravity. [5]

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SECTION H *Manufacturing, Management and Design*

Answer not more than two questions from this section

21 You are a member of a team investigating the design of a new supermarket shopping trolley.

(a) With examples, describe how you would use the Kano model to help in the design of this product. Comment on the strengths and weaknesses of this design tool. [8]

(b) Sketch an outline of a design process that you might follow to design this product, explaining the reasoning behind your choice. [5]

(c) During the design process, you plan to make a range of models and prototypes. Describe the types of prototype that might be appropriate at different stages of the process. For each type of prototype, comment on how it would be used to advance the design. [7]

22 Cambridge Display Technology (CDT) was established in 1989 to commercialise intellectual property relating to polymer light-emitting diodes.

(a) CDT has changed its business model several times since 1989. List three of these models, and for each outline the resources that would be needed to make the business model work. [6]

(b) CDT has raised over \$150m of funding from different sources since 1989. Key amongst these funding sources was equity investment from business angels and venture capitalists. Describe what is meant by the terms:

(i) equity investment;

(ii) business angel;

(iii) venture capitalist.

What particular features would venture capitalists look for in a business plan from a company seeking funding? [8]

(c) Equity investment was not the only type of funding available to support the start-up and growth of CDT. List and describe three non-equity types of funding that would also be available to a company such as CDT. [6]

23 (a) Describe four strategies, giving one practical example of each, that would allow fixed costs to be converted into variable costs in manufacturing a new product. [6]

(b) A new product is to be made in a dedicated factory with a maximum production capacity of 700 units per month. Fixed costs are £10,000 per month. The variable costs of production are £50 per unit made. All units made are sold.

(i) If each unit can be sold for £75, determine the monthly break-even production volume. [3]

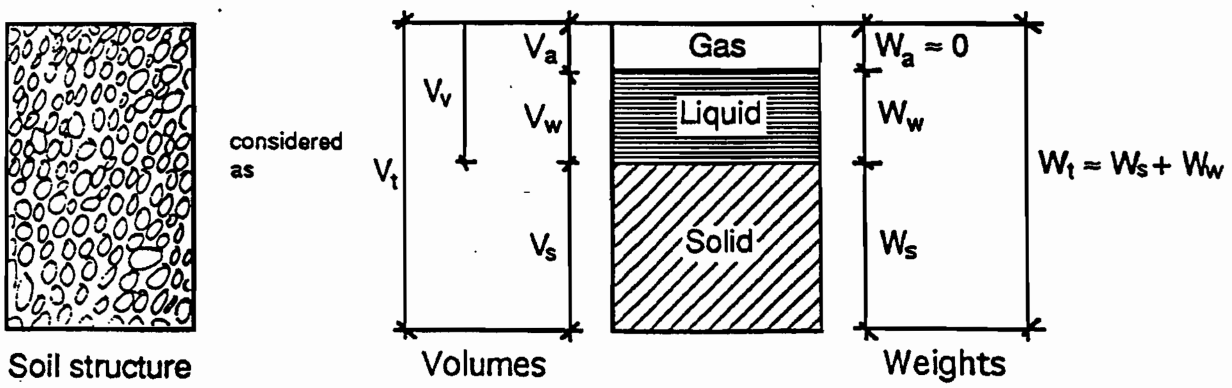
(ii) If only 300 units can be sold at the full price of £75 and thereafter the unit price is dropped by £0.10 for each extra unit sold, determine the most profitable monthly level of production Q_{max} and the corresponding profit. [7]

(iii) If all the costs of production could be converted to variable costs, what variable cost per unit would give the same profit, for the same level of production Q_{max} as that calculated in part (ii) above? Comment on your answer. [4]

END OF PAPER

Data sheet – Soil Mechanics

General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w G_s / e)$
Unit weight of water	$\gamma_w = 9.81\ \text{kN/m}^3$ (although we assume $10\ \text{kN/m}^3$)
Unit weight of soil	$\gamma = W_t / V_t = \left(\frac{G_s + S_r e}{1 + e}\right)\gamma_w$
Buoyant (effective or submerged) unit weight	$\gamma' = \gamma - \gamma_w = \left(\frac{G_s - 1}{1 + e}\right)\gamma_w$ (soil saturated)
Unit weight of dry soil	$\gamma_d = W_s / V_t = \left(\frac{G_s}{1 + e}\right)\gamma_w$
Relative density	$I_d = \frac{(e_{max} - e)}{(e_{max} - e_{min})}$

where e_{max} is the maximum voids ratio achievable in the quick tilt test (for sands), and
e_{min} is the minimum voids ratio achievable by vibratory compaction (for sands).

Classification of particle sizes

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D	equivalent diameter of soil particle
D_{10}, D_{60} etc	particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

Stress components

Principle of effective stress (saturated soil):

$$\begin{array}{lccll} \text{total stress} & \sigma & = & \text{effective stress } \sigma' + & \text{pore water pressure } u \\ & \tau & = & \tau' + & 0 \end{array}$$

and

σ_v = vertical stress

σ_h = horizontal stresss

τ = shear stress

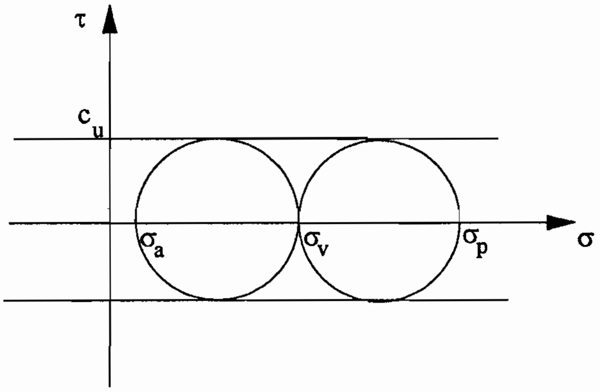
Strength of clays (undrained behaviour only)

Under constant volume (undrained) conditions only, the strength of clays can be characterised by the *undrained shear strength* c_u which is mobilized when the shear stress $\tau = c_u$. This conforms to Tresca's criterion, and the active and passive total horizontal stresses, σ_a and σ_p respectively, are given by

$$\sigma_a = \sigma_v - 2\,c_u$$

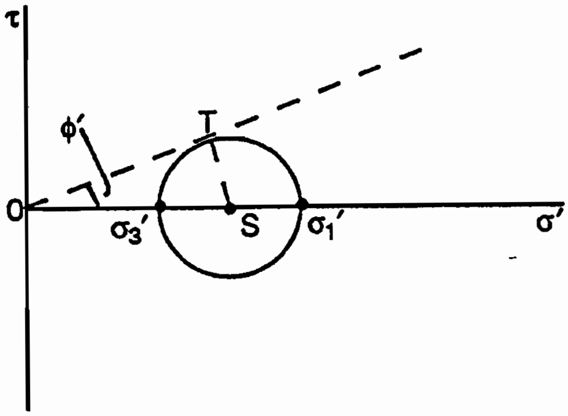
$$\sigma_p = \sigma_v + 2\,c_u$$

where σ_v is the total vertical stress.



Strength of sands

Mobilised angle of shearing ϕ'
where $\tau = \sigma' \tan \phi'$



$$\sin\phi' = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

$$\therefore \phi' = \sin^{-1}\left[\frac{\left(\sigma_1'/\sigma_3'\right) - 1}{\left(\sigma_1'/\sigma_3'\right) + 1}\right]$$

Earth pressure coefficient K: $\sigma_h' = K\sigma_v'$

Active pressure: $\sigma_v' > \sigma_h'$ $\therefore$ $\sigma_1' = \sigma_v'$

$\sigma_3' = \sigma_h'$

$K_a = (1 - \sin\phi')/(1 + \sin\phi')$

Passive pressure: $\sigma_h' > \sigma_v'$ $\therefore$ $\sigma_1' = \sigma_h'$

[We assume principal stresses $\sigma_3' = \sigma_v'$

are horizontal and vertical] $K_p = (1 + \sin\phi')/(1 - \sin\phi') = \dfrac{1}{K_a}$

Angle of shearing resistance:

at peak strength ϕ'_{max} at $\left(\sigma_1'/\sigma_3'\right)_{max}$

at critical state ϕ'_{crit} after large strains.

Sand strength data: friction hypothesis

In any shear test on soil, failure occurs when $\phi' = \phi'_{max}$ and

$$\phi'_{max} = \phi'_{crit} + \phi'_{dilatancy}$$

where ϕ'_{crit} is the ultimate angle of shearing resistance of a random aggregate which deforms at constant volume, so the dilatancy, which indicates an increase in volume during shearing, approaches zero $(\phi'_{dilatancy} -> 0)$ as $\phi'_{max} -> \phi'_{crit}$.

ϕ'_{crit} is a function of the mineralogy, size, shape and distribution of particles. For a particular soil it is almost independent of initial conditions. Typical values (±2°):

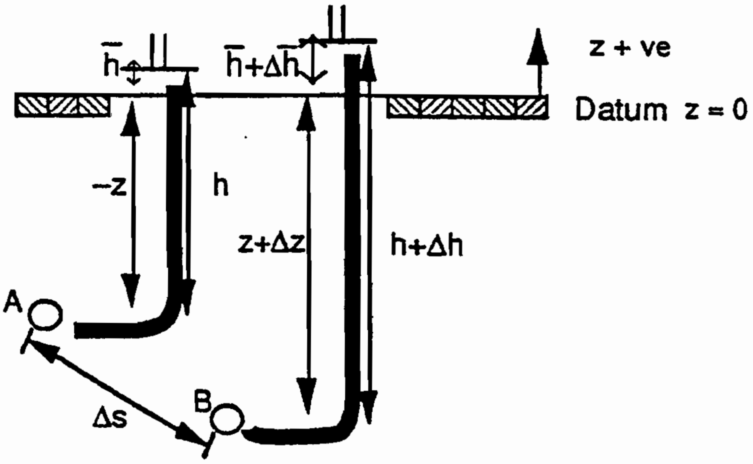
	ϕ'_{crit}	ϕ'_{max}	
feldspar	40°		
quartz	33°	53°	($I_d = 1$, and mean effective stress OS < 150 kPa)
mica	25°		

Seepage

Excess pore water pressure

Head $h = u/\gamma_w$

Potential $\bar{h} = h + z$



Total pore water pressure head at A: $u = \gamma_w h = \gamma_w \left(\bar{h} - (-z)\right)$

B: $u + \Delta u = \gamma_w (h + \Delta h) = \gamma_w \left(\bar{h} + z + \Delta\bar{h} + \Delta z\right)$

[Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w \left(\bar{h} + \Delta\bar{h}\right)$]

Hydraulic gradient A–B $i = -\dfrac{\Delta\bar{h}}{\Delta s} = -\dfrac{\Delta\bar{u}}{\gamma_w \Delta s}$

Darcy's law $v = ki$

v = average or superficial seepage velocity

k = coefficient of permeability

Typical permeabilities

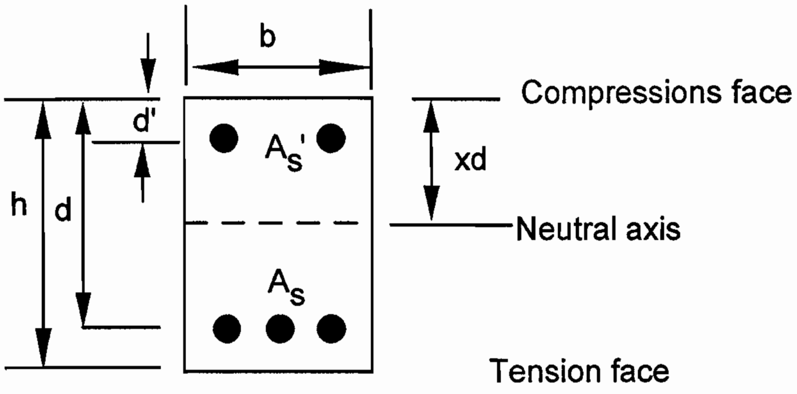
$D_{10} > 10\text{mm}$: non-laminar flow

$10\text{ mm} > D_{10} > 1\mu\text{m}$: $k \cong 0.01(D_{10}\text{in mm})^2$ m/s

clays : $k \cong 10^{-9}$ to 10^{-11} m/s

Design of reinforced concrete

Data sheet for use in Part IB Civil Engineering Elective Course.



Design Stresses

Cube strength for concrete f_{cu}. At failure in bending, stress in concrete $= 0.4f_{cu}$ over whole area of concrete in compression.

Tensile yield stress of steel f_y. At failure in bending, stress in bars in tension $= 0.87f_y$, stress in bars in compression $= 0.75f_y$.

Design Equations

Moment capacity of singly reinforced beam

$$\begin{aligned} M &\le 0.15\, f_{cu} b d^2 \\ M &= 0.87 f_y A_s d(1 - x/2) \\ x &= 2.175(f_y/f_{cu}).(A_s/bd) \quad (\le 0.5 \text{ to avoid over reinforcement}) \end{aligned}$$

Moment capacity of doubly reinforced beam

$$\begin{aligned} M &= 0.15 f_{cu} b d^2 + 0.75 f_y A'_s (d - d') \\ 0.87 f_y A_s &= 0.75 f_y A'_s + 0.2 f_{cu} b d \end{aligned}$$

Shear capacity of all beams

Total shear capacity $V = (v_c + v_s)bd$

Where, $v_c = 0.68(100\,A_s/bd)^{0.33}.(400/d)^{0.25}$ $\left(N/mm^2\right)$

and $v_s = 0.87 f_y\, A_{sq}/(bs)$

in which s = shear link spacing, A_{sq} is total area of all shear bars in a link and A_s is the total area of effective longitudinal *tension* steel at the section.

Standard bar sizes

Diameter (mm)	6	8	10	12	16	20	25	32	40	50
Area (mm^2)	28	50	78	113	201	314	491	804	1256	1963

Available steel types

Deformed high yield steel	$f_y = 460\ N/mm^2$
Plain mild steel	$f_y = 250\ N/mm^2$

Lap and anchorage lengths 40 bar diameters

Density of reinforced concrete: $24\ kN/m^3$

Reinforcement areas per metre width

	Spacing of bars (mm)									
	75	100	125	150	175	200	225	250	275	300
Bar Dia. (mm)										
6	377	283	226	189	162	142	126	113	103	94.3
8	671	503	402	335	287	252	224	201	183	168
10	1050	785	628	523	449	393	349	314	285	262
12	1510	1130	905	754	646	566	503	452	411	377
16	2680	2010	1610	1340	1150	1010	894	804	731	670
20	4190	3140	2510	2090	1800	1570	1400	1260	1140	1050
25	6550	4910	3930	3270	2810	2450	2180	1960	1790	1640
32	10700	8040	6430	5360	4600	4020	3570	3220	2920	2680
40	16800	12600	10100	8380	7180	6280	5580	5030	4570	4190
50	26200	19600	15700	13100	11200	9820	8730	7850	7140	6540
Areas calculated to 3 significant figures according to B.S.I recommendations										

2 May 2006

Part IB Data Sheet: Electrical Engineering Elective Transistor Design Summary Sheet

Gauss's Theorem

$\varepsilon_0\varepsilon_r E_1 - \varepsilon_0\varepsilon_r E_2 =$ charge per unit area enclosed between upper surface 1 and lower surface 2.

FET Design Summary

- $\tau_t \rightarrow$ switching time as 1st approx. (scattering limited transit time).
- $\tau_{eff} = \tau_t + R_{load}C_{eff(output)} \rightarrow$ switching time as 2nd approx.
- $L = v_s\tau_t$ (source-drain spacing).
- $I_{sat} = eNv_s\,Wd_s = eNWLd_s/\tau_t$
- Aspect ratio W/L (technology?).
- $(1/2)eN(d_s)^2/\varepsilon_0\varepsilon_r =$ (Max Gate Voltage)
- $E'_{peak} = eNd_s/\varepsilon_0\varepsilon_r < E_{breakdown}$
- Minimum Drain Source Voltage ~ E_sL (E_s is the field required to reach limiting velocities).

Mutual Conductance

$$g_{mo} \sim I_{sat}/V_{gate(max)}$$

Mutual conductance reduces with frequency as $g_m(\omega) \approx g_{mo}/(1 + j\omega\tau_t)$;

$$v_{out} = g_m(\omega)\,R\left(1 + j\omega RC_{eff(out)}\right) \approx g_{mo}R/[1 + j\omega(\tau_t + RC_{eff(out)})]$$
$$= g_{mo}R/[1 + j\omega\tau_{eff}]$$

Capacitances for FET

Parallel Plate Capacitance: $\varepsilon_0\varepsilon_r$ Area/spacing

Used for rough estimates of parasitic capacitance.

Effective Capacitances for FET

$C_{eff(out)} \rightarrow C_{gate/drain} + C_{drain/source} + C_{load}$;

$C_{eff(in)} \rightarrow MC_{gate/drain} + C_{gate/source\,(proximity)} + C_{gate/source\,(electronic)}$;

$C_{electronic} = g_{mo}\tau_t$; $M = (1 + g_{mo}R_{load})$.

Time Constants for FET

$\mu = e\tau/m^*$ relates mean free time τ and mobility.

Transit time τ_t over distance L and scattering limited velocity v_s are related by $\tau_t = L/v_s$.

$v_{out} \approx g_{mo}R/[1 + j\omega\tau_{eff}] = g_{mo}R/[1 + j\omega/(2\pi f_t)]$

$1/(2\pi f_\tau) = \tau_t + RC_{eff(out)} = \tau_{eff}$ The transition frequency is f_t.

10% to 90% rise time is $T = 2.2\,\tau_{eff} = (2.2/2\pi)(1/f_t) = 0.35/f_t$.

Electrical Engineering Elective:
Tunnel Barrier Design Summary Sheet

Schrodinger's Equation

Complex Wave $\Psi = A\exp(-j2\pi f\,t + j2\pi x/\lambda) = A\exp(-j\omega t)\exp(jkx);$

<momentum> $\Psi = p\Psi = (h/\lambda)\Psi = -j(h/2\pi)\partial\Psi/\partial x;$

<Total energy> $\Psi = E\Psi = hf\Psi = j(h/2\pi)\partial\Psi/\partial t$

$(h/2\pi) \rightarrow \hbar;\ \ h = 6.625\times 10^{-34}\,\text{J/s}.$

Schrodinger's equation:-

$$E\Psi = (1/2m)[-j\hbar\partial/\partial x]^2\Psi + e\phi\Psi$$

Tunneling (Rectangular barrier $\varepsilon\phi$)

Propogating waves outside barrier with incident kinetic energy $U_{\text{incident}} = (\hbar k_i)^2/2m^*$

Evanescent waves inside barrier: $-(\hbar k)^2/2m^* = (\hbar k_i)^2/2m^* - e\phi$

Technology Design Summary

Diffusion

Constant Surface Concentration:

$$C(x,t) = C_s\ \operatorname{erfc}\left[\frac{x}{2(Dt)^{1/2}}\right]$$

Constant Total Dopant:

$$C(x,t) = \frac{S}{(\pi Dt)^{1/2}}\exp\left[\frac{-x^2}{4Dt}\right]$$

W I Milne
13.3.06