

ENGINEERING TRIPOS PART IB

Paper 1 MECHANICS

Numerical Answers

Section A

- 1) (b) $\dot{\theta} = \sqrt{\frac{2aL}{k^2 + L^2}}$
- (c) $\frac{ma(k^2 + 3L^2)}{k^2 + L^2}$, greater than
- 2) (a) $\ddot{\theta} = \frac{4g}{5L}$
- (b) $\frac{\sqrt{2}}{5}mg$
- (c) $\frac{8mgL}{135}$, $\frac{2}{3}L$ from top of slab
- 3) (a) $9mL^2$
- (b) $\omega_{BC} = \frac{\omega}{2}$ clockwise, $\omega_{CD} = \omega$ anti-clockwise
- (c) $T = 22mL^2\omega^2$

Section B

4) (a) (i) $a \cos \theta \mathbf{e}_1 + a \sin \theta \mathbf{k}$

(iii) $-a \left(\ddot{\theta} \sin \theta + (\dot{\theta}^2 + \dot{\phi}^2) \cos \theta \right) \mathbf{e}_1$
 $+ a \left(\ddot{\phi} \cos \theta - 2\dot{\theta} \dot{\phi} \sin \theta \right) \mathbf{e}_2$
 $+ a \left(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta \right) \mathbf{k}$

(b) (ii) $ma^2 \dot{\phi} \cos^2 \theta$

5) (b) (i) $a\theta = b\phi$

(ii) $V = mg \left[(a+b) \cos \theta - \frac{4b}{3\pi} \cos \left(1 + \frac{a}{b} \right) \theta \right]$

(iii) $\frac{a}{b} > \frac{3\pi}{4} - 1$

6) Normal forces on wheels:

	Front left,	Front right,	Rear left,	Rear right
(a)	2250 N,	2250 N,	2750 N,	2750 N
(b)	2062.5 N,	1812.5 N,	3187.5 N,	2937.5 N
(c)	5000 N,	0,	5000 N,	0

ENGINEERING TRIPOS PART IB

Tuesday 5 June 2007 9.00 to 11.00

Paper 3; MATERIALS

Numerical Solutions

- 3 (d) $E \alpha \Delta T = 189 \text{ MPa}$,
New fracture stress = 223 MPa
- 5 (c) Hoop stress = 40 MPa
Longitudinal stress = 20 MPa
Minimum yield stress = 100 MPa in the hoop direction
Minimum yield stress = 50 MPa in the longitudinal direction
- (d) Need to increase the yield stress by at least a factor of 2 in the hoop
direction
- 6 (c) Diffusion distance in 1 hour = 1.53 μm . Therefore, homogenisation is
complete

ANSWERS

- 1 (a) 3.725 J/K
 (b) (ii) $\alpha > 0.105$
- 2 (a) (ii) 0.834, 1161.3 kJ/kg (iii) 15.1 kJ/kg (iv) 0.361
- 3 (b) 330.4 K
 (c) 6.15 bar
 (d) 37.4 kJ per kg air
 (e) 0.0118 kg H₂O per kg air
- 4 (a) Reynolds number and Strouhal number
 (b) 60 m/s
 (c) 2 Hz
 (d) 0.926
 (e) see crib
 (f) see crib
- 5 (a) see crib
 (b) $u = \frac{1}{2\beta} \frac{dp}{dx} \left(r + \frac{R_1 R_2}{r} - (R_1 + R_2) \right)$
 (c) R_2/R_1
- 6 (a) $U_2 = 4U_1$
 (b) $U_3 = 3U_1$
 (c) $P_2 = P_1 + 9\rho U_1^2$
 (d) $F = -4h(P_1 - P_3 + 12\rho U_1^2)$
 (e) see crib

Answers to 1B paper 5 electrical

1 c 600Ω , $18.6 \text{ k}\Omega$

d $V_E = -560 \text{ V}$ $V_{CC} = 15\text{-}20 \text{ V}$

2 b $I_E = 18.7 \text{ mA}$, $V_E = 2.81 \text{ V}$, $V_C = 8.51 \text{ V}$

c $I_E = 13.73 \text{ mA}$, $V_E = 2.059 \text{ V}$

d $V_C = 12.36 \text{ V}$

e gain = -3.65 , $R_i = 6.5 \text{ k}\Omega$

3b $I_L = 13.12 \text{ kA}$, $E = 27.003 \text{ kV}$, $\delta = 20.49^\circ$

c $I = 17.155 \text{ kA}$

d $I = 14.037 \text{ kA}$

4b delta; $P = 3.97 \text{ kW}$, $Q = 22.45 \text{ kVA}$

star $P = 3.44 \text{ kW}$, $Q = 2.58 \text{ kVA}$

$S_{\text{tot}} = 23.64 \text{ kW}$, $\cos \text{ ph } 0.33$

c $V = 559 \text{ V}$

d $V = 426.2 \text{ V}$

5d $\omega_s = 157 \text{ rad/s}$, $s = 0.31$ $\omega = 108 \text{ rad/s}$ $T = 99.8 \text{ Nm}$

6d $T = 0.065$

7 $L = 10^{-7} \text{ H/m}$, $C = 4.10^{-11} \text{ F/m}$, $a/b = 1.648$, reflections = 5

Answers

- 1 (b) GM = 9.54 dB, PM = 32.6 deg
Min at ≈ 0.9 rad/sec, Gain = 1.76
- (d) GM = 12.6 dB, PM = 53.7 deg
Min at ≈ 2.1 rad/sec, Gain = 0.91
- 2 (a) $\bar{y}(s) = \frac{K(s)G(s)}{1+K(s)G(s)}\bar{r}(s) + \frac{G(s)}{1+K(s)G(s)}\bar{d}(s)$
- (b) (i) $0.123 \cos(7t + 2.47)$
(ii) 0.5
(iii) $0.5 + 0.123 \cos(7t + 2.47)$
- 3 (b) $3 \leq k < \infty$
(c) 2.43
(d) $k_d = 2.12$
- 4 (c) SNR = 34.19 dB
(d) 77.8 kHz
- 5 -
- 6 (a) (ii) $\frac{\pi}{j} [\delta(\omega - \omega_o) - \delta(\omega + \omega_o)] e^{j\phi\omega/\omega_o}$
(b) (i) $f_s = 4$ kHz, $f_o = 1, 5, 9, 13, \dots$ kHz

Numerical solutions, IB Paper 7, 2007

1. (a) $\frac{\pi}{2}(e^{-a^2} - e^{-b^2})$
(b) $\frac{\pi}{2}(e^{-a^2} - e^{-b^2})$
(c) $e^{-r^2}\hat{\mathbf{e}}_z$
(d) -
2. (a) -
(b) $64D/3$
(c) $2D(2 - x^2 - y^2)$
(d) $64D/3$
3. (a) -
(b) -
(c) $k = n(n + 1)$
4. (a) $a = 2$
(b) -
(c) $x = 1/9, y = 8/3$
(d) -
5. (a) np
(b) -
(c) $p^3(n^2 - n) + p^2n$
(d) pn
6. (a) $(\lambda/(s + k))^k$
(b) -
(c) $k/\lambda, k/\lambda^2$

Answers

- Q3: b) i) $N=2$ for stiff clay and $N=8.5$ for soft clay
ii) $\sigma_T = 175$ kPa
- Q4: b) FoS against sliding = 1.21
d) FoS against sliding = 0.92
- Q5: a) Sagging Moment = 281 kNm Hogging Moment = 500 kNm
c) Steel area for sagging = 1703 mm^2 Steel area for hogging = 3208 mm^2
d) Spacing of Shear reinforcement = 345 mm

Part IB 2007 Paper 8 Selected Topics
Section C – Mechanics Materials and Design

Answers:

6. (c) $4/9\rho V^2 A$

7 (b)

σ_{\min}	σ_{\max}	$\Delta\sigma$	σ_m
-10	60	70	25
10	20	10	15
6	50	44	28

All stresses in MPa

(c) 0.49 years (clearly too short – need to redesign)

Numerical answers for Part IB 2007
Paper 8 Section D (Aerothermal Engineering)

9. (a) 42100 feet (c) 246, 0.0159 kg/passenger-km
10. (c) 52.0 kN, 0.907
11. (b) 82.0 %, 17.5 kg/s (c) 1.91 m (d) 485 kJ/kg, 6 (e) 14.7, 84.6 %

The above results are presented with an accuracy of 3 significant figures.

Numerical answers for Part IB 2007
Paper 8 Section E (Electrical Engineering)

Q 13

(a) $2 \times 10^{21} \text{ m}^{-3}$

(b) 10^6 V/m
 $2 \times 10^{-6} \text{ m}$

(c) 1.28 mA

(d) 1.51 V

Q14

(a) $2.82 \text{ ohm}^{-1} \text{ m}^{-1}$

(b) $7.1 \times 10^{-3} \text{ eV}$

(c) $1.76 \times 10^{-7} \text{ m}$

(d) $3.5 \times 10^{-12} \text{ s}$

Paper 8 Section G Engineering for the Life Sciences 2006/7 Crib

Question 20

(a) (i) $7/8$

(c)(i) $N_A \propto m^{4/3}$