ENGINEERING TRIPOS   PART IB

PAPER 2: STRUCTURES

SECTION A

1 (a) One redundancy

(b)

$$\mathbf{t} = \frac{1}{\sqrt{2}} \begin{bmatrix} 2\gamma H \\ V + \gamma H \\ V - \gamma H \end{bmatrix} \quad \text{where} \quad \gamma = \frac{\sqrt{2}}{2+\sqrt{2}}$$

(c)

$$\delta_v = \frac{\sqrt{2}VL}{AE}, \quad \delta_h = \frac{\sqrt{2}\gamma HL}{AE}$$

2 (b)

$$\sigma = \frac{Mb}{2I} \quad \text{where} \quad I \approx \frac{2}{3}b^3 t, \quad \tau = \frac{T}{2b^2 t}$$

at first yield

$$\sigma^2 + 4\tau^2 = Y^2 \ \text{(Tresca)}, \quad \sigma^2 + 3\tau^2 = Y^2 \ \text{(von-Mises)}$$

3 (a.ii) Vertical reaction is $4EI\alpha T/\pi R^2$

(b) Maximum bending moment is $EI\alpha T_0/R$

SECTION B

4 (a.i) $F = 6M_p/L$

(a.ii) $F = 4M_p/L$

(b.i) $F = 6M_p/L$

(b.ii) $F = 4M_p/L$

(c) $F = 6M_p/L$ (in other spans, F is larger)

5 (a.i) $H = 8\lambda M_p/L$

(a.ii) $V = (3 + \lambda)M_p/L$

(a.iii) $H = (2 + 4\lambda)M_p/L$ (for hinges at corners and supports)

(a.iv) $H/2 + V = (3 + \lambda)M_p/L$ (for hinges on supports, right-side corner and mid-span beam) OR $H + V = 3(1 + \lambda)M_p/L$ (where right-side corner hinge moves to left-hand corner)

(b) $H/2 + V = (3 + \lambda)M_p/L$ (for hinges on bottom support, mid-span beam and mid-height right-side column) OR $H + V = (4 + 2\lambda)M_p/L$ (for hinges on both supports and 1/3 along span of beam)

6 (a) $p = (2m/L^2)/(1 - \beta)^2$

(b) $p = (2m/L^2)/|2\beta - 1|$

(c) $p = (2m/L^2)/|2/3 - \beta|$

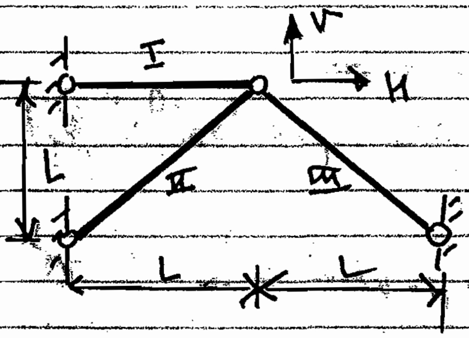
KAS June 2007

SECTION A

1 a) Redundancies: $s-m = b+r-D.j$; $b=3$, $j=4$, $r=2\times 3$, $D=2$

$s-m = 3+6-2\times 4 = 1$ (no mechanisms) $\Rightarrow \underline{s=1}$

b)



bar tensions $\underset{\sim}{t} = [t_I \; t_{II} \; t_{III}]^T$

Set t_I as redundant tension

$\underset{\sim}{t} = \underset{\sim}{t_0} + x\underset{\sim}{s}$

eqlm set, self-stress.

$\underset{\sim}{t_0}$: set $t_I = 0$ $\Rightarrow$ nodal eqlm gives $\underset{\sim}{t_0} = [0 \;\; V+H \;\; V-H]/\sqrt{2}$

$\underset{\sim}{s}$: set $t_I = 1$, $V, H = 0$ $\Rightarrow$ $\underset{\sim}{s} = [1 \;\; -1/\sqrt{2} \;\; 1/\sqrt{2}]^T$

$$\underset{\sim}{t} = \frac{1}{\sqrt{2}}\left[\begin{bmatrix}0\\V+H\\V-H\end{bmatrix} + x\begin{bmatrix}\sqrt{2}\\-1\\1\end{bmatrix}\right]$$

"x" is a proportioning constant consistent with compatibility requirements – solved later

Need flexibility matrix: $$\underset{\sim}{F} = \frac{L}{AE}\begin{bmatrix}1&0&0\\0&\sqrt{2}&0\\0&0&\sqrt{2}\end{bmatrix}$$

Define bar extension vector: $\underset{\sim}{e} = [e_I \; e_{II} \; e_{III}]^T = \underset{\sim}{F}\cdot\underset{\sim}{t}$, by definition. The constant x is furnished by

$$\underset{\sim}{s}^T\underset{\sim}{e} = \underset{\sim}{0}.$$

$$\underset{\sim}{e} = \underset{\sim}{F}\underset{\sim}{t} = \frac{L}{AE}\begin{bmatrix}1&&\\&\sqrt{2}&\\&&\sqrt{2}\end{bmatrix}\cdot\frac{1}{\sqrt{2}}\left[\begin{bmatrix}0\\V+H\\V-H\end{bmatrix} + x\begin{bmatrix}\sqrt{2}\\-1\\1\end{bmatrix}\right] = \frac{L}{\sqrt{2}AE}\begin{bmatrix}\sqrt{2}x\\\sqrt{2}(V+H)-\sqrt{2}x\\\sqrt{2}(V-H)+\sqrt{2}x\end{bmatrix}$$

$$\underset{\sim}{s}^T\underset{\sim}{e} = [1 \;\; -1/\sqrt{2} \;\; 1/\sqrt{2}]\cdot\frac{L}{AE}\begin{bmatrix}x\\V+H-x\\V-H+x\end{bmatrix} = \frac{L}{AE}\left[x - \frac{1}{\sqrt{2}}(V+H-x) + \frac{1}{\sqrt{2}}(V-H+x)\right]$$

$$\underset{\sim}{s}^T\underset{\sim}{e} = \frac{L}{\sqrt{2}AE}\left[\sqrt{2}x - \cancel{V} - H + x + \cancel{V} - H + x\right] = 0$$

$$\Rightarrow x[2+\sqrt{2}] = 2H \Rightarrow \underline{x = \frac{2H}{2+\sqrt{2}}}$$

$$\Rightarrow \underset{\sim}{t} = \frac{1}{\sqrt{2}}\left[\begin{bmatrix}0\\V+H\\V-H\end{bmatrix} + \frac{2H}{2+\sqrt{2}}\begin{bmatrix}\sqrt{2}\\-1\\1\end{bmatrix}\right] = \frac{1}{\sqrt{2}}\begin{bmatrix}\frac{2\sqrt{2}}{2+\sqrt{2}}H\\V+H\left(1-\frac{2}{2+\sqrt{2}}\right)\\V-H\left(1-\frac{2}{2+\sqrt{2}}\right)\end{bmatrix}$$

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$$1-\frac{2}{2+\sqrt{2}} = \frac{2+\sqrt{2}-2}{2+\sqrt{2}} = \frac{\sqrt{2}}{2+\sqrt{2}} \quad : \text{define } \underline{\gamma = \frac{\sqrt{2}}{2+\sqrt{2}}}$$

$$\Rightarrow \underset{\sim}{t} = \frac{1}{\sqrt{2}}\begin{bmatrix} 2\gamma H \\ V+\gamma H \\ V-\gamma H \end{bmatrix} \qquad [u = \sqrt{2}\cdot\gamma\cdot H]$$

c) For deflection, use $\delta_{reqd} = \underset{\sim}{t}^* \cdot \underset{\sim}{e}$
virtual force in required dir

$$\delta_V \Rightarrow \underset{\sim}{t}^* = \underset{\sim}{t}\Big|_{V=1,\,H=0} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\underset{\sim}{e} = \frac{L}{AE}\begin{bmatrix} u \\ V+H-u \\ V-H+u \end{bmatrix} = \frac{L}{AE}\begin{bmatrix} \sqrt{2}\gamma H \\ V+H(1-\sqrt{2}\gamma) \\ V-H(1-\sqrt{2}\gamma) \end{bmatrix}$$

$$\delta_V = \frac{1}{\sqrt{2}}\cdot\frac{L}{AE}\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} \sqrt{2}\gamma H \\ V+H(1-\sqrt{2}\gamma) \\ V-H(1-\sqrt{2}\gamma) \end{bmatrix} = \underline{\frac{\sqrt{2}VL}{AE}}$$

$$\delta_H \Rightarrow \underset{\sim}{t}^* = \underset{\sim}{t}\Big|_{V=0,\,H=1} = \frac{1}{\sqrt{2}}\gamma\cdot\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$\delta_H = \underset{\sim}{t}^* \cdot \underset{\sim}{e} = \frac{\gamma}{\sqrt{2}}\cdot\begin{bmatrix} 2 & 1 & -1 \end{bmatrix}\cdot\frac{L}{AE}\begin{bmatrix} \sqrt{2}\gamma H \\ V+H(1-\sqrt{2}\gamma) \\ V-H(1-\sqrt{2}\gamma) \end{bmatrix}$$

$$= \frac{\gamma L}{\sqrt{2}AE}\left[2\sqrt{2}\gamma H + \cancel{V} + H(1-\sqrt{2}\gamma) - \cancel{V} + H(1-\sqrt{2}\gamma)\right]$$

$$= \frac{\gamma L}{\sqrt{2}AE}\left[\cancel{2\sqrt{2}\gamma H} + H - \cancel{\sqrt{2}\gamma H} + H - \cancel{\sqrt{2}\gamma H}\right] = \underline{\frac{\sqrt{2}\gamma HL}{AE}}$$

On the whole well answered in the early parts, and most candidates correctly used virtual work to determine final displacements. Part (b) threw up most problems in view of calculating the bar forces.

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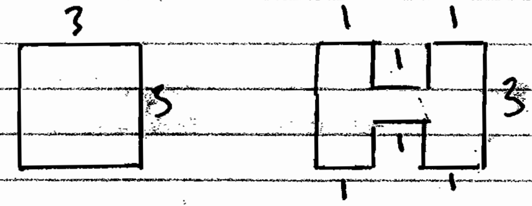
2a)



All plates have same thickness, t, of same material

$A_e \propto$ 9 ; 7

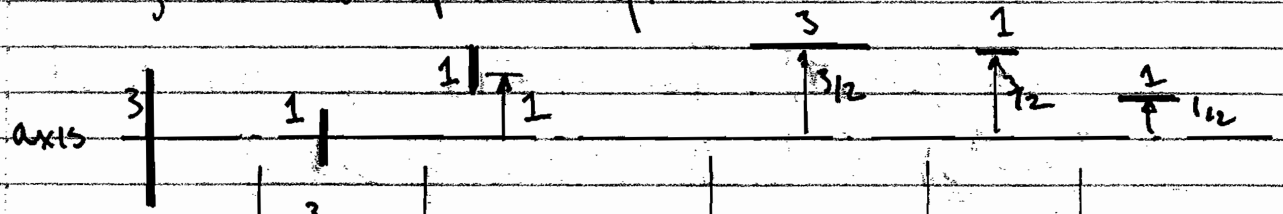
$\oint ds/t \propto$ 12 ; 16

$A_e^2/\oint ds/t \propto \frac{81}{12}$; $\frac{49}{16}$

ratio of torsional stiffness $= \frac{81}{12} : \frac{49}{16}$ OR $1 : \frac{49}{\cancel{16}_4} \cdot \frac{\cancel{12}^3}{\cancel{81}_{27}}$

$= \underline{\underline{1 : \frac{49}{108}}}$

For bending, consider contribution of individual plates w.r.t. horizontal axis of bending.



	(A)	(B)	(C)	(D)	(E)	(F)
$I \propto$	$\frac{3^3}{12} = \frac{27}{12}$	$\frac{1^3}{12} = \frac{1}{12}$	$\frac{1}{12} + \underbrace{1 \cdot 1^2}_{\parallel \text{axis}} = \frac{13}{12}$	$\underbrace{3 \cdot \left(\frac{3}{2}\right)^2}_{\parallel \text{axis}} = \frac{27}{4}$	$\frac{9}{4}$	$\frac{1}{4}$

For square section $I \propto \left[2\times(A) + 2\times(D)\right] = 2\left[\frac{27}{12} + \frac{27}{4}\right] = 54/3$

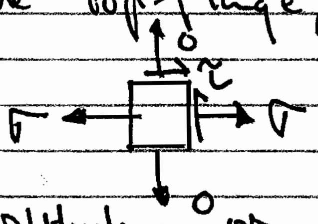
For H-section $I \propto \left[2\times(A) + 4\times(C) + 2\times(F) + 4\times(E)\right] = 2\cdot\frac{27}{12} + 4\cdot\frac{13}{12} + 2\cdot\frac{1}{4} + 4\cdot\frac{9}{4}$

$= \frac{220}{12}$

[N.B. (B) element not needed]

ratio $= \frac{54}{3} : \frac{220}{12} \equiv 1 : \frac{\cancel{220}^{55} \cdot 3}{\cancel{12}_4 \cdot 54} = \underline{\underline{1 : 55/54}}$

b) box-section of side-length b carries b.m. M and torque T: in the top-flange, an element is stressed according to



$\sigma = \frac{M \cdot b/2}{I}$; $\tau = \frac{T}{2 \cdot \underbrace{b^2}_{A_e} t}$ ← wall thickness.

(thro' thickness, or out-of-page stress = 0)

(thin-walled formulae)

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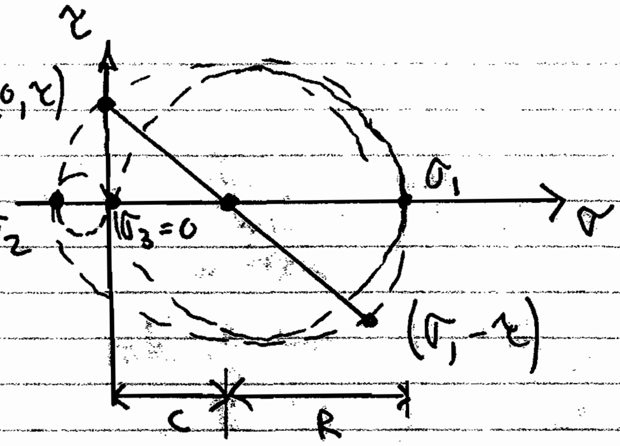
b) contd. Three Mohr's circles;

Plot $(0,\tau)$, $(\sigma,-\tau)$ as in data book convention



$\Rightarrow$ $\sigma_1 = C+R$, $\sigma_2 = C-R$, $\sigma_3 = 0$ } principal stresses

where C is centre and R is radius of largest circle.

Tresca $\Rightarrow$ max. difference in principal stresses $= Y$, uniaxial yield

$$\Rightarrow 2R = Y \quad (\sigma_1 - \sigma_2 = Y) \quad \Rightarrow \quad \underline{4R^2 = Y^2}$$

Von Mises $\Rightarrow$ $(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2 = 2Y^2$

$$\Rightarrow (C+R+C-R)^2 + (C-R)^2 + (C+R)^2 = 2Y^2 \Rightarrow \underline{\underline{2C^2 + 6R^2 = 2Y^2}}$$

However from Mohr's circle $\underline{\underline{C = \frac{\sigma}{2}, \; R^2 = \frac{\sigma^2}{4} + \tau^2}}$

Tresca $\Rightarrow$ $\sigma^2 + 4\tau^2 = Y^2$ (at first yield) — I

Von Mises $\Rightarrow$ $\underline{\sigma^2 + 3\tau^2 = Y^2}$

von Mises always lower irrespective of σ & τ value

Recall: $\sigma = \frac{Mb}{2I}$, $\tau = T/2b^2t$

$$I \sim \frac{b^4}{12} - \frac{(b-2t)^4}{12} = \frac{b^4}{12}\left[1 - \left(1-\frac{2t}{b}\right)^4\right] \simeq \frac{2}{3}b^3t \quad \text{ignoring high-order terms.}$$

$\Rightarrow$ $\sigma \sim \frac{3M}{4b^2t}$, $\tau = \frac{T}{2b^2t}$ .. substitute into I for governing relationships betw M & T.

Many candidates failed to exploit the small thickness assumption, leading to problems throughout. The Mohr's circle approach was done rather well, but final expressions involving moment & torque were largely absent due to the ignoring of small thicknesses once more.

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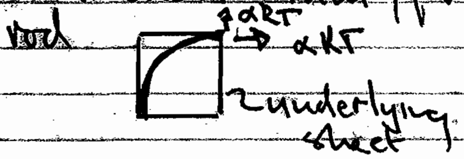
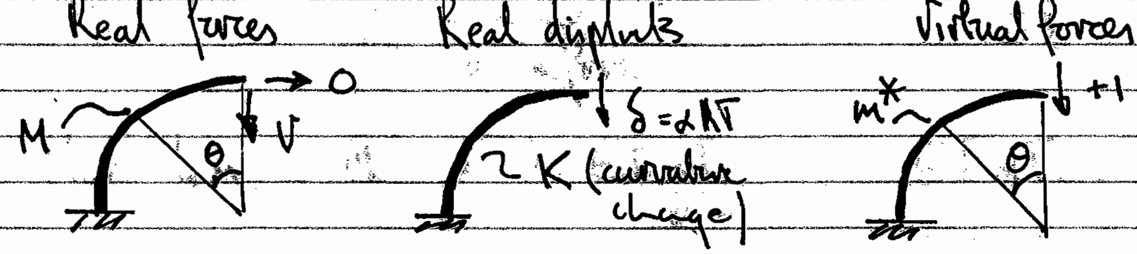
3a) (i) Unconstrained displacement. If the rod were "bonded" to an underlying isotropic sheet of the same material of side-length R, which is also heated, then the sheet would expand $\underbrace{\alpha R T}_{\alpha T \times R}$ in orthogonal direction. Thus, for the curved rod

αT thermal strain, R length



(ii) Constraint (vertical) becomes active ~~too~~ after heating.



Real $K = \frac{V \cdot R\sin\theta}{EI}$, where $VR\sin\theta = M$ (bending moment)

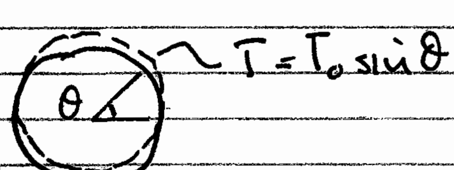
Virtual b.m, m^*, $= R\sin\theta \cdot 1$ ← virtual force ↓

V.W. eqn $\Rightarrow$ $1 \cdot \delta = \int_0^{\pi/2} m^* \cdot K \underbrace{ds}_{Rd\theta}$

$$\Rightarrow \quad 1 \cdot \alpha R T = \int_0^{\pi/2} R\sin\theta \cdot \frac{VR\sin\theta}{EI} \cdot R d\theta$$

$$\Rightarrow V = \frac{4EI \cdot \alpha T}{\pi R^2}$$

b) Draw/sketch variation in temp rise $\Rightarrow$



Symmetry $\Rightarrow$ no axial force/b.m. at $\theta = 0$, or π, but there can be S.F.

$\Rightarrow$ $T = T_0 \sin\theta$ can be represented by case in part (a) but where temperature rise varies around quadrant

P.T.O.

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In this case, the $\overbrace{\text{displacement equivalent}}^{=\Delta,\ \text{say}}$ to δ in (a) is calculated from the compatibility condition.

$$\int_0^{\pi/2} \underbrace{\alpha T}_{\text{absolute}} \cdot \underbrace{\sin\theta}_{\text{vert. compnt}} \underbrace{ds}_{\text{arc-length}}$$

$$\Rightarrow \Delta = \int_0^{\pi/2} \alpha T_0 \cdot \sin\theta \cdot \sin\theta \, R\,d\theta = \frac{\pi}{4} \cdot \alpha T_0 \cdot R.$$

$$\Rightarrow 1 \cdot \Delta = \int_0^{\pi/2} \underbrace{m^* K}_{\text{as before}} \, ds \Rightarrow V = \frac{EI\alpha T_0}{R^2}$$

$$\text{Max b.m.} = RV = R \cdot \frac{EI\alpha T_0}{R^2} = \underline{\underline{EI\alpha T_0 / R}}$$

Unpopular and poorly solved: only one correct solution. Candidates were diverted from correctness by referring to (straight beam) databook cases, which do not help in any way, or by not exploiting virtual work. Part (a.i) was the only completely solved part of this question.

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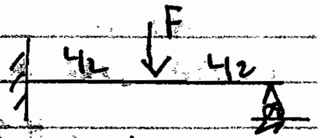
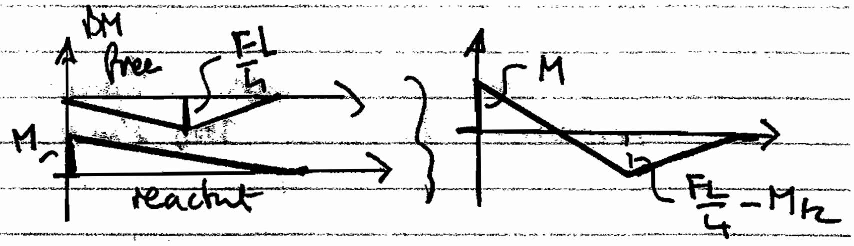
SECTION B

4 a) (i)



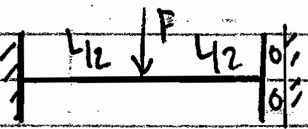
choose root moment as reactant



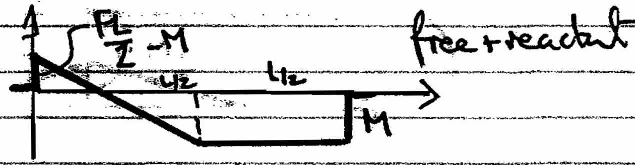
$$M \le M_p \text{ and } \left|\frac{FL}{4} - \frac{M}{2}\right| \le M_p$$

Set $M = M_p \Rightarrow FL/4 \le \frac{3M_p}{2} \Rightarrow \underline{F_{MAX} = 6M_p/L}$

(ii)



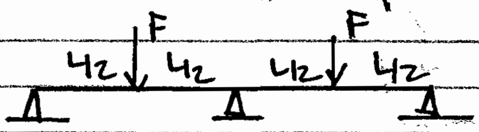
choose tip moment as reactant.



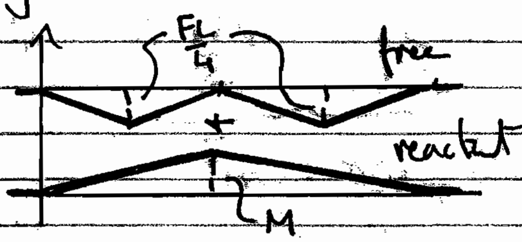
$$\left|\frac{FL}{2} - M\right| \le M_p \quad M \le M_p \Rightarrow \underline{F_{MAX} = \frac{4M_p}{L}}$$

b) Can treat the structure as a two span only, for pin does not transmit moment ⇒ single redundancy

(i)



⇒



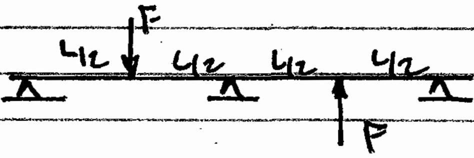
choose redundancy as B.M over internal support

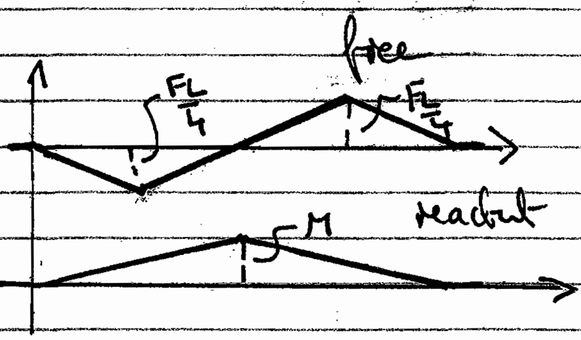
Salient points: $\left|\frac{FL}{4} - M/2\right| \le M_p$ (both centre spans)

$M \le M_p$

$\Rightarrow \underline{F_{MAX} = 6M_p/L}$

(ii)



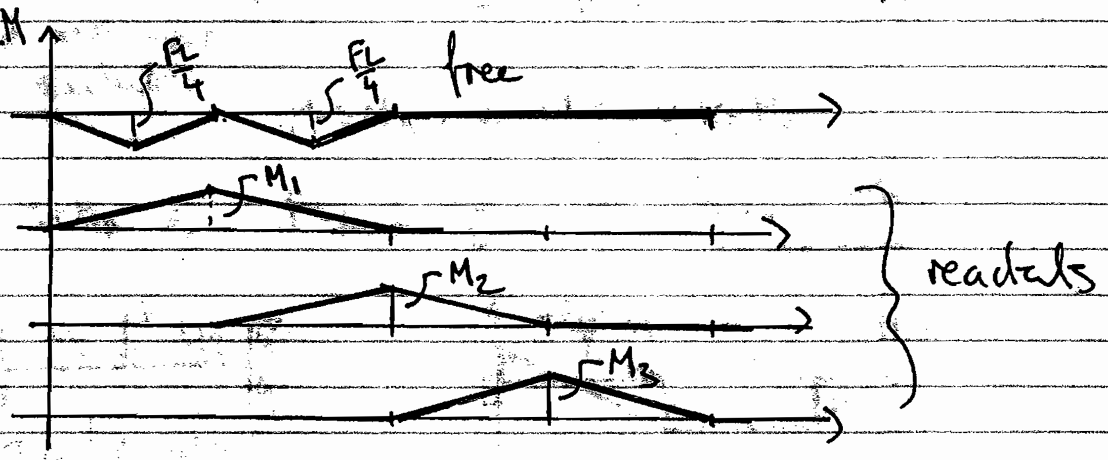


choose moment over support as redundancy.

Salient points: $\left|-\frac{FL}{4} + M/2\right| \le M_p$; $M \le M_p$ $\left|\frac{FL}{4} + \frac{M}{2}\right| \le M_p$.

F_{MAX} satisfied when $M = 0 \Rightarrow \underline{F_{MAX} = \frac{4M_p}{L}}$ [There is no b.m over support due to symmetry / anti

c) When the continuum connection exists there are 3 redundancies, which are set to be b.m's over internal supports.



Salient points: $\left|M_{1/2} - \frac{FL}{4}\right| \leq M_p$; $M_1 \leq M_p$; $\left|\frac{M_1}{2} - \frac{FL}{4} + \frac{M_2}{2}\right| \leq M_p$

$M_2 \leq M_p$ $M_3 \leq M_p$.

M_3 does not affect F, so can discount: F governed by the two relationships

$$\underbrace{\left|\frac{M_1}{2} - \frac{FL}{4}\right| \leq M_p}_{F_{MAX} = \frac{6M_p}{L} \text{ when } M_1 = M_p} \; ; \; \underbrace{\left|\frac{M_1 + M_2}{2} - \frac{FL}{4}\right| \leq M_p}_{F_{MAX} = \frac{8M_p}{L}, \text{ when } M_1 = M_2 = M_p}$$

HOWEVER, left-most span governs $\Rightarrow$

$$F_{MAX} = \frac{6M_p}{L}$$

Common mistakes: a(ii), two states of self-stress assumed when there was only one redundancy. assuming zero self-stress from the outset in b(ii) without declaring antisymmetry argument; assuming parts (a) + (b) to be related without quantifying so; not checking all spans in (c). Many did not spot B.M. distribution in a(ii) and algebraic errors, when manipulating inequalities, were high. Generally well done: note, that there were lots of "$FL/2$" rather than "$FL/4$"...

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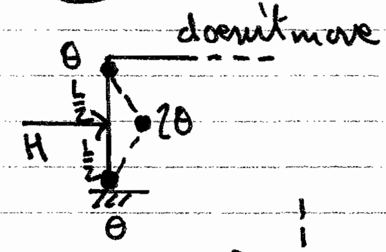
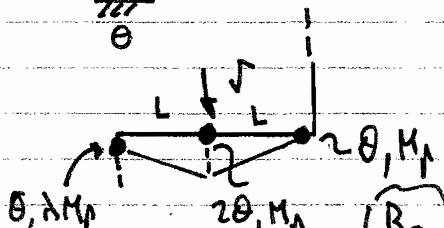
5a) (i) Beam



Hinges all form as A/D since fully plastic moment is λM_p, $|\lambda|<1$.

$\Rightarrow H \cdot \frac{L}{2}\theta + V \cdot 0 = 4M_p \cdot \lambda\theta \Rightarrow \underline{\underline{H = \frac{8\lambda M_p}{L}}}$ — (A)

(ii)

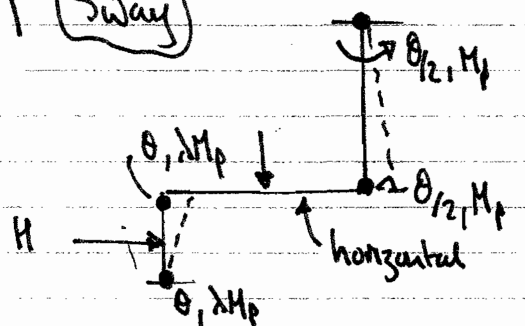


Beam

$V \cdot L \cdot \theta + H \cdot 0 = M_p[\lambda\theta + 2\theta + \theta]$

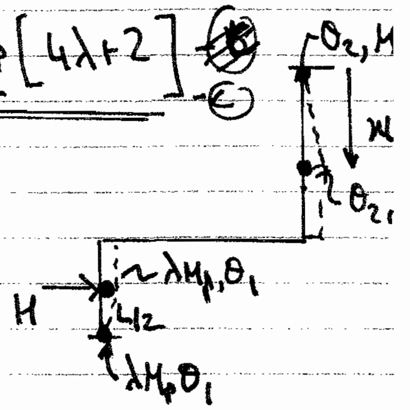
$\Rightarrow \underline{\underline{V = \frac{M_p}{L}[3+\lambda]}}$ — (B)

(iii) Sway



$\Rightarrow H \cdot \frac{L}{2}\theta = M_p\left[\lambda\theta + \lambda\theta + \frac{\theta}{2} + \frac{\theta}{2}\right]$

$\Rightarrow \underline{\underline{H = \frac{M_p}{L}[4\lambda+2]}}$ — ~~(B)~~ (C)



Another, more general mechanism has:

x is general distance from top on right

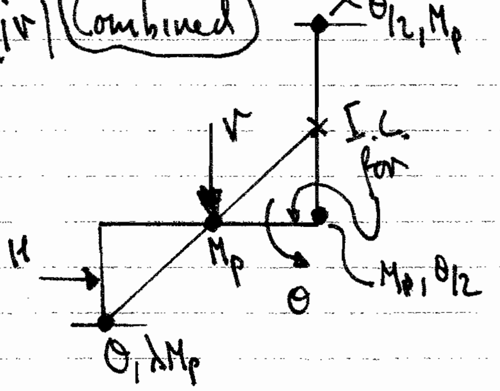
Compatibility $\Rightarrow$ $\underline{\frac{L}{2}\theta_1 = xL \cdot \theta_2}$
(top beam horizontal)

$\Rightarrow H \cdot \frac{L}{2}\theta_1 = M_p[\lambda\theta_1 + \lambda\theta_1 + \theta_2 + \theta_2] \Rightarrow H = \frac{M_p}{L}\left[4\lambda + \frac{2}{x}\right]$

~~Maximum when~~ Minimum H found by max x (= 2)

$\Rightarrow H = \frac{M_p}{L}[4\lambda + 1]$.

(iv) Combined

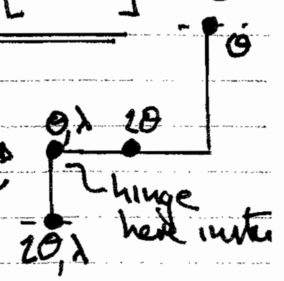


$H\frac{L}{2}\theta + V \cdot L\theta = M_p\left[\lambda\theta + 2\theta + \frac{\theta}{2} + \frac{\theta}{2}\right]$

$\Rightarrow \underline{\underline{\frac{H}{2} + V = \frac{M_p}{L}[\lambda + 3]}}$ — (D)

Also acceptable

$\Rightarrow \underline{\underline{H + V = \frac{3M_p}{L}[1+\lambda]}}$



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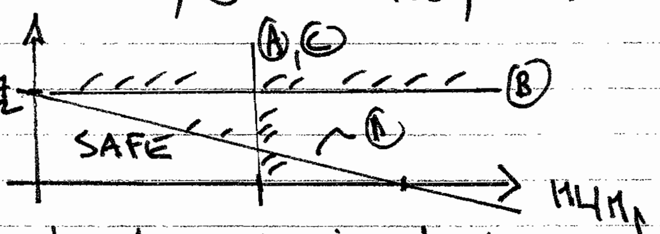
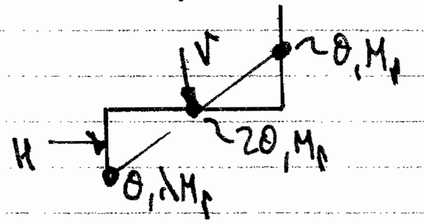
For $\lambda = 1/2$, (A): $H = 4M_p/L$, (B) $V = 7M_p/2L$, (C) $H = 4M_p/L$, (D) $\frac{H}{2} + V = 7M_p/2$



Note (B) & (D) overlap when $H = 0$

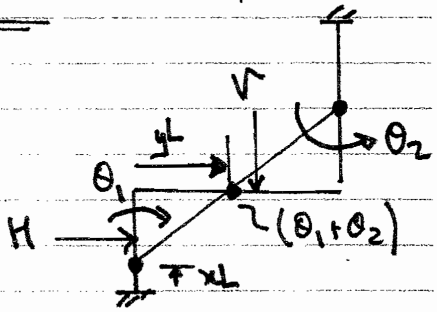
(b) The previous combined mechanism hints at a possible hinge on bar CD, coincident with I.C.



$$H\cdot\frac{L}{2}\theta + V\cdot L\theta = M_p[\lambda\theta + 2\theta + \theta]$$

$$\Rightarrow H/2 + V = \frac{M_p}{L}[3+\lambda] \quad \text{— (E); as (D)}$$

Another possible, more general solution has



x, y are general parameters, and compatibility dictates all three hinges are colinear, and that $\theta_1/\theta_2 = (2-y)/y$ [similar Δ's]

Int. W.D. $\Rightarrow H\cdot L[\frac{1}{2} - x]\theta_1 + V\cdot L\theta_2$; Ext. WD $= M_p[\lambda\theta_1 + \theta_2 + (\theta_1+\theta_2)]$

$$\Rightarrow H[\tfrac{1}{2} - x][(2-y)/y] + V = M_p/L\left[(1+\lambda)\cdot\left(\frac{2-y}{y}\right) + 2\right] \qquad 0<y<1;\ 0<x<$$

Check $x=0$, $y=1$, as (iv), then $H\cdot\frac{1}{2}\cdot 1 + V = \frac{M_p}{L}[\lambda+1+2]$ ✓

Also, a popular solution $x=0$, $y=2/3$ (hinge at top root) $\Rightarrow H + V = \frac{M_p}{L}[2\lambda +$

Parts (i)–(iii) posed few problems: HOWEVER, the combined mechanism in (iv) had many errors, most noticeably, compatibility between rotations incorrectly identified. Many candidates forgot that in (ii), the hinge forms in the weaker, λ<1, beam. Few candidates correctly identified part (b) and offered solutions where, again, bodies moved incompatibly. The general x, y, solution above, would not be expected, although the most common correct solution had $x=0$, $y=2/3$ (and not (D))

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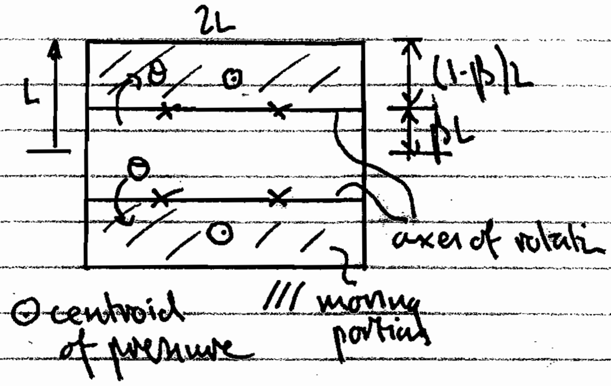
6 a)



two sides

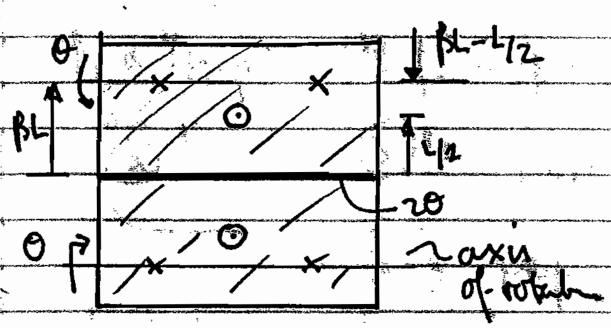
$$2\left[\underbrace{\cdot\, p \cdot 2L\cdot(1-\beta)L}_{\text{force}} \times \underbrace{\frac{(1-\beta)L}{2}}_{\text{lever}}\,\theta\right] = 2m\cdot 2L\cdot\theta$$

Work done internal = external

$$\Rightarrow \quad p(1-\beta)^2 \cdot L^3 = 2mL$$

$$\Rightarrow \quad p_1 = \frac{2m}{L^2}\cdot\frac{1}{(1-\beta)^2}$$

b)



$$2\cdot\left[\underbrace{p\cdot 2L\cdot L}_{\text{force}}\cdot\left(\beta L-\frac{L}{2}\right)\cdot\theta\right]$$

$$= m\cdot 2L\cdot 2\theta$$

$$\Rightarrow 4p\cdot L^3(\beta - 1/2) = 4mL$$

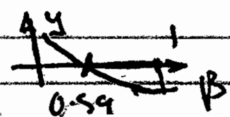
$$\Rightarrow \quad p_2 = \frac{2m}{L^2(2\beta-1)}$$

$\beta > 1/2$, falls inwards & vice versa

$\underline{\underline{p_2 < p_1}} \Rightarrow \frac{1}{2\beta-1} < \frac{1}{(1-\beta)^2} \Rightarrow (1-\beta)^2 - (2\beta - 1) < 0$

$\Rightarrow \beta^2 - 2\beta + 1 - 2\beta + 1 < 0$ (= y, say)

roots are $\beta = 2 \pm \sqrt{2} = 0.585, \underbrace{3.414}_{N/A}$

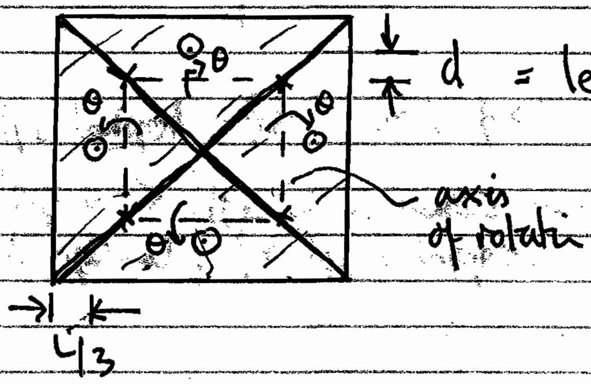


$|\beta| < 1$

$\Rightarrow \underline{\underline{0.59 < \beta < 1}}$ *

* when $\beta < 1/2$, there is another limit – see end.

c)



d = lever arm $= L - \beta L - L/3 = \frac{2L}{3} - \beta L$

(depending on β, it falls inwards or upwards at centre)

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$$4\left[p \cdot \frac{L^2}{4} \cdot L\left(\tfrac{2}{3}-\beta\right) \cdot \theta\right] = \underbrace{4m\left[\theta \cdot L + \theta \cdot L\right]}_{\text{projected length of yield line onto axes of rotation}}$$

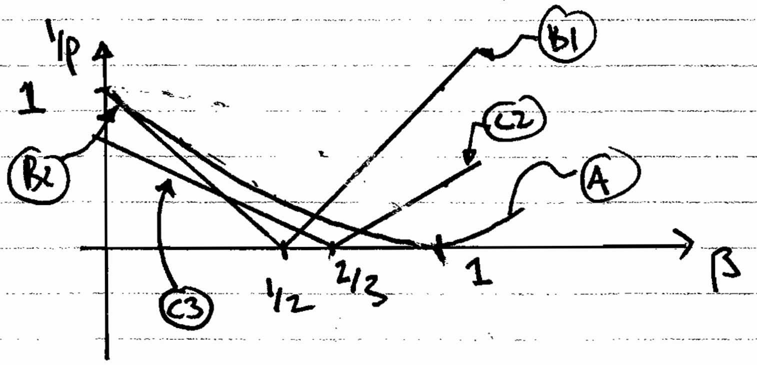
$$\Rightarrow \quad p_3 = \frac{2m}{L^2} \cdot \frac{1}{2/3 - \beta}$$

Summary:

$\underbrace{p_1 \propto 1/(1-\beta)^2}_{\text{all } \beta}$ (A), $\quad p_2 \propto 1/2\beta - 1$ (B1) $\sim \beta \geq 1/2$, $\quad p_3 \propto 1/{2/3 - \beta}$ (C1), $\quad \beta \leq 2/3$,

$p_2 \propto 1/1-2\beta$ (B2), $\beta \leq 1/2$, $\quad p_3 \propto 1/\beta - 2/3$ (C2), $\quad \beta \geq 2/3$

same proportionality constant throughout: to see if p_3 is smaller plot $1/p$ in range $0 \leq \beta \leq 1$, and consider largest value of $1/p$



(C2)+(C3) never dominate in any range $\Rightarrow p_3 > p_1, p_2 \therefore p$ cannot be improved in (c).

Many candidates got (a) straightforwardly.
Part (b) was less well-answered, especially in view of calculating the work done by the pressure: often, the displacement of the yield line was equated, incorrectly, to the displacement of the centre of pressure. Few candidates got part (c) correct: calculation assumed $\beta = 1$ from the outset but the calculation of w.d. by pressure via "pyramid" method as taught in lectures was improperly formulated from outset. Once more, identifying the centre of pressure and calculating its displacement would simplify matters greatly. There was no competitive advantage from using the projection method for calculating energy dissipated, for the crack patterns were straightforward