1B paper 5 Electrical Crib

SECTION A

Q1. A differential signal is one which is presented using two signal conductors, carrying signals which are mutually inverted (they may be described as 180 degrees out of phase, but the signals are not usually generated by shifts of phase. A common mode signal is typically applied using one signal conductor and a common earth return which is often common to many signals. A differential amplifier is designed to be sensitive to the difference between the signals applied to its two input terminals. It produces (ideally) zero response when identical signals are applied to the inputs.

The advantage of using a differential amplifier in circumstances of high interference is that interference is very often generated as a common-mode signal. If a differential amplifier is used, the differential signal inputs will normally be affected nearly identically by such common-mode sources, so that the interference signals applied to the amplifier inputs will typically be identical. A good differential amplifier will have a high common-mode rejection ratio (CMRR)

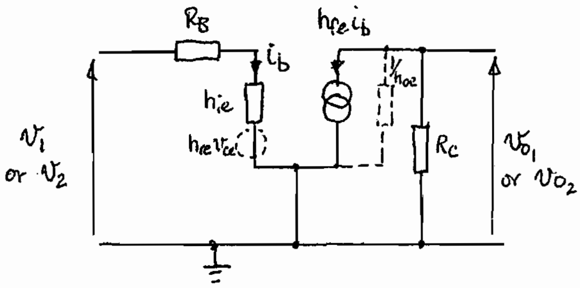
CMRR = Differential gain/Common-mode gain --- which should be large

and will not respond significantly to such signals. When applying this principle, it is necessary to reconfigure the signal source such that it generates its signal in differential mode. This typically means isolating it from earth, and using a twin cable (often a twisted-pair) to convey the differential signal to the amplifier.

(b) Since the circuit is symmetrical, the analysis for both cases can be simplified by considering only a half-circuit. In **differential mode**, equal but opposite signals are applied as v_1 and v_2. The net change in emitter current flowing through R_T is thus zero, and the potential at A remains fixed. Hence for small-signal analysis, the point A may be treated as earth. In **common mode**, each input receives exactly the same magnitude and polarity of signal, and the current in R_T changes. We can regard each half of the circuit as possessing an emitter resistor $2R_T$, the combination in parallel summing to R_T. The circuit shows h_{re} and $1/h_{oe}$ but these are not required here.

Many candidates incorrectly placed R_B *between input and ground.*

(b - i)



(c - i) Differential circuit

$$v_1 = i_b(R_B + h_{ie})$$

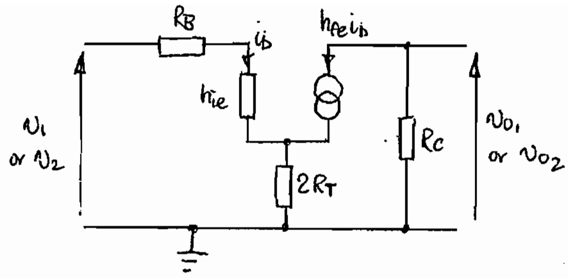
$$v_o = -h_{fe}R_C i_b$$

Hence

$$\left.\frac{v_o}{v_1}\right|_{diff} = -\frac{h_{fe}\,R_C}{R_B + h_{ie}} \qquad (1)$$

(b – ii)

(c - ii) Common mode circuit



$$v_1 = i_b(R_B + h_{ie}) + i_b \times 2R_T(h_{fe}+1)$$

$$v_o = -h_{fe}R_C i_b$$

Hence

$$\left.\frac{v_o}{v_1}\right|_{com} = -\frac{h_{fe}R_C}{R_B + h_{ie} + 2R_T(h_{fe}+1)} \quad (2)$$

$$\text{Hence CMRR} = \frac{G_{diff}}{G_{common}} = \frac{R_B + h_{ie} + 2R_T(1+h_{fe})}{R_B + h_{ie}} \quad (3)$$

With the values given, from (1), $\boldsymbol{R_C} = \frac{R_B + h_{ie}}{h_{fe}} \times 80 = \frac{1500 \times 80}{200} =$ **600 Ω**

From (3), $5000 = \frac{1000 + 500 + 2R_T(1+200)}{1000+500}$ Hence $\boldsymbol{R_T} = \frac{4999 \times 1500}{402} =$ **18.65 kΩ**

(d) With I_C = 15 mA for each transistor, 30 mA flows through R_T. If point A is to be at –1 V, $\mathbf{V_{EE}}$ must be: –1 – 30 × 18.65 ≈ **–560 V** .

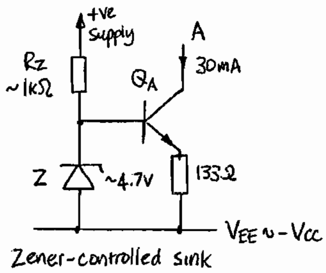
For V_{DD}, the potential drop across each R_C must be 0.6 × 15 = 9 V. However, to operate correctly the transistor must have a V_{CE} of at least a few volts, in order to deliver the stated h_{fe}, and to allow a useful swing. Allowing (say) 6-11 V for V_{CE}, V_{CC} needs to be about **15-20 V**.

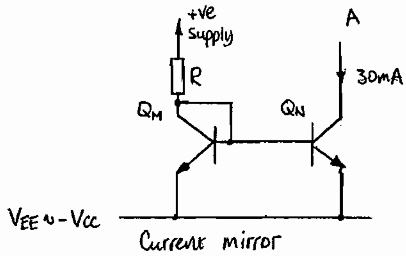
There is an enormous asymmetry about these results. The very large V_{EE} is required to deliver the specified CMRR. Clearly such a supply would not be easily implemented in portable equipment.

(e) The disparity just discussed is not necessary. Use of a constant-current sink (as shown below left) offers a convenient source of steady current, and a very high impedance at Q_A's collector to small signals, of order hundreds of kΩ (*derivation not required).* Z and R_Z keep the base of Q_A (and hence the emitter) fixed in potential relative to the new supply V_{EE}, which need be no more than a few volts negative of zero, conveniently -V_{CC}. The choice of emitter resistor then determines the current sunk from the circuitry connected at A. This sink is used in place of R_T.

Not many candidates were able to suggest a suitable circuit, though it is covered in an Examples Sheet.

An alternative (*suggested by a few candidates*) is the use of a current mirror (below right). Here the current flowing in R is 'mirrored' provided the potential at the collectors of Q_N and Q_M are not too different. The current from A is thus determined by the choice of R, of the +ve supply, and of the relative dimensions of Q_N and Q_M.

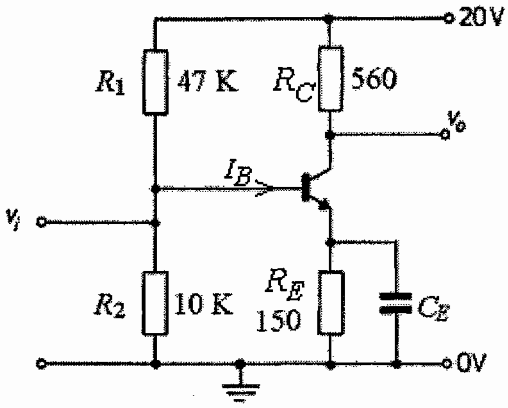




Q2 (a) R_1 and R_2 form a potential divider whose function (together with emitter resistor R_E) is to determine the operating point of the transistor, i.e. to determine I_B and V_{CE} before a signal is applied.

Maintaining V_B significantly > 0.7 V and choosing R_1 and R_2 so that the divider current >> I_B means that the operating point will vary only slightly as h_{FE} varies across devices.

Note that the input impedance for small signals is reduced owing to the shunting effect of the parallel combination of R_1 and R_2. Choice of values is thus a compromise.



(b) If $h_{FE} = \infty$, then we may assume $I_B = 0$ (*not, as some suggested, that* $I_C = \infty$). Hence, the voltage V_B is given by

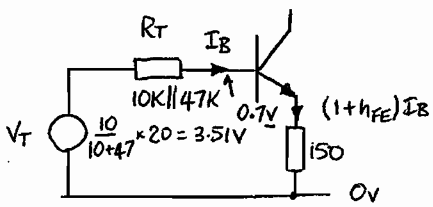
$$V_B = \frac{10\times10^3}{(10+47)\times10^3}\times 20 = 3.51 \text{ V}. \quad \text{Thus } \mathbf{V_E} = 3.51 - 0.7 = \underline{\mathbf{2.81\ V}}.$$

Hence $\mathbf{I_E} = 2.81 / 150 = \underline{\mathbf{18.72\ mA}}$.

Since $I_B = 0$, I_C is also 18.72 mA, and the voltage drop across R_C is given by $I_C \times R_C$. The collector voltage $\mathbf{V_C}$ is therefore $20 - I_C \times R_C$ or $20 - 0.01872 \times 560 = \underline{\mathbf{9.51\ V}}$

(c) Where $h_{FE} = 150$, a finite base current flows and V_B is no longer as in (b).
To determine V_B, the simplest approach is to determine the Thevenin equivalent of the bias circuit R_1 and R_2. Note that a current 150 I_B flows in R_C and 150 I_B + I_B flows in R_E.

Many candidates assumed that V_B *was the same as in section (b).*



10 kΩ || 47 kΩ = 8.25 kΩ

By KVL, (currents in mA, resistance in kΩ)

$3.51 = 8.25 \times I_B + 0.7 + 151\ I_B \times 0.15$

$2.81 = (8.25 + 22.65)\ I_B$

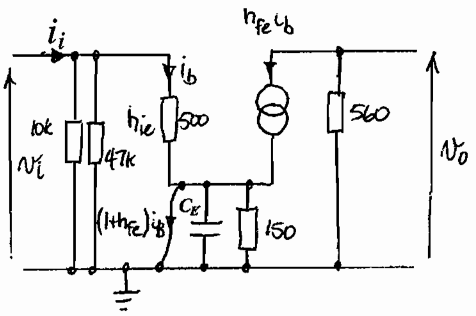
Hence $I_B = 2.81 / 30.9$ mA = 90.9 μA

$\mathbf{I_E} = 151\ I_B = 151 \times 0.0909$ mA = $\underline{\mathbf{13.73\ mA}}$

$\mathbf{V_E} = 13.73\ R_E = 13.73 \times 0.15 = \underline{\mathbf{2.059\ V}}$

$V_C = 20 - (150 \times 0.0909) \times 0.56 = 20 - 7.64 =$ **12.36 V**

(d) The small-signal equivalent circuit is shown below: the bypass capacitor C_E provides a direct path to earth for signals on the emitter, so we show a short-circuit.



Gain =

$$\frac{v_o}{v_i} = -\frac{560\,h_{fe}}{h_{ie}} = \frac{200}{500} \times 560 = -224$$

(e) With C_E omitted, the emitter-ground short is removed, with the capacitor itself. A current $(1 + h_{fe})\,i_b$ now flows thru emitter resistor R_E. Hence using KVL at the input:

$v_i = R_E(1+h_{fe})i_b + h_{ie}i_b$. This gives: $\quad i_b = \dfrac{v_i}{(1+h_{fe})R_E + h_{ie}} \qquad (1)$

and at the output: $\quad v_o = -R_C\,h_{fe}\,i_b \qquad (2)$

Hence **the gain** $\quad \dfrac{v_o}{v_i} = \dfrac{R_C\,h_{fe}}{(1+h_{fe})R_E + h_{ie}} = \dfrac{560 \times 200}{500 + 201 \times 150} =$ **-3.65**

The input resistance R_i is defined as $\dfrac{v_i}{i_i}$.

This comprises three parallel elements: the small-signal resistance of the base-emitter junction, given by $\dfrac{v_i}{i_b}$, in parallel with the bias resistors R_1 and R_2. From (2),

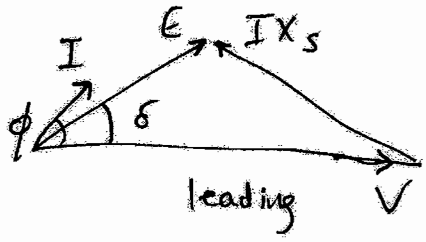
$\dfrac{v_i}{i_b} = (1+h_{fe})R_E + h_{ie} = 500 + 201 \times 150 = 30.65\ \text{k}\Omega$. Including the parallel resistance due to the potential divider gives:

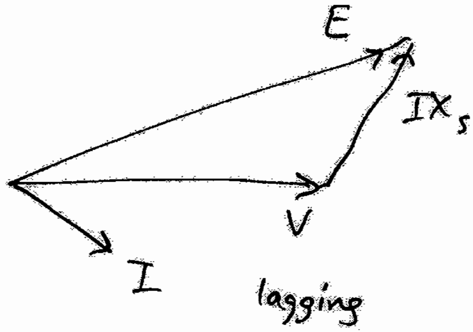
R_i = 30.65 kΩ || 8.25 kΩ = **6.50 kΩ**.

Many candidates assumed that the input resistance was either h_{ie} + 150, or was just the parallel combination of R_1 and R_2.

SECTION B

Q3 a/





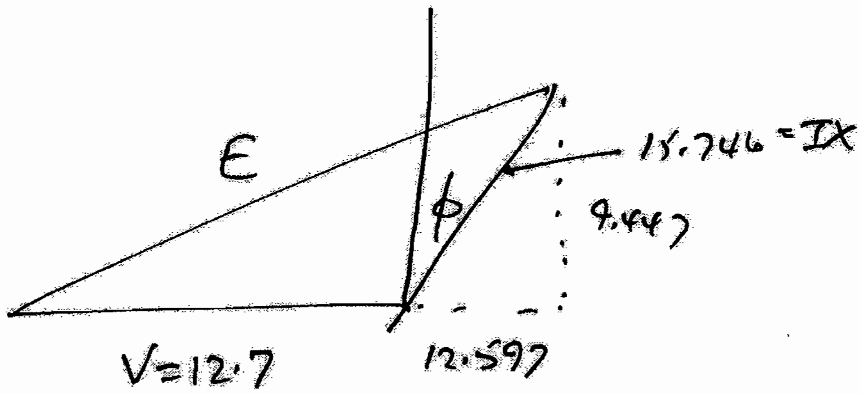
b/ $V_{ph} = 22/\sqrt{3} = 12.7$ kV

$P = 3\ I\ V \cos\phi$, so

$I_{ph} = 300 \times 10^6 / 3 \times 12.7 \times 0.6 = 13122$ A, as star, **$I_L = I_{ph} = 13.12$ kA**

$\cos\phi = 0.6$, so $\sin\phi = 0.8$

$IX_s = 13122 \times 1.2 = 15746$ V

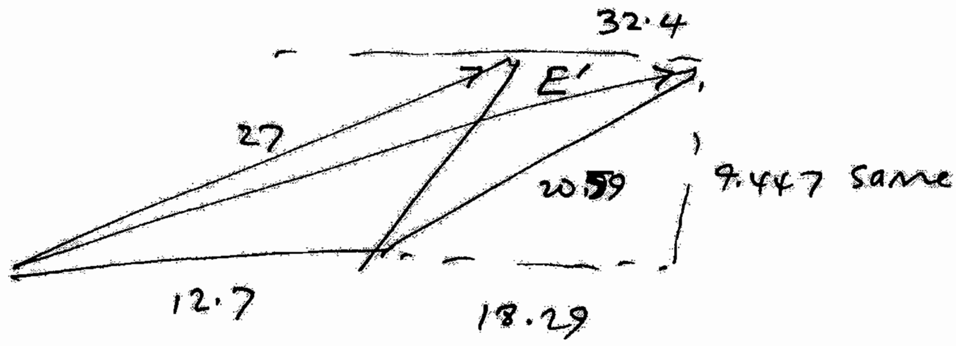


$15746 \times 0.8 = 12597$
$15746 \times 0.6 = 9447$

$E^2 = ((12.7 + 12.597)^2 + 9.447^2)^{1/2}$

E = 27.003 kV Line E = 27.003 x 1.732 = 46.769 kV.

$\tan\delta = 9.447 / 25.297$ **$\delta = 20.48$ degrees** c/

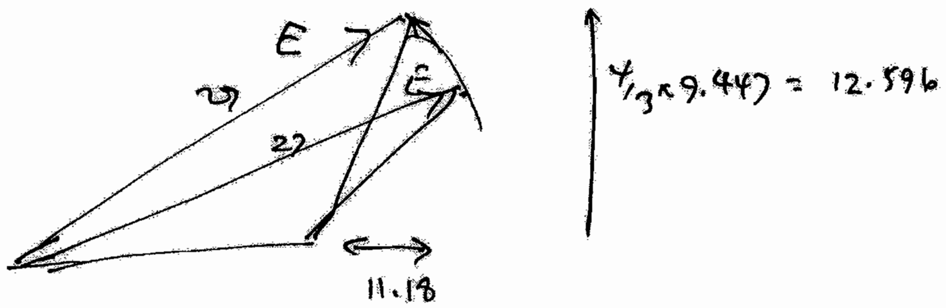


Increase E_{ph} by 20% from 27 kV to 32.40 kV.

IX cos ϕ remains same.
IX sin ϕ = 18.29 kV, so IX = 20.586 hence

I = 17.155 kA.

d/



IX cos ϕ now increased by factor 4/3 to 12.596 kV

E = 27.003 kV still.

V + IX sinϕ = 23.883 kV   IX sinϕ = 11.18

so $IX = (11.18^2 + 12.597^2)^{1/2} = 16.844$

I = 14.037 kA.

Q4 (a) generators are more efficient.
Balanced load = zero return current losses

b/ star
$V_{ph} = 415 / \sqrt{3} = 240$

$Z = (32^2 + 24^2)^{1/2} = 40$

Hence $I_{ph} = 240 / 40 = 6$ A

$P = 3.\ 6^2.\ 32 = 3.44$ kW
$Q = 3.\ 6^2.\ 24 = 2.58$ kVAR

Delta
$V_{ph} = V_{line}$

$Z = (5^2 + 25^2)^{1/2} = 25.5$ ohms I = 415 / 25.5 = 16.28 A

$P = 3.\ x\ 16.28^2\ x\ 5 = 3.97$ kW
$Q = 3\ x\ 16.28^2\ x\ 25 = 19.87$ kVA.

P tot = 3.44 + 3.97 = 7.41 kW
Q tot = 2.58 + 19.87 = 22.45 kVA

S tot = 23.64 kVA

$\cos \phi = 7.41 / 22.45 =$ **0.33**

c/ $S = \sqrt{3}\,VI = 23.64$ I line = 32.9 A.

P line = 3 x 32.9^2 x 0.5 = 1.623 kW
Q line = 5 P = 8.118 kVA

Hence
P tot = 9.033

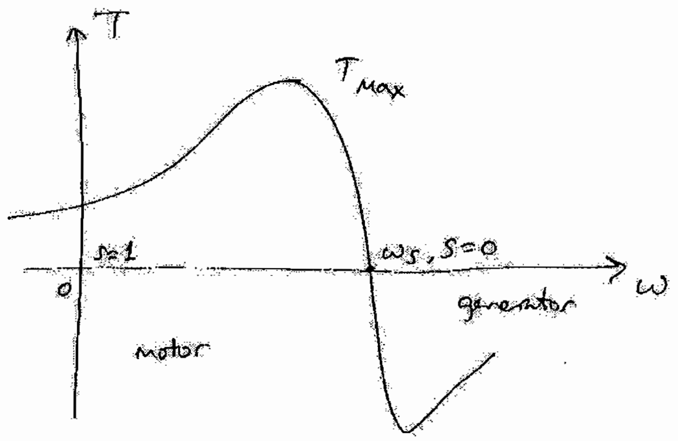
Q tot = 30.57 S tot = 31.87 kW $V = S/\sqrt{3}.I_L$ = **559 V**

d) P loads stay same as in b = 7.41 kW, Q loads = 0

$I = P/\sqrt{3}.V$ = **10.31 A**
P line = 3 x10.3^2 x 0.5 = 0.159 kW Q line = 3 x 10.3^2 x 2.5 = 0.796 kVA
P tot = 7.569, Q tot = 0.796 S tot = 7.611

$V = S/\sqrt{3}.I_L$ = **426.2 V**

Q5 a/



b/ total power = $3 I'_2{}^2 R'_2/s\,(1-s)$

$P_{out} = T\omega_r = T\,\omega_s\,(1-s)$

Equating these,

$$T = \frac{3}{\omega_s} I'_2{}^2 R'_2 / s$$

also, $I'_2{}^2 = \dfrac{V_2{}^2}{(R_1 + R'_2/s)^2 + (X_1 + X'_2)^2}$

substitute into equn for T to get the result.

c/ max torque occurs when I'_2 is a max.. Hence make the loss R'_2/s = Z of rest of load.

$Z = (R_1^2 + (X_1 + X_2)^2)^{½}$

Hence $$s = \frac{R'_2}{\sqrt{(R_1{}^2 + (X_1 + X'_2)^2}}$$

d) 4 poles = 2 pole pairs **w_s = 157 rad/s**
substitute in, gives **s = 0.31**. $w = (1\text{-}s)w_s$ = **108 rad/s**

415 V or 240 V per phase.

$$T = \frac{3x240^2}{((1+(1.4/0.31))^2 + 4.4^2)^{1/2}} \cdot \frac{1.4}{0.31x50x3.1416} = \mathbf{99.8\ Nm}$$

SECTION C

Q6 Plane electromagnetic waves have the form,

$E_x (z,t) = E_0 \exp(j (wt - kz))$

$H_y (z,t) = H_0 \exp(j (wt - kz))$

Impedance of free space is defined as $\eta_0 = E_x/H_y = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ ohms

It is comparable to the transmission line impedance, $Z = \sqrt{\frac{L}{C}}$ ohms

b) from Maxwells' equns,

$$\frac{\partial E_x}{\partial z} = -\mu_0 \frac{\partial H}{\partial t}$$

- $kE_x = \mu_0\, \omega/k$

but we also know

$w/k = v = 1/\sqrt{\mu\varepsilon}$

Hence, $E_x/ H_y = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ so $H_y = E_{x0} / \sqrt{\frac{\mu_0}{\varepsilon_0}} .\exp(j(wt - kz))$

c) by analogy with transmission lines, $T = \frac{2Z_0}{Z_0 + Z_L}$

So for 1-2 interface $T_{12} = \frac{2\sqrt{\mu_r}}{\sqrt{\mu_r} + 1}$

At 2-3 interface $T_{23} = \frac{2}{1 + \sqrt{\mu_r}}$

So $T_{1\text{-}3} = \frac{4\sqrt{\mu_r}}{(1+\sqrt{\mu_r}\,)^2}$

$P = ½ \operatorname{Re}(E_x H)$ so $\mathbf{T_{power}} = ½\, \mathbf{T}^2 / \mathbf{Z} = \frac{8\mu}{(1+\sqrt{\mu}\,)^4} / \eta_0$

For $\mu_r = 81$, transmission coefficient **T = 0.065**

Q7 a) Amperes Law

$\oint$ H. dl = I (include diagram of Ampere circuit)

L = Φ/I

Φ = 1. ∫ B(r).dr

Thus, H(r) = I/2πr
B = μ_0 H = μ_0 I / 2πr

Φ = μ_0 ∫ I/2πr / dr = (μ_0 I)/(2π). ln (a/b)

so L = μ_0 /(2π). ln (a/b)

b) for a transmission line, $Z=\sqrt{\frac{L}{C}}$ = 50 ohm,

wave velocity $v = \frac{1}{\sqrt{LC}}$ = 5. 10^8 m/s.

(wave velocities can be above speed of light, phase velocities cannot)

thus, L = Z/v = 10^{-7} H/m, C = 1/Z.v = 4.10^{-11} F/m

L = μ_0/2π . ln(a/b) = 4π.10^{-7}/2π. Ln (a/b) = 10^{-7}.

Thus ln (a/b) = ½, a/b = $e^{1/2}$ = **1.648**

c) F = 50 /(10+50) = 5/6 into the line

At sending end, voltage reflection coef
R = (10 – 50)/(10 + 50) = 2/3 power = 4/9

At load end, voltage reflection coef
R = (100 – 50)/(100 + 50) = 1/3 power = 1/9

After a set of reflections at each end, the pulse amplitude is

1/9. 4/9 smaller.

We need $(1/9)^m . (4/9)^n < 10^{-3}$ Or m+n > 4.5 or **5**