ENGINEERING TRIPOS PART IB

Monday 4 June 2007 9 to 11

Paper 1

MECHANICS

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

The answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper
Single-sided graph paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 A heavy rigid bar AB is connected to block BCDE by a frictionless hinge at B, with the whole system resting on a smooth table, as shown in plan view in Fig. 1. The system is initially at rest and AB makes a right angle with BE. The block then begins to move with an acceleration of magnitude a across the table in the direction shown. The block does not rotate.

Bar AB is of mass m with radius of gyration k about its centre of mass G which is a distance L from B.

(a) Show that the angular acceleration of AB is given by

$$\ddot{\theta} = \frac{aL\cos\theta}{(k^2 + L^2)} \ ,$$

where θ is the change in angle of the rod from the horizontal. [8]

(b) Calculate the angular velocity of AB just before it strikes the side BE of the block. [6]

(c) Obtain an expression for the force exerted by AB on the hinge at B just before AB strikes the side BE of the block. Is this greater than, less than or the same as when the block first began to move? [6]

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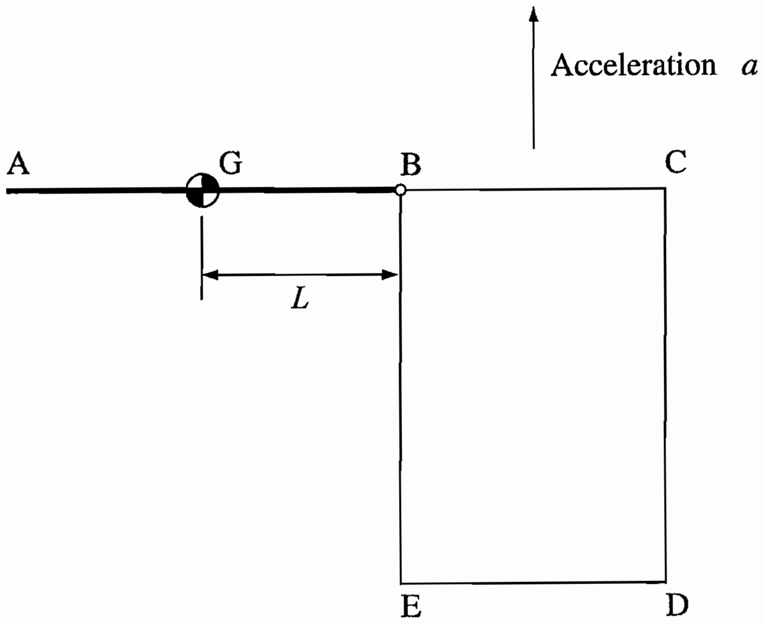

Fig. 1

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2 A pre-fabricated uniform concrete wall slab of mass m and length $2L$ is supported solely by two flexible steel cables, as shown in Fig. 2.

The right hand cable between the slab and the support breaks suddenly. Assume that the slab is initially motionless, the cables are light and inextensible and that the left hand cable remains intact. Immediately after the cable breaks:

(a) Show that the initial angular acceleration $\ddot{\varphi}$ of the rod is of magnitude $3g/5L$, where g is the acceleration due to gravity, and calculate the initial angular acceleration, $\ddot{\theta}$ of the unbroken cable. [10]

(b) Calculate the tension in the remaining cable. [2]

(c) Calculate the magnitude and position of the maximum bending moment in the slab. [8]

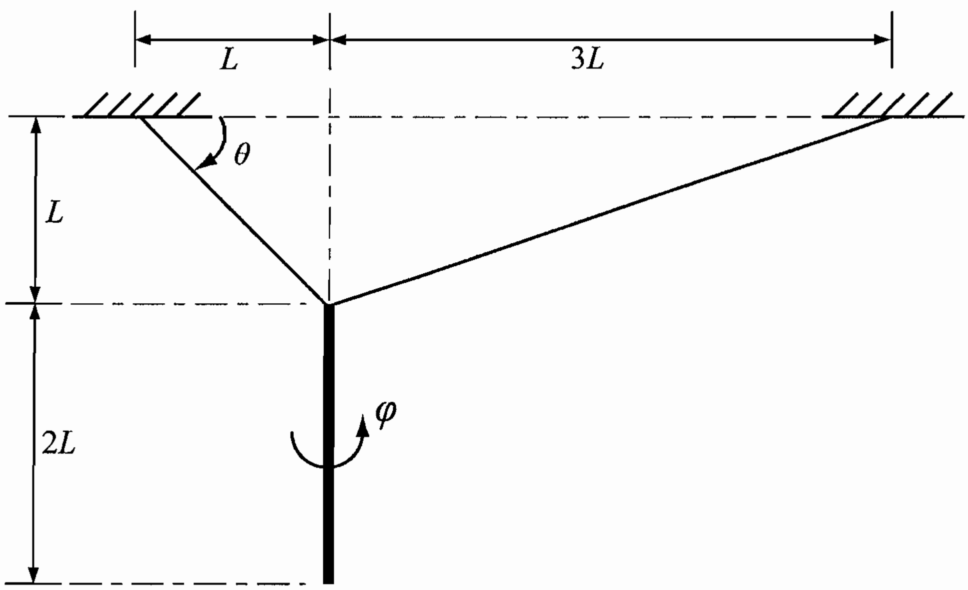

Fig. 2

3 The mechanism sketched in Fig. 3 allows a sole driven link AB of mass m to cause a uniform thin disc of mass $8m$ to rotate about its centre at D via a connecting rod BC of mass $2m$, which is pinned to the disc at C. Assume that all joints are frictionless.

At the instant shown, ABC forms a right angle and the driven link AB has angular velocity ω and angular acceleration $\omega^2/2$.

(a) Calculate the moment of inertia of the disc about its axis D (the axis points out of the page). [2]

(b) Using appropriate velocity and acceleration diagrams, calculate the angular velocities of the connecting rod BC and the disc, and show that the acceleration of point C is of magnitude $\sqrt{10}L\omega^2$. [12]

(c) Assuming the links to be uniform rigid bars, calculate the torque T required at A to drive the mechanism. [6]

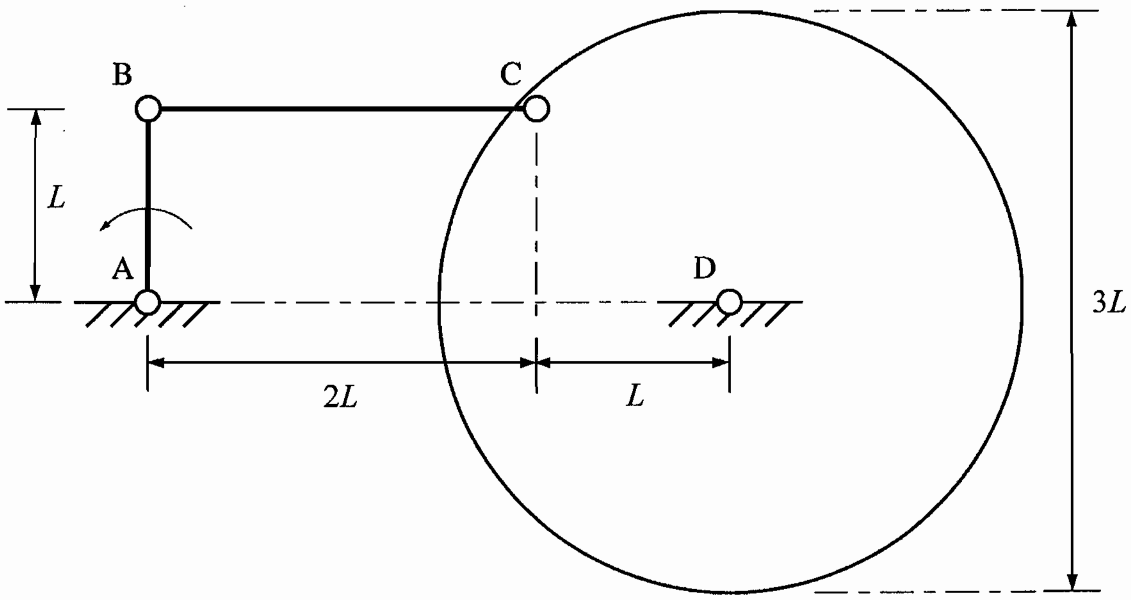

Fig. 3

SECTION B

4 A particle P is attached by an inextensible string of length a to the origin O of a fixed Cartesian co-ordinate frame (**i**, **j**, **k**). P is spinning about the vertical axis **k** at a varying inclination to the (**i**, **j**) plane. The position of P at any instant is defined by the angles ϕ and θ as shown in Fig. 4. Perpendicular unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ lie within the plane **i**, **j** and rotate with a varying angular velocity $\dot{\phi}\mathbf{k}$ to form a rotating reference frame for P.

(a) In terms of $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{k}$, ϕ and θ :

(i) find the position vector OP ; [2]

(ii) show that the absolute velocity of P is

$$\mathbf{v}_P = -a\dot{\theta}\sin\theta\,\mathbf{e}_1 + a\dot{\phi}\cos\theta\,\mathbf{e}_2 + a\dot{\theta}\cos\theta\,\mathbf{k} \;;$$ [2]

(iii) find the absolute acceleration of P. [6]

(b) The only force acting on P is the tension in the string OP. The particle P has mass m, but you should neglect gravity.

(i) Show that the kinetic energy of the particle is

$$T = \frac{1}{2}ma^2\left(\dot{\theta}^2 + \dot{\phi}^2\cos^2\theta\right) .$$ [2]

(ii) Calculate the moment of momentum of the particle about the **k** axis. [4]

(iii) Show that the results for (b)(i) and (b)(ii) are consistent with your answer to (a)(iii). [4]

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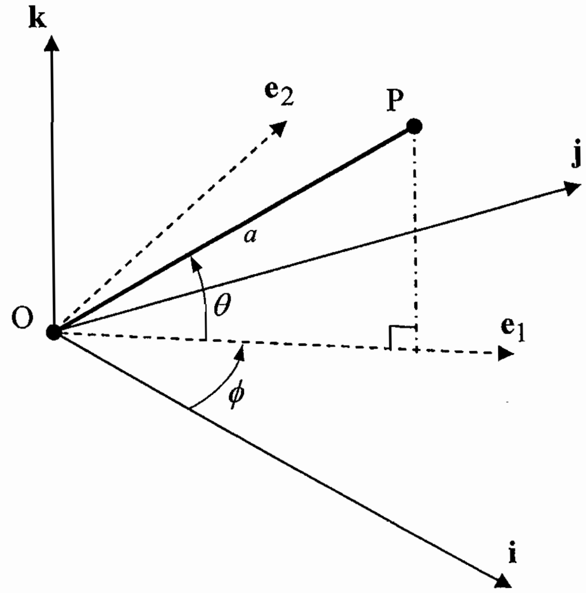

Fig. 4

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5 (a) Consider qualitatively how the potential energy V of a frictionless particle in a valley, or on a hill, as shown in Fig. 5a, varies with angular displacement θ . Explain in energy terms why the particle is in equilibrium if $\theta = 0$ and show that this equilibrium is stable if

$$\frac{d^2V}{d\theta^2} > 0 \text{ at } \theta = 0 \text{ .}$$ [4]

(b) A half cylinder of weight mg and radius a rolls in a vertical plane upon another larger half cylinder of radius b as shown in Fig. 5b. The contact between the half-cylinders is rough so that no slip occurs between them.

(i) Find the relationship between the angle of roll ϕ of the upper half cylinder and the angle to the contact point θ . [2]

(ii) Find a function for the potential energy of the upper half-cylinder in terms of θ . Show that $\theta = 0$ is a position of equilibrium. [8]

(iii) Find the condition for stability at the equilibrium position. [6]

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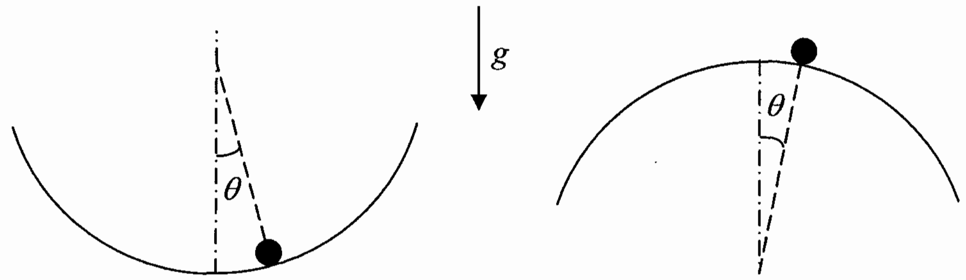

Fig. 5a

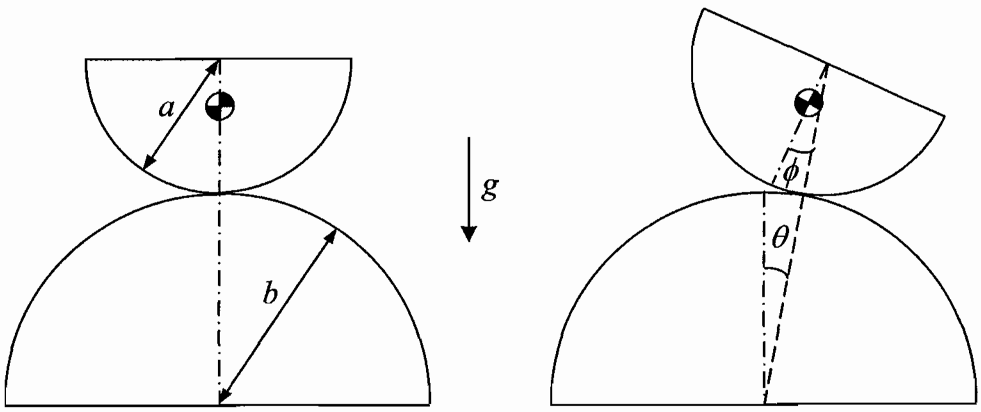

Fig. 5b

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6 A rear-engined, four-wheeled, racing car has a wheelbase of 4 m and an axle width of 2 m, as shown in Fig. 6. The car may be modeled as two parts: a chassis and a rotating engine unit. The chassis has a mass of 800 kg and its centre of gravity G lies at the midpoint of the four wheels, 0.5 m above the ground. The engine, E, can be considered to be a rotor spinning about the car's longitudinal axis with a mass of 200 kg and a radius of gyration of 0.1 m. The engine is in the rear part of the car, 1 m in front of the back axle. The engine rotates at 2000 rad s^{-1}, clockwise when viewed from behind the car, if the car is travelling forwards at 40 m s^{-1}.

Find the normal force acting between each of the four wheels and the ground when the car is travelling:

(a) In a straight line at a constant speed; [4]

(b) In a straight line and accelerating at 5 m s^{-2}; [8]

(c) Around a right-hand corner of radius 80 m at a constant speed of 40 m s^{-1}. [8]

You should assume that $g = 10\ \text{m s}^{-2}$.

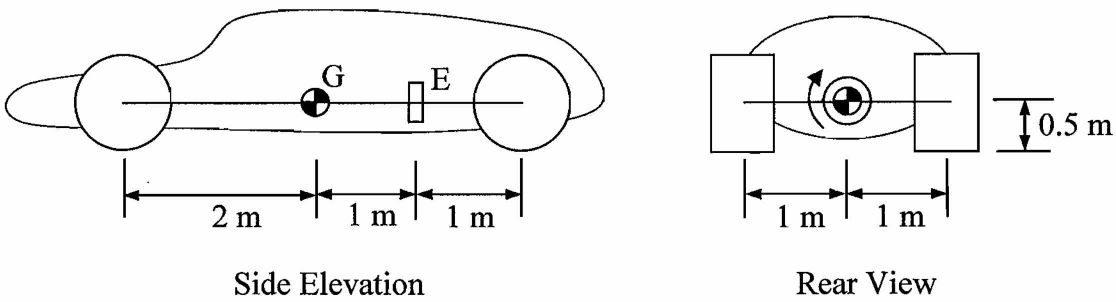

Fig. 6

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