ENGINEERING TRIPOS PART IB

Monday 4 June 2007 2 to 4

Paper 2

STRUCTURES

Answer not more than ***four*** *questions, which may be taken from either section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

1 The pin-jointed structure in Fig. 1 has linear elastic bars of equal cross-sectional area A , made of the same material of Young's modulus E . There are no initial stresses and self-weight is to be ignored.

(a) Determine the number of redundancies. [2]

(b) Using the Force Method, determine a set of bar forces in equilibrium with the applied forces, H and V , as shown. Compute the actual bar forces in the structure. [14]

(c) Find the components of displacement of the pin-joint at which the loads are applied. [4]

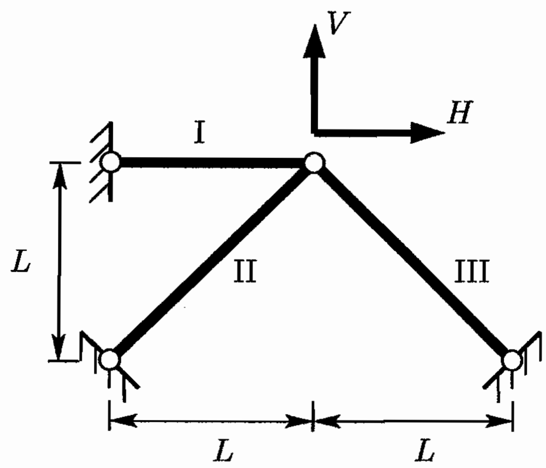


Fig. 1

2 (a) Figure 2 shows two different thin-walled cross-sections of the dimensions indicated. They are made of the same plate material, of thickness t . With respect to the square cross-section, show that the ratio of torsional stiffness of the square cross-section to the H-cross-section is 1:(49/108), and that the ratio of bending stiffness about a horizontal axis is 1:(55/54). [7]

(b) A thin-walled box section has a square cross-section of side-length b and wall-thickness t , and carries a bending moment about a horizontal axis M and a torque T . For this loading, draw the Mohr's circles of stress for an element of material in the top flange, assuming that M results in a positive moment. Determine the relationships between M and T at first yield according to (i) Tresca's criterion and (ii) von-Mises' criterion. [13]

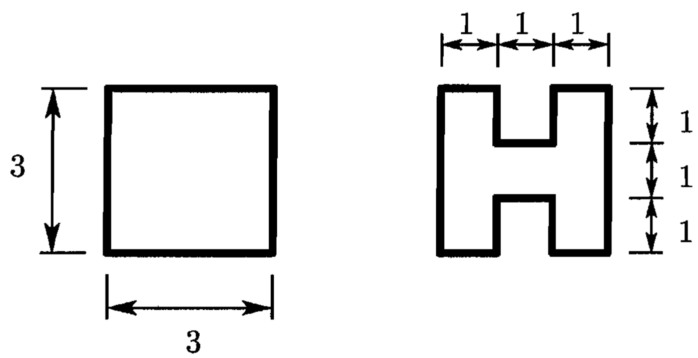


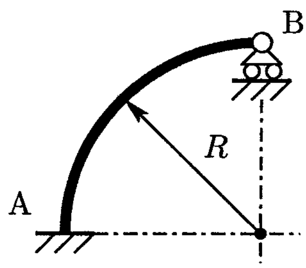
Fig. 2

3 (a) The planar quadrant rod AB shown in Fig. 3(a) undergoes a uniform rise in temperature T above ambient conditions. The rod has a uniform cross-section and is initially unstressed. The linear elastic bending stiffness is EI and the linear coefficient of thermal expansion is α .

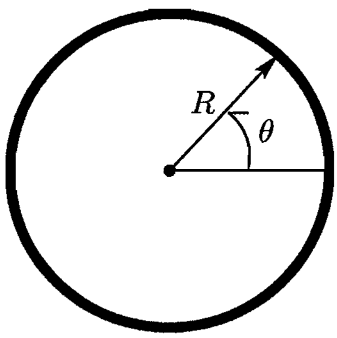
(i) If the rod end B is initially unconstrained, show that it undergoes a vertical displacement of αRT . [4]

(ii) The rod end B is now constrained vertically before heating, as shown in Fig. 3(a). Determine the reaction at the constraint after heating. [10]

(b) An isolated ring, shown in Fig. 3(b), experiences a *variable* rise in temperature $T = T_0 \sin\theta$ where θ is the angle subtended around the ring. Note that T is uniform through the thickness of rod. Find the maximum bending moment. [6]



(a)



(b)

Fig. 3

SECTION B

4 The structures shown in Figs. 4(a) - (c) have a uniform fully-plastic moment M_p and are unstressed when unloaded. All forces are vertical and are applied half-way between the supports, as indicated.

(a) Perform a lower bound analysis to determine the maximum, safe value of F for:

(i) the propped cantilever in Fig. 4(a); [3]

(ii) the cantilever in Fig. 4(b), whose tip can deflect but not rotate. [3]

(b) The four-span continuous beam in Fig. 4(c) is interrupted by a pin over the middle support. Perform a lower bound analysis to find the maximum safe load when:

(i) $F_1 = F_2 = F$; [4]

(ii) $F_1 = F$, $F_2 = -F$. [4]

(c) When $F_1 = F_2 = F$, and the pin is replaced by a continuous connection, find the maximum safe value of F . [6]

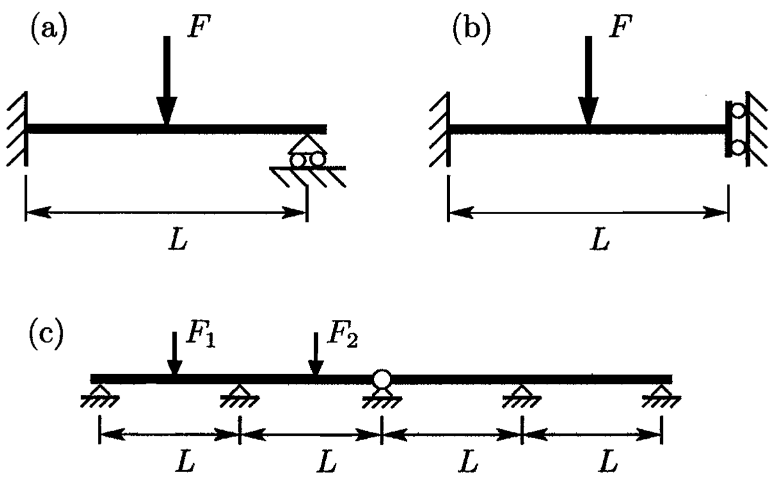


Fig. 4

5 A stepped frame ABCD carries two loads H and V as shown in Fig. 5. The fully plastic moment is M_p in the longer beams BC and CD, and λM_p in the shorter beam AB where $0 < \lambda < 1$. The frame is initially unstressed and deforms within its own plane.

(a) Perform an upper bound analysis to determine the collapse conditions for:

(i) a beam mechanism located on AB; [3]

(ii) a beam mechanism located on BC; [3]

(iii) one sway mechanism; [3]

(iv) one combined mechanism with four plastic hinges. [3]

For $\lambda = 0.5$, combine all results into a single interaction diagram. [2]

(b) By considering a combined mechanism with *three* plastic hinges, determine the corresponding collapse conditions for all values of H and V. [6]

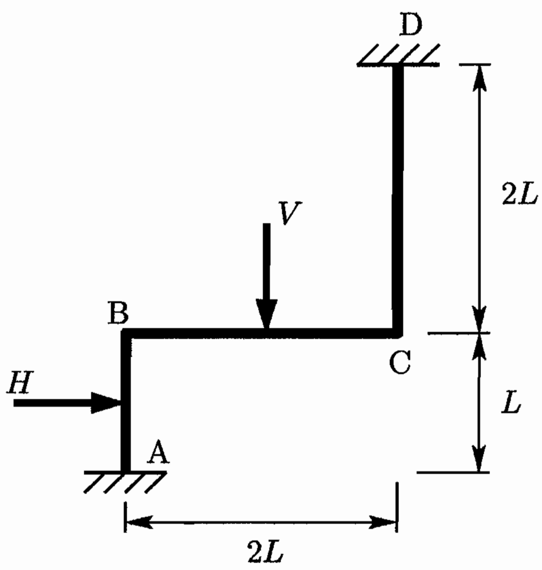


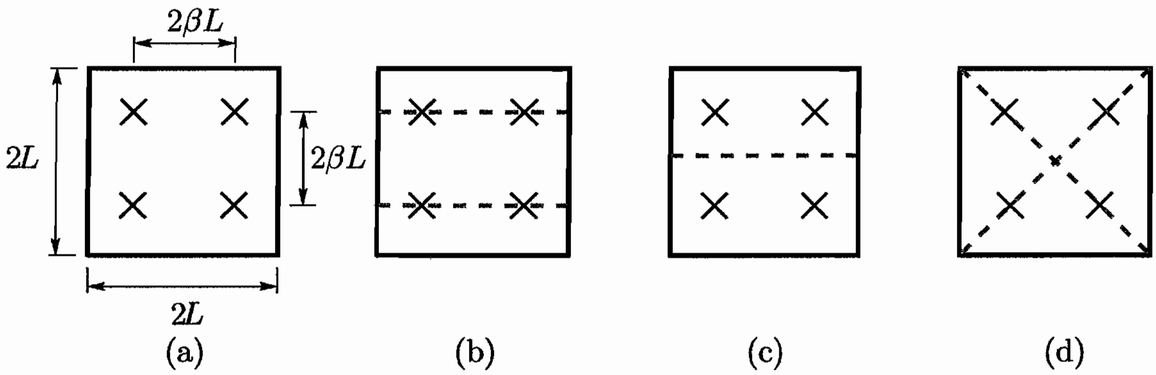
Fig. 5

6 The square slab in Fig. 6(a) has side-length $2L$ and is shown in plan view. It carries a uniformly distributed pressure loading p and is supported on four *internal* columns, arranged in a square layout of side-length $2\beta L$ of the same centre as the slab. Each column performs as a point support. The slab is initially unstressed and has uniform moment of resistance per length m both for hogging and sagging.

(a) By considering the collapse mechanism of two yield lines shown in Fig. 6(b), find the critical value of p . [6]

(b) By considering the collapse mechanism of a single yield line shown in Fig. 6(c), find the critical value of p and find the value of β which results in a smaller collapse load compared to the result found in (a). [6]

(c) By considering the collapse mechanism shown in Fig. 6(d), show that the critical value of p cannot be improved compared to the results found in (a) and (b). [8]



Note: cracks are shown as dashed lines

Fig. 6

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