ENGINEERING TRIPOS . PART IB

Tuesday 5 June 2007 2 to 4

Paper 4

THERMOFLUID MECHANICS

Answer not more than ***four*** *questions.*

Answer not more than ***two*** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Thermofluids Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

*Answer not more than **two** questions from this section.*

1 *It should be noted that parts (a) and (b) of this question are unrelated.*

(a) A mass of 0.02 kg of air at a temperature of 300 K is confined to a cylinder of volume 0.01 m^3 by a thermally non-conducting piston. The back face of the piston is exposed to a vacuum, and friction between the piston and cylinder may be neglected. The air is compressed *reversibly* to a volume of 0.002 m^3, the compression process being so fast that there is negligible heat transfer to the cylinder walls. The piston then remains stationary while the air temperature falls to 300 K as heat is transferred to the cylinder. The cylinder has a large heat capacity and its temperature may be assumed to remain constant at 300 K throughout the process. There is no heat transfer between the cylinder and the environment.

For the complete process, calculate the entropy change of the thermodynamic system comprising the air and cylinder. Explain *precisely* the physical reason for this entropy change. [10]

(b) On a cold day, a cyclic heat pump is to be used to raise the temperature of a house from the environment temperature T_0 to a comfortable temperature T_1. The PER (performance energy ratio) of the pump is α times the PER of a thermodynamically reversible heat pump operating between T_0 and the instantaneous temperature of the house. The heat capacity of the house is C and heat loss from the house to the environment can be neglected.

(i) Show that W, the minimum work input to the heat pump needed to raise the temperature of the house from T_0 to T_1, is given by

$$W = \frac{C}{\alpha}\left[(T_1 - T_0) - T_0 \ln\left(\frac{T_1}{T_0}\right)\right].$$

[6]

(ii) Suppose the heat pump is driven by a cyclic heat engine with thermal efficiency 0.30. If $T_0 = 275$ K and $T_1 = 293$ K, find the minimum value of α such that the engine-pump combination is thermodynamically more efficient than simply burning fuel to supply the required amount of heat. [4]

2 *In this question you may use either the steam tables, the steam chart, or a combination of both. If you use the steam chart, be aware that marks will be awarded for accuracy of property determinations.*

An electricity generating power plant is based on the following steady-flow steam cycle. Saturated liquid water at a pressure of 0.04 bar is supplied to a feed pump where it is compressed isentropically to a pressure of 150 bar. The water then enters a boiler and superheater where it is heated at constant pressure, leaving as superheated steam at a temperature of 500 °C. The steam is then expanded adiabatically in a turbine with an isentropic efficiency of 0.83 to a pressure of 0.04 bar. Finally, the steam is condensed at constant pressure to saturated liquid water.

(a) (i) Sketch the cycle on an enthalpy-entropy $(h - s)$ diagram. Include the saturation line and mark the critical point on the diagram.

(ii) Find the dryness fraction at turbine exit and the turbine work output per kg of steam circulating.

(iii) Estimate the feed pump work input per kg of steam circulating.

(iv) Calculate the cycle efficiency. [12]

(b) (i) Give two reasons why the high wetness at turbine exit is unacceptable.

(ii) Suggest a modification to the steam cycle to alleviate this problem.

(iii) State, giving reasons, the likely effect of the modification on the cycle efficiency.

(iv) Explain briefly the physics of how the condenser is able to maintain a sub-atmospheric back pressure of 0.04 bar on the turbine. [8]

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3 Figure 1 is a schematic diagram of an air conditioning unit for a jet aircraft with a pressurised cabin. When flying at high altitude, air enters the compressor of the jet engine at a pressure of 0.25 bar and a temperature of 220 K (state 1). The air is compressed with an isentropic efficiency of 0.88 to a pressure of 0.90 bar (state 2) where a small fraction of the flow is extracted and fed to the air conditioning unit. This consists of a compressor and turbine on the same shaft. The air is compressed to pressure p_3 and temperature T_3 (state 3), cooled at constant pressure (state 4), and finally expanded in the turbine to a pressure and temperature of 0.75 bar and 20 °C (state 5). The air is then fed directly to the cabin. It may be assumed that the compression and expansion processes in the air conditioning unit are reversible and adiabatic, and that air behaves as a perfect gas with the properties given in the thermofluids data book. Mechanical losses in the turbine-compressor unit may be neglected.

(a) Sketch the circuit from state 1 to state 5 on a temperature-entropy $(T-s)$ diagram including all relevant constant pressure lines. [5]

(b) Calculate the temperature T_2 of the air extracted from the jet engine compressor. [2]

(c) Calculate the pressure p_3. [8]

(d) Calculate the heat transferred in the cooler per unit mass of air delivered to the cabin. [2]

(e) For comfort, the cabin air must be humidified. Assuming the air entering the jet engine compressor to be dry, calculate the mass of water which must be added per unit mass of cabin air supplied in order to provide a relative humidity of 60 %. [3]

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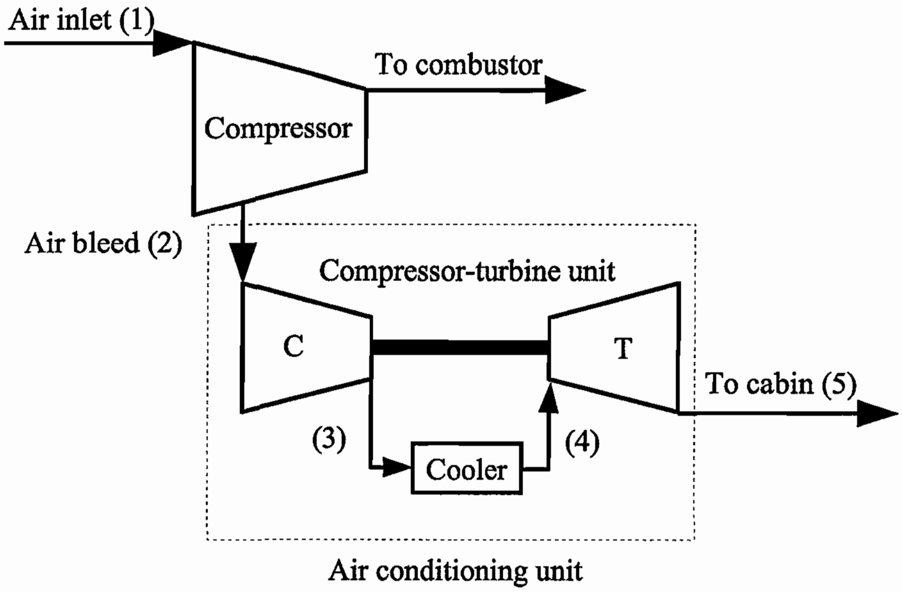

Fig. 1

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SECTION B

*Answer not more than **two** questions from this section.*

4 A long cylindrical pipe, fully-immersed in a steady ocean current, is found to vibrate with a certain frequency f due to the fluid flow. Previous studies of this phenomenon show that the frequency of vibration of a long cylinder depends on the speed of the flow V, the diameter of the cylinder d, the dynamic viscosity of the fluid μ and the density of the fluid ρ.

(a) Find a non-dimensional frequency and the non-dimensional group, or groups, on which it depends. [4]

(b) A pipe of 0.1 m diameter is fully-immersed in a 1 ms^{-1} ocean current. In order to predict the vibration frequency in this situation, tests are to be made in a wind-tunnel using a cylinder of 0.025 m diameter. What speed should be used in order to achieve full dynamic similarity? (The properties of air and seawater are given below). [4]

(c) The frequency of vibration of the cylinder in the wind-tunnel at this velocity is found to be 480 Hz. What is the frequency at which the real pipe will vibrate in the 1 ms^{-1} ocean current? [4]

(d) The peak force per unit length is found to depend on the same variables. Find the ratio of the peak force per unit length experienced by the pipe in the ocean to the peak force per unit length measured in the wind-tunnel under the conditions of full dynamic similarity. [4]

(e) Assuming that the wind-tunnel can be run at any speed desired, what limits the maximum ocean current speed that can be modelled using the 0.025 m diameter cylinder in the wind-tunnel whilst maintaining full dynamic similarity? [2]

(f) If the pipe was near the surface of the ocean what additional phenomena might be important? What additional non-dimensional group, or groups, would need to be considered? [2]

The dynamic viscosity and density of seawater are 0.001 kg $m^{-1}s^{-1}$ and 1000 kg m^{-3}, respectively, and of air are 1.8×10^{-5} kg $m^{-1}s^{-1}$ and 1.2 kg m^{-3}, respectively.

5 A viscous Newtonian fluid flows between two concentric pipes as shown in Fig. 2. The outside radius of the inner pipe is R_1 and the inside radius of the outer pipe is R_2 . The flow is fully developed in the sense that the streamwise velocity component u , the shear stress in the fluid τ and the streamwise pressure-gradient dp/dx are all independent of x , the streamwise distance. The flow is laminar and the dynamic viscosity of the fluid is μ . The streamwise pressure-gradient does not vary in the radial direction.

(a) By considering the forces on a small ring-shaped element of fluid, or otherwise, show that the streamwise shear stress in the fluid τ is related to the streamwise pressure gradient by

$$\frac{1}{r}\frac{d(\tau r)}{dr} = \frac{dp}{dx} ,$$

where r is the radial co-ordinate. [4]

(b) The inner pipe is heated in such a way that the viscosity is not uniform across the radius but instead varies as $\mu = \beta r$, where β is a constant. Find an expression for the streamwise velocity u in terms of R_1 , R_2 , dp/dx , β and r . [8]

(c) Find the ratio of the drag-force per unit streamwise length acting on the outer pipe to the drag-force per unit streamwise length acting on the inner pipe in terms of the same variables. [8]

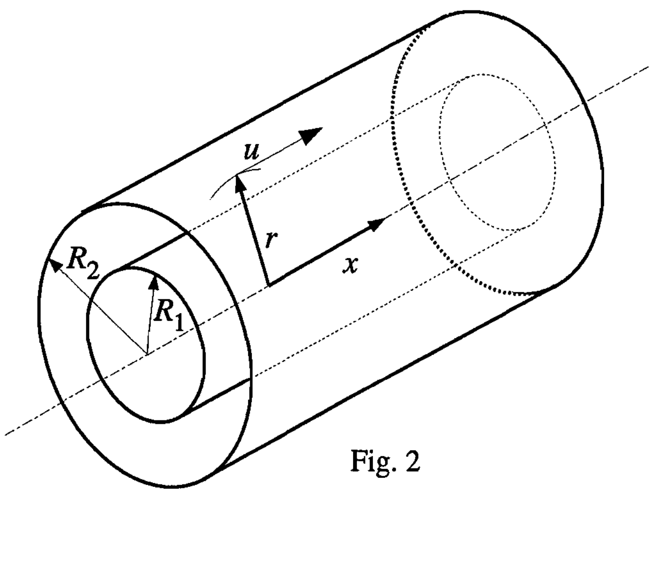

Fig. 2

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6 Fluid flows through a large duct of height $4h$ as shown in Fig. 3. Part of the upstream section of this duct is divided into two separate ducts of height h and $3h$ respectively. At station **1**, far upstream, the velocity and pressure are uniform in each duct. In the larger duct the velocity is U_1 and the pressure is P_1 and in the smaller duct the velocity is $9U_1$ and the pressure is also P_1 . At some distance from the exit of the smaller duct, a flat plate of height h is mounted on the floor perpendicular to the flow. Fluid from the two upstream ducts mixes turbulently and passes through the gap between this flat plate and the top of the duct. At station **2**, just upstream of the gap, the pressure is uniform across the whole height of the duct and the velocity U_2 is uniform across the gap. At station **3**, far downstream, the velocity U_3 and pressure P_3 are both uniform over the whole duct height. The fluid throughout has a uniform density of ρ , the flow is incompressible and friction forces may be neglected. The duct has unit width (i.e. in the direction 'into the page').

(a) What is the velocity U_2 in the gap between the plate and the top of the duct in terms of U_1 ? [1]

(b) What is the velocity U_3 at the station far downstream in terms of U_1 ? [1]

(c) What is the pressure P_2 at station 2 in terms of P_1 , ρ and U_1 ? [4]

(d) Find an expression for the force F that the plate exerts on the fluid in terms of P_1 , P_3 , U_1 , ρ and h . [10]

(e) How would the inclusion of friction change the value of the pressure P_3 ? [4]

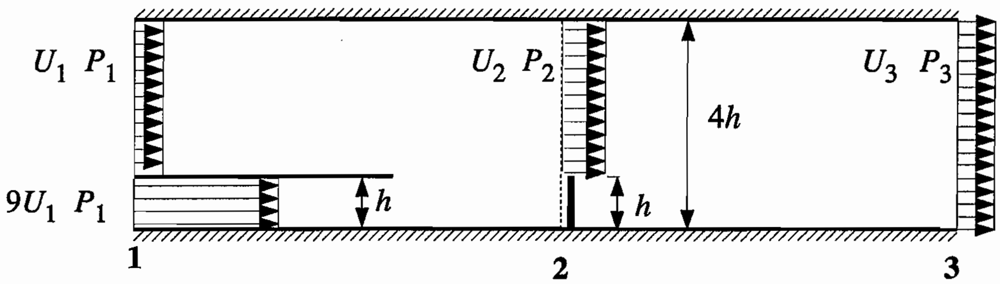

Fig. 3

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