ENGINEERING TRIPOS PART IB

Wednesday 6 June 2007 2 to 4

Paper 5

ELECTRICAL ENGINEERING

*Answer not more than **four** questions.*

*Answer at least **one** question from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

*Answer at least **one** question from this section.*

1. (a) The differential amplifier shown in Fig. 1 is to be used to amplify a signal from a strain gauge, typically of the order 1 mV. There is a significant unwanted common-mode signal arising from ac mains interference. Explain briefly with the aid of a diagram the terms *differential signal* and *common-mode signal*, and explain why a differential amplifier would be a good choice for this application. [3]

(b) Sketch small-signal equivalent circuits of Fig. 1 for the following:

(i) differential gain, defined as $(v_a - v_b)/(v_1 - v_2)$;

(ii) common-mode gain, defined as v_a/v_1. [4]

(c) Hence find expressions for the differential and common-mode gain. Show also that the common-mode rejection ratio CMRR is given by

$$\text{CMRR} = \frac{R_B + h_{ie} + 2(1+h_{fe})R_T}{R_B + h_{ie}}$$

R_B is chosen as 1 kΩ. If h_{ie} = 500 Ω and h_{fe} = 200, find suitable values for R_C and R_T to provide a differential gain of 80 and a common-mode rejection ratio of 5000. The small-signal parameters h_{oe} and h_{re} may be taken as zero in these calculations. [6]

(d) The transistor's operating point should be a collector current of 15 mA. Determine suitable supply voltages, V_{CC} and V_{EE}, if the point marked A in Fig. 1 is desired to be at −1 V when no signals are applied. Comment on the values you obtain. [4]

(e) In order to improve the performance of the circuit, it is required to replace R_T with a constant current source to achieve the highest possible CMRR. Sketch a suitable circuit for this function. What other advantages would this circuit offer? [3]

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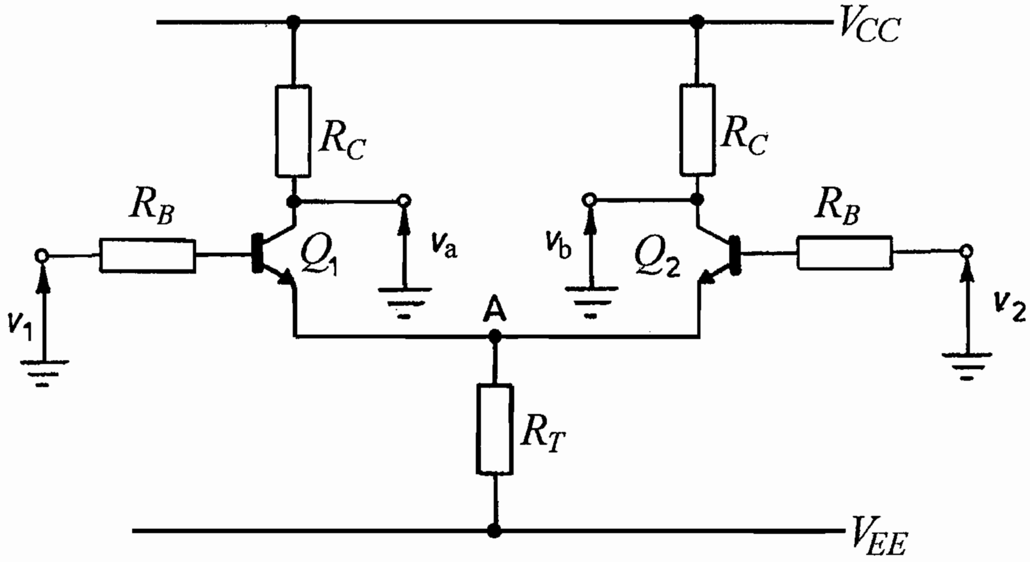

Fig. 1

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2 Figure 2 shows the circuit diagram of a Class A bipolar transistor amplifier.

(a) What is the function of resistors R_1 and R_2, and what are the most important factors that influence their choice? [2]

(b) If the transistor is initially assumed to have infinite dc current gain h_{FE}, and the operating base-emitter voltage V_{BE} is 0.7 V, determine the quiescent emitter current I_E. Determine also the quiescent voltages at the emitter V_E and at the collector V_C. [4]

(c) In fact, the transistor has a minimum value for h_{FE} of 150. Determine the values I_E, V_E and V_C with this value of h_{FE}. You may assume that the value of V_{BE} remains at 0.7 V. [2]

(d) The transistor has small-signal parameters h_{fe} equal to 200 and h_{ie} equal to 500 Ω. Both h_{oe} and h_{re} may be taken as zero. Sketch a small-signal equivalent circuit valid for mid-band frequencies, and determine the small-signal gain of the amplifier v_o/v_i assuming that C_E behaves as a short circuit to the signals being amplified. [7]

(e) With the capacitor C_E omitted from the circuit, determine:

(i) the new small-signal voltage gain;

(ii) the small-signal input resistance. [5]

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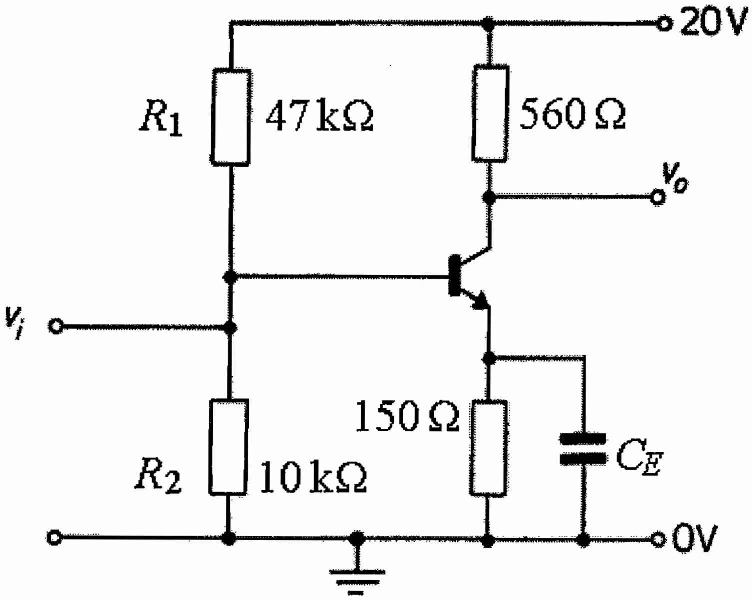

Fig. 2

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SECTION B

*Answer at least **one** question from this section.*

3 (a) Sketch a labelled phasor diagram for one phase of a synchronous generator when delivering power to an infinite bus under the following conditions:

(i) leading power factor of 0.8;

(ii) lagging power factor of 0.8.

Indicate clearly the current phasor in each case. [3]

(b) A 2-pole star-connected 22 kV, 1000 MVA synchronous generator has a synchronous reactance of 1.2 Ω per phase. It delivers 300 MW into an infinite bus at a lagging power factor of 0.6. Calculate:

(i) the stator line current;

(ii) the excitation line voltage;

(iii) the load angle. [7]

(c) The excitation voltage of the generator described in (b) is increased by 20%, with the power of the prime-mover remaining unchanged. Calculate the new value of stator current. [5]

(d) The prime-mover power of the generator in (b) is increased to 400 MW, and the excitation voltage remains as in (b). Calculate the new value of stator current. [5]

4 (a) Discuss the main reasons why electrical power is generated as three phase alternating voltages. [2]

(b) A 415 V, 50 Hz, 3-phase power line supplies two separate loads, one being star-connected with an impedance of (32 + 24j) Ω in each arm, and the other being delta-connected with an impedance of (5 + 25j) Ω in each arm.

Calculate the power and reactive power supplied to each load. Calculate the total power and power factor for the combined loads. [8]

(c) The transmission line supplying these loads has an impedance of (0.5 + 2.5 j) Ω per line. Calculate the line voltage at the supply end, assuming the voltage at the load end is 415 V. [6]

(d) What would be the new supply line voltage if the power factor of the combined loads is corrected to unity.

[4]

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5 (a) Draw a typical torque - speed curve for a three-phase induction motor. [2]

(b) Figure 3 shows a simplified equivalent circuit per phase of an induction motor, in which the magnetising reactance and iron loss resistance can be ignored. Show that the output torque of the motor is given by

$$T = \frac{3V^2}{(R_1 + R'_2/s)^2 + (X_1 + X'_2)^2} \cdot \frac{R'_2}{s\,\omega_s}$$

where the symbols have their usual meaning. [5]

(c) Using the maximum power transfer theorem or otherwise, show that the maximum torque developed by the motor occurs at a slip of

$$s = \frac{R'_2}{\sqrt{R_1^{\,2} + (X_1 + X'_2)^2}}$$ [4]

(d) A 3-phase, four-pole induction motor is star-connected to a 415 V, 50 Hz supply. The parameters of the motor are

$R_1 = 1\ \Omega$, $R'_2 = 1.4\ \Omega$, $X_1 = 2\ \Omega$, $X'_2 = 2.4\ \Omega$

Calculate the maximum torque developed by the motor, and the rotation speed at which this occurs.

[9]

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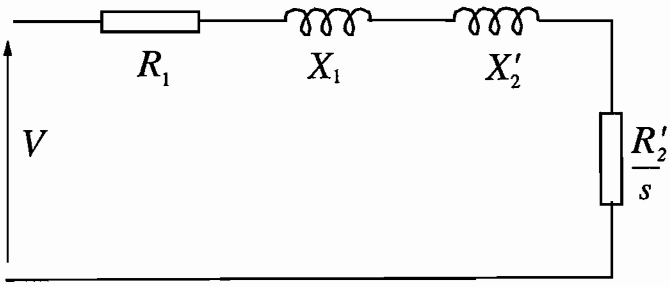

Fig. 3

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SECTION C

*Answer at least **one** question from this section.*

6 (a) Explain the term *impedance of free space*, given by $\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$, as applied to plane electromagnetic waves propagating in free space, and compare it to an equivalent term for a transmission line. [3]

(b) The electric field of a plane electromagnetic wave is given by

$$\bar{\mathbf{E}} = \boldsymbol{u}_x E_0 \exp[j(\omega t - \beta z)]$$

Using Maxwell's equations or otherwise, derive an expression for its magnetic field intensity, **H.** [4]

(c) Figure 4 shows a plane electromagnetic wave of peak electric field strength $\bar{\mathbf{E}}_1$ propagating through a region of air (region 1) and then incident normally on a flat, finite slab of material (region 2). The opposite side of the slab is also air (region 3). The material has a relative permeability of μ_r and a relative permittivity of 1.

By analogy to transmission line theory or otherwise, derive expressions for the transmitted electric field at the boundary between regions 1 and 2, and show that the electric field which propagates through the slab into region 3 is given by

$$\mathbf{E} = \frac{4\sqrt{\mu_r}}{\left(1+\sqrt{\mu_r}\right)^2}\mathbf{E}_1$$ [7]

(d) Hence show that the power per unit area which propagates into region 3 is

$$P = \frac{8\mu_r}{\left(1+\mu_r^{1/2}\right)^4}\frac{E_i^2}{\eta_0}$$

Find the fraction of incident power transmitted for the case of $\mu_r = 81$. [6]

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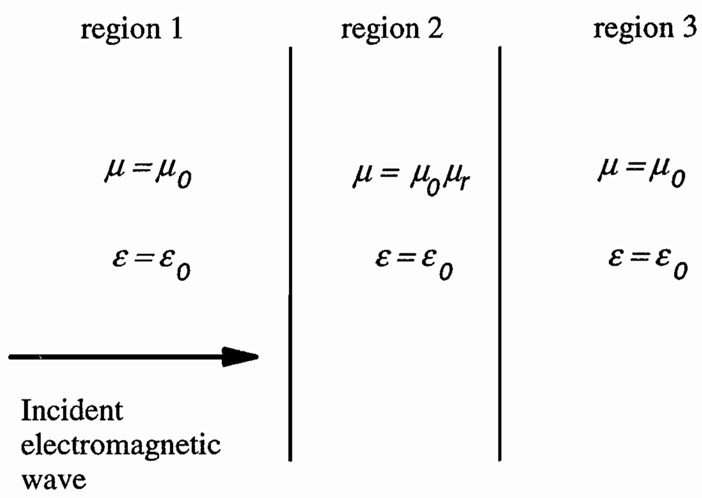

Fig. 4

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7 (a) A coaxial cable has an outer radius of a and an inner radius of b. Using Ampere's law, derive an expression for the inductance per unit length of the cable. [6]

(b) When this coaxial cable acts as a transmission line, it has a characteristic impedance of 50 Ω and a wave velocity of 5×10^8 ms^{-1}. Determine the capacitance and inductance per unit length. If the relative permeability of the insulator is 1, calculate the radius ratio of the cable, *a/b*. [7]

(c) This transmission line has a length of 20 m, and it is terminated by a resistor of 100 Ω. A sharp pulse of 1 V is fed from a source of internal resistance 10 Ω. After how many reflections will the power of the pulse fall to less than 0.1 % of its initial value inside the line ? [7]

END OF PAPER