ENGINEERING TRIPOS PART IB

Thursday 7 June 2007 2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than ***four*** *questions.*

Answer not more than ***two*** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Additional copy of Fig. 2.
Additional copy of Fig. 4.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

*Answer not more than **two** questions from this section.*

1 (a) Explain briefly what is meant by the terms *gain margin* and *phase margin*. [3]

(b) A system has a transfer function

$$G(s) = \frac{2}{s(s+1)(s+2)}$$

and a controller connected as shown in Fig. 1. Assuming that $K(s) = 1$, use the Nyquist diagram for $G(s)$ shown in Fig. 2 to estimate:

(i) the gain and phase margins;

(ii) the frequency in radians/sec at which $|1 + K(s)G(s)|$ is minimum and the magnitude of the closed-loop frequency response at that frequency. [5]

(c) A phase compensator is now used to replace $K(s)$ such that

$$K(s) = \frac{1+2s}{1+s/4} \, .$$

Sketch the Nyquist diagram for $K(s)G(s)$ on the copy of Fig. 2 provided. [6]

(d) Using your sketch, re-estimate the quantities found in part (b) for the compensated system. Based on these estimates, what can be inferred about the effect of the compensator on the closed-loop step response? [6]

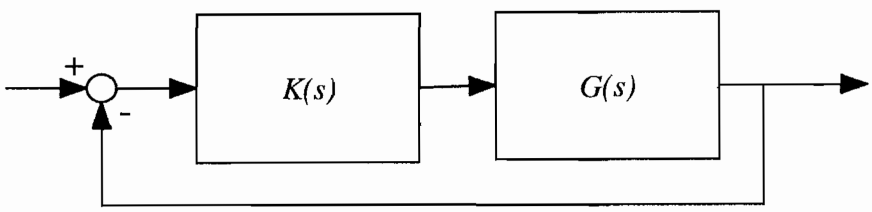

Fig. 1

(cont.

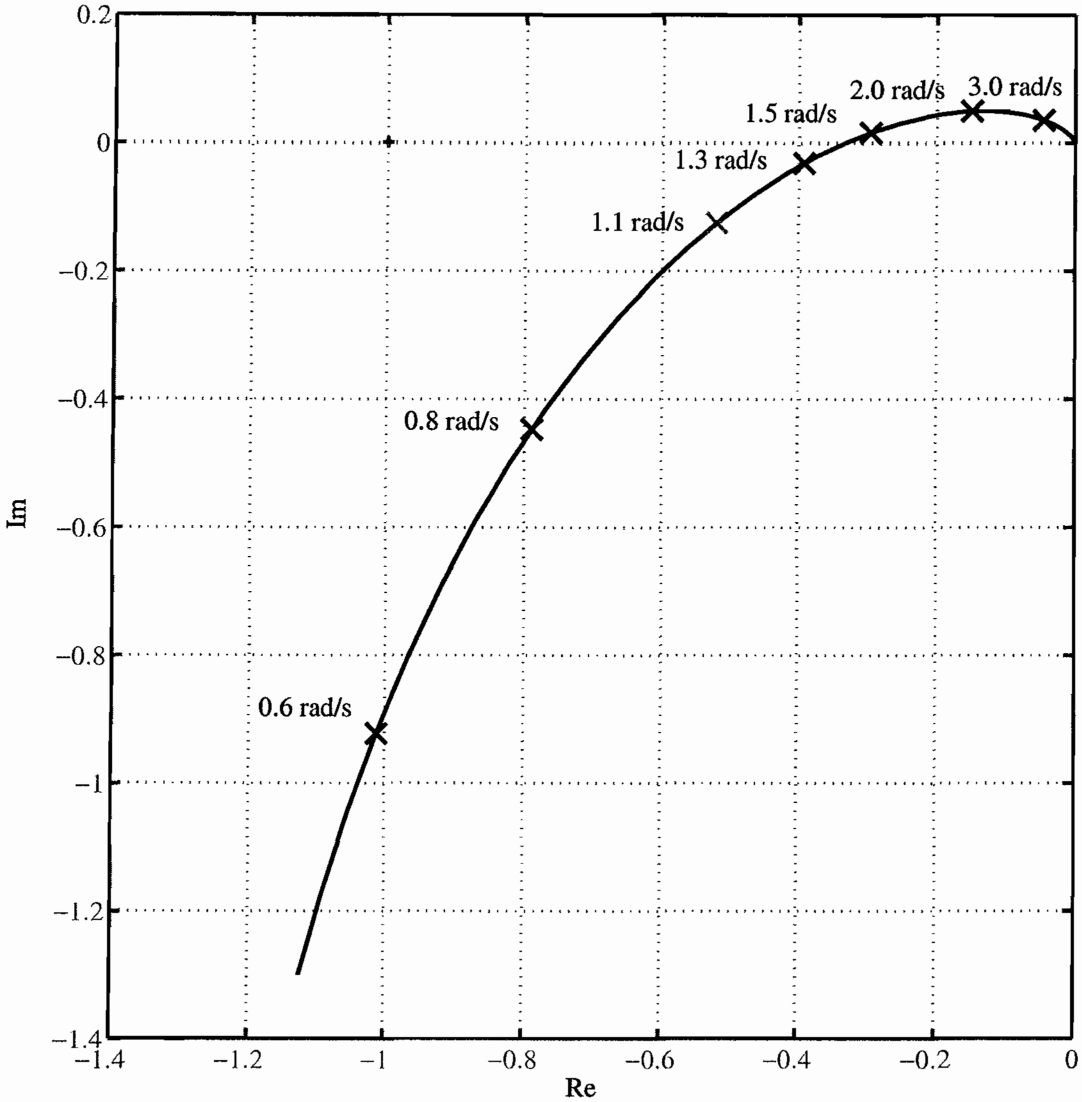

Fig. 2

Note: an additional copy of Fig. 2 is supplied at the end of this paper. This should be annotated with your constructions and handed in with your answer to this question.

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2 Figure 3 shows a feedback control system in which

$$G(s) = \frac{20}{s(s^2+6s+20)}$$

and Fig. 4 shows the magnitude of the frequency response of $G(s)$. The control system is to be designed such that the steady-state response to disturbances is zero and the frequency range over which the gain of the closed-loop system exceeds 0.5 should extend to at least 7 radians/sec. You may assume that the closed loop system is always stable.

(a) Derive an expression for $\bar{y}(s)$ as a function of the reference input $\bar{r}(s)$ and the disturbance $\bar{d}(s)$. [4]

(b) For $K(s) = 2$, find the steady state response of $y(t)$ when:

(i) $r(t) = \cos(7t)$ and $d(t) = 0$;

(ii) $r(t) = 0$ and $d(t) = H(t)$;

(iii) $r(t) = \cos(7t)$ and $d(t) = H(t)$,

where $H(t)$ is the unit step function. [5]

(c) The proportional controller is now extended by adding integral and derivative terms such that $K(s) = 2 + 1/s + 3s$. By expressing $K(s)$ as the product of a quadratic and the term $1/s$, draw the magnitude response of $K(s)G(s)$ on the additional copy of Fig. 4. [6]

(d) Verify that the control system now meets the design requirements. [5]

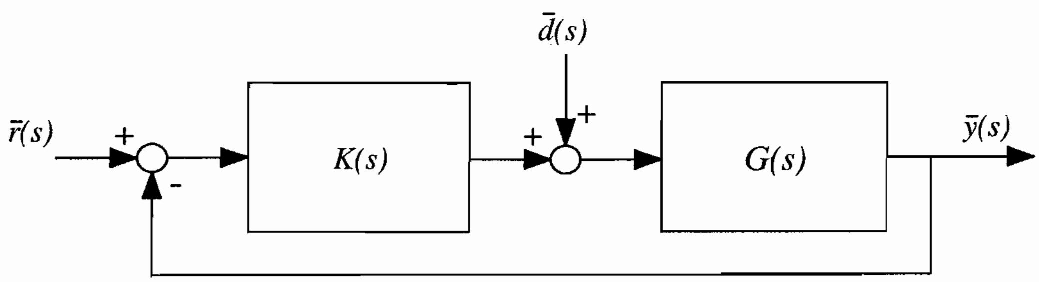

Fig. 3

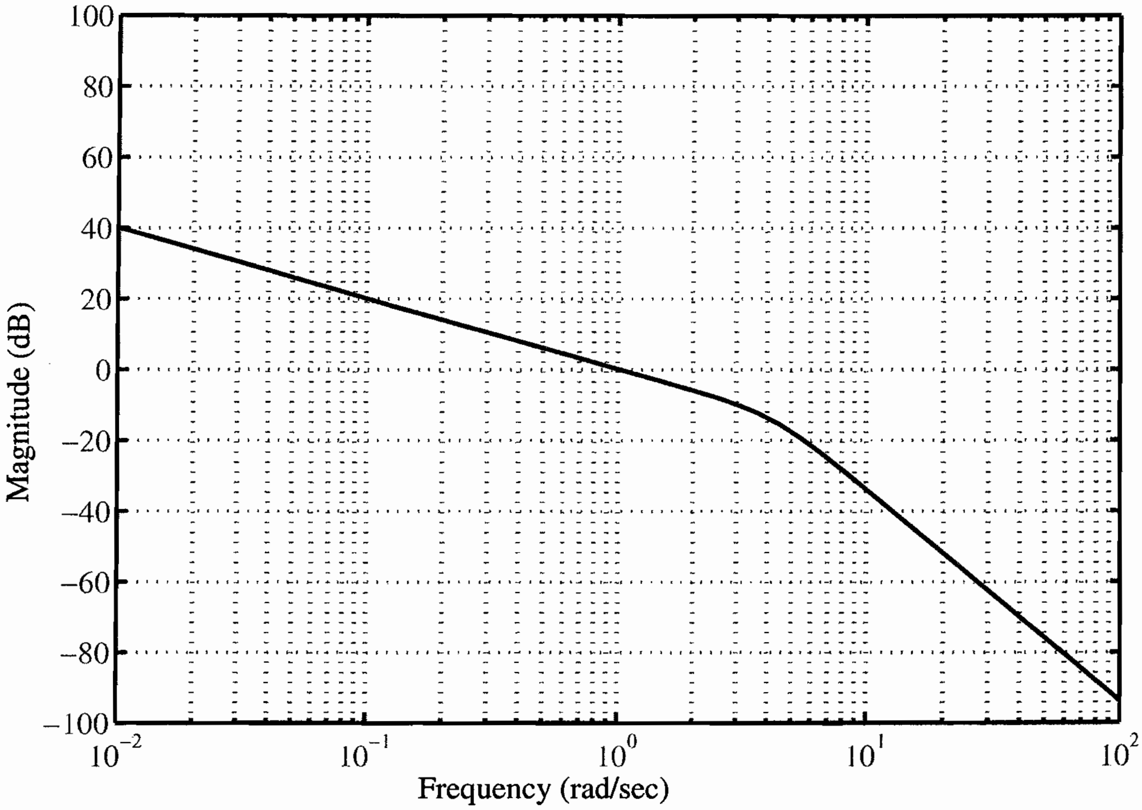

Fig. 4

Note: an additional copy of Fig. 4 is supplied at the end of this paper. This should be annotated with your constructions and handed in with your answer to this question.

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3 (a) For any system with a proper rational transfer function, state the conditions in terms of pole positions for the system to be unstable. [4]

(b) The feedback control system shown in Fig. 5 has

$$G(s) = \frac{1}{(s-1)(s+3)}\,.$$

If $K(s) = k$, show that the poles of the closed-loop system are given by

$$s = -1 \pm \sqrt{4-k}\,.$$

Hence, determine the range of k for which the feedback system is stable. [6]

(c) When $k = 20$, compute the damping factor of the closed-loop system and hence sketch the system step response. Your sketch should show the peak overshoot and the approximate time at which the peak occurs. [5]

(d) Show that adding a derivative term to the controller so that $K(s) = k + k_d s$ can increase damping. Using the same value $k = 20$, find the value of k_d required to give a damping factor of 0.5. [5]

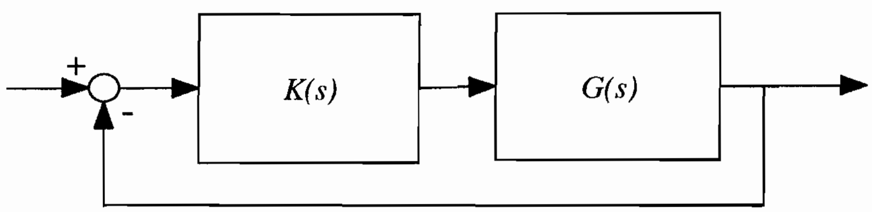


Fig. 5

SECTION B

*Answer not more than **two** questions from this section.*

4 The signal $x(t)$ shown in Fig. 6 is a periodic zero-mean triangular waveform with repetition frequency 1 kHz and peak amplitude 1 volt.

(a) The signal $x(t)$ is low-pass filtered with cut-off frequency 10 kHz and sampled at 20 kHz. Sketch the resulting spectrum. [5]

(b) Show that the rms noise voltage introduced by an ideal uniform quantiser with a step size δV is given by $\delta V / \sqrt{12}$. [5]

(c) If the quantiser is 8-bit and the range is $\pm$ 5 volts, calculate the signal-to-noise power ratio (SNR) of the quantised signal in decibels. How could this SNR be improved without increasing the number of quantisation levels? [5]

(d) Calculate the bit rate of the signal sampled in part (a) and quantised in part (c). Determine the minimum bandwidth that a communications channel should have to reliably support this bit rate at an SNR of 5dB. [5]

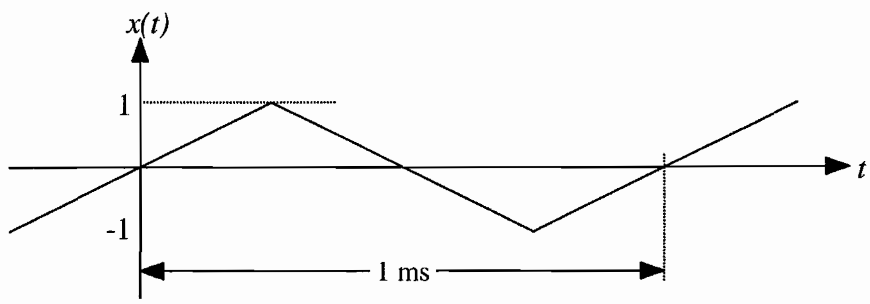


Fig. 6

5 (a) With regard to amplitude modulation schemes, explain what is meant by *single side-band (SSB)* and *double side-band (DSB)* operation and briefly discuss their relative merits. [4]

(b) By considering the case of $x(t) = \cos(\omega t)$, show that simple multiplication of $x(t)$ by a carrier wave $\cos(\omega_c t)$ where $\omega_c \gg \omega$ leads to double side-bands. [4]

(c) By considering the transmission of speech with a bandwidth of 300 Hz to 3 kHz, explain why it is not practical to obtain SSB modulation by directly filtering $x(t)\cos(\omega_c t)$ to suppress one of the side-bands. [4]

(d) Fig. 7 shows a block diagram of an alternative amplitude modulation scheme. The modulation signals are $m_1(t) = 2\cos(Bt)$ and $m_2(t) = \cos((\omega_c - B)t)$, where B is the cut-off frequency of the lowpass filter, and ω_c is the carrier frequency. If $x(t) = \cos(\omega t)$ and $\omega < B \ll \omega_c$, derive an expression for the output $y(t)$ and hence demonstrate that this is an SSB scheme. [8]

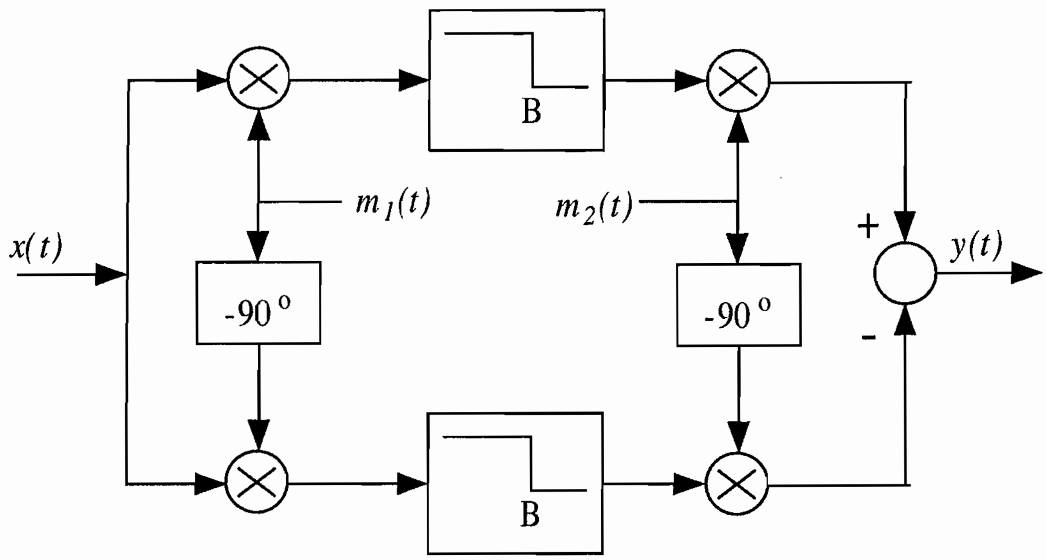


Fig. 7

6 (a) (i) Show that if $x(t) \rightleftharpoons X(\omega)$ where $X(\omega)$ is the Fourier Transform of $x(t)$, then

$$x(t-t_o) \rightleftharpoons e^{-\mathrm{j}\omega t_o} X(\omega)\,.$$

[3]

(ii) Show that the Fourier Transform of $\sin(\omega_o t)$ is

$$\frac{\pi}{\mathrm{j}}[\delta(\omega-\omega_o)-\delta(\omega+\omega_o)]$$

and find the Fourier Transform of $\sin(\omega_o t+\phi)$. [5]

(b) (i) The signal $x(t)=\sin(2\pi f_o t)$ is sampled at times $t=0$ ms, $t=0.25$ ms, $t=0.5$ ms and $t=0.75$ ms to produce the sequence $x_n=\{0,1,0,-1\}$. What is the sampling frequency f_s and what are the possible values of f_o for $f_o>0$? [5]

(ii) Assuming that $f_o \leq \frac{1}{2} f_s$, calculate the discrete Fourier Transform (DFT) of x_n. To what frequencies do the individual terms in the DFT correspond? [5]

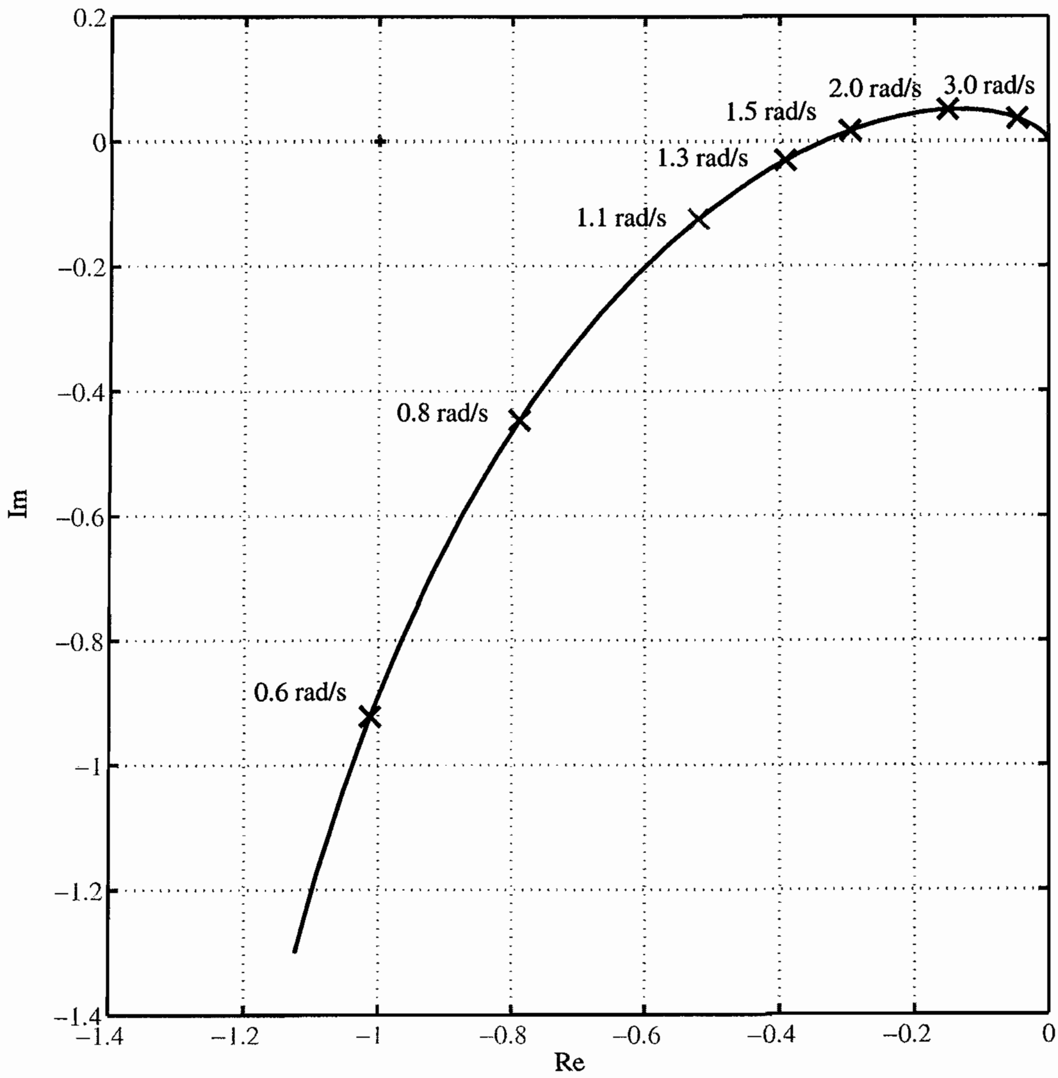
(iii) Explain why there is more than one non-zero term in the DFT even though $x(t)$ is a pure sinusoid. [2]

END OF PAPER

ENGINEERING TRIPOS PART IB, PAPER 6, 7 June 2007

Candidate Number:

This copy of Fig. 2 should be annotated with your constructions and handed in with your answer to question 1.



ENGINEERING TRIPOS PART IB, PAPER 6, 7 June 2007

Candidate Number:

This copy of Fig. 4 should be annotated with your constructions and handed in with your answer to question 2.