

ENGINEERING TRIPOS PART IB

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Friday 8 June 2007 2 to 4.30  
2 to 3.30 Foreign Language Option

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Paper 8

SELECTED TOPICS

*Answer one question from Section A.*

*If you are not taking the Foreign Language option, answer four questions, taken from only two of Sections B-H. Not more than two questions from each section may be answered.*

*If you are taking the Foreign Language option, answer two questions from one of Sections B-H.*

*All questions carry the same number of marks.*

*The approximate number of marks allocated to each part of a question is indicated in the right margin.*

*Answers to questions in each section should be tied together and handed in separately.*

Section A (Business Economics)	2
Section B (Civil and Structural Engineering)	3
Section C (Mechanics, Materials and Design)	7
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Attachments: Data Sheet for Section B (6 pages)	
Data Sheet for Section E (2 pages)	
Additional copy of Fig. 3	

<p><b>You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator</b></p>
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SECTION A *Business Economics*

*Answer not more than one question from this section.*

- 1 (a) Under what circumstances would you expect a monopoly to charge a higher price than it would if the industry was operating under perfect competition? [5]
- (b) Under what circumstances might monopolies provide economic and social benefits? [5]
- (c) Using the Keynesian Consumption Function model, explain the potential impact of cutting income taxes on the level of aggregate demand. [5]
- (d) Using either the Life Cycle model or the Permanent Income model, explain the potential impact of cutting income taxes on the level of aggregate demand. [5]
- 2 (a) Describe the kinked demand curve theory. [6]
- (b) What is the principal-agent problem? Describe one example of the problem. [5]
- (c) What impact would the following have on the level of investment in the macroeconomy:
- (i) a decrease in the rate of interest; [3]
- (ii) an expected fall in future consumption; [3]
- (iii) a reduction in the real exchange rate? [3]

SECTION B *Civil and Structural Engineering*

*Answer not more than two questions from this section.*

*You may refer to the data sheet at the end of the examination paper.*

3 (a) A deep excavation is to be undertaken in layered ground, with alternating clay and sand layers. The ground water level is close to the ground surface. The sides of the excavation are to be supported by vertical walls installed before any ground is excavated.

(i) Describe two alternative types of wall that might be used to support the excavation sides and how each type would be constructed. [4]

(ii) Describe two different types of problems that potentially could be encountered with water during excavation. [4]

Your answers should be illustrated with sketches.

(b) A tunnel for a metro in a city is to be constructed at a depth of 25 m below ground level. Over the first part of its length the tunnel will be in a stiff clay with an undrained shear strength  $S_u = 250 \text{ kN m}^{-2}$  and unit weight  $\gamma = 20 \text{ kN m}^{-3}$ . Over the second part the tunnel will be in a soft clay with an undrained shear strength  $S_u = 50 \text{ kN m}^{-2}$  and unit weight  $\gamma = 17 \text{ kN m}^{-3}$ .

(i) Describe the implications for attempting to construct the tunnel with an open face for both parts, by reference to the stability ratio for each case. [4]

(ii) If the part of the tunnel in the soft clay is to be constructed with a closed-face tunnelling machine, what would be the face pressure required to maintain a stability ratio of at least 5? [4]

(c) Sketch the shape of a settlement trough at the ground surface associated with construction of a tunnel in soft ground. Explain why a masonry building directly above the tunnel centreline may not experience as much damage as a building off-set from the centreline. [4]

(TURN OVER

4 A tunnel structure has been constructed in a shallow river estuary, as shown in Fig. 1. The tunnel is sufficiently heavy to resist uplift. The lowest 3 m of the tunnel is in the stiff clay of the river bed, for which the unit weight  $\gamma = 20 \text{ kN m}^{-3}$ , the undrained shear strength  $S_u = 70 \text{ kN m}^{-2}$  and the critical state angle of friction is  $\phi' = 20^\circ$ . The depth of water in the river estuary is 2 m.

In the short term, immediately after construction, one side of the tunnel is backfilled with a loose silty sand to the top of the structure. The sand has unit weight  $\gamma = 19 \text{ kN m}^{-3}$  below the water, and  $\gamma = 16 \text{ kN m}^{-3}$  above the water. The critical state angle of friction of the sand is  $\phi' = 30^\circ$ . A surcharge of  $60 \text{ kN m}^{-2}$  is applied at the surface of the backfill.

It should be assumed that the tunnel moves sufficiently to mobilise active pressures on the backfilled side and passive pressures on the other side (the river side), and that its walls are smooth.

- (a) Calculate the total horizontal stresses acting in the short term on each side of the tunnel structure and sketch the stress distribution as a function of depth. [8]
- (b) Ignoring any friction on the base of the tunnel, calculate the factor of safety against sliding in the short term. [4]
- (c) Soon after the tunnel is completed, sand seams in the clay on the backfilled side cause it to soften rapidly such that long-term conditions are reached. Calculate the total and effective horizontal stresses on the backfilled side. [5]
- (d) Assuming that undrained conditions remain in the clay on the river side of the tunnel, estimate the reduction in factor of safety against sliding due to the softening of the clay on the backfilled side. [3]

(cont.)

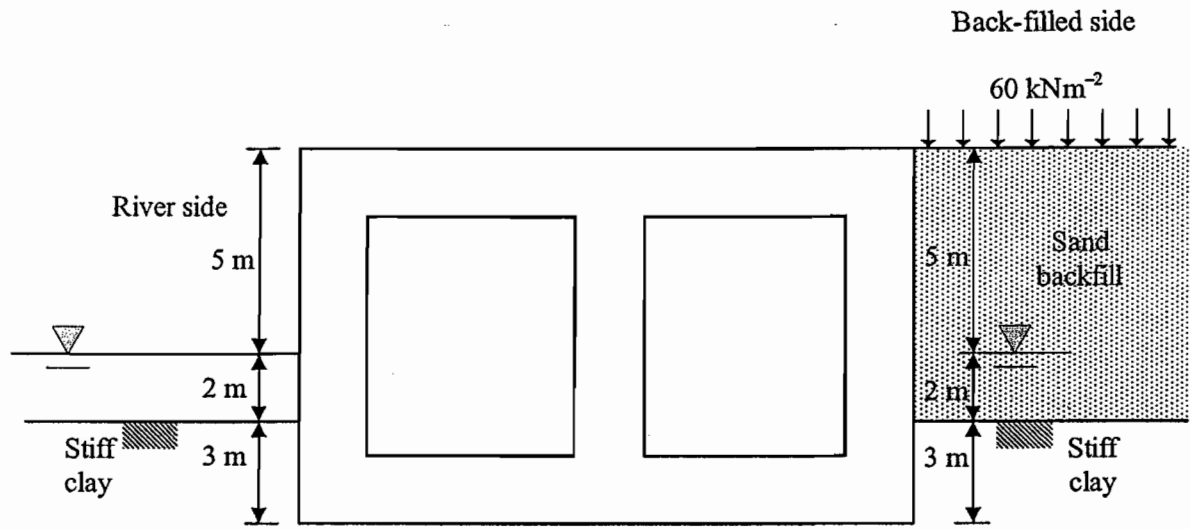


Fig. 1

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5 A reinforced concrete beam 250 mm wide with an effective depth of 500 mm, is continuous over two spans with the ends simply supported, as shown in Fig. 2. Each span is 10 m long. It is loaded with a uniformly distributed load of  $40 \text{ kN m}^{-1}$  (which includes the beam's own dead weight). A structural analysis shows that the reaction at the central support is 500 kN. All loads include appropriate partial factors of safety. The reinforcing steel has a characteristic yield strength of  $460 \text{ MN m}^{-2}$  and the concrete has a characteristic cube strength of  $40 \text{ MN m}^{-2}$ .

(a) Draw the shear force and bending moment diagrams for the beam and find the value and location of the maximum bending moments in both sagging and hogging. [3]

(b) Show that the beam can be singly-reinforced for sagging bending but must be doubly-reinforced for hogging bending. [3]

(c) Determine a suitable reinforcement layout at the locations of maximum hogging and sagging moment. [7]

(d) Find the location of the maximum shear force and design suitable shear reinforcement at that location. [4]

(e) Choose suitable overall cross-section dimensions, and choose the layout of the flexural-reinforcing bars throughout the beam. Produce a sketch showing how this steel, and the shear steel, will be laid out. [3]

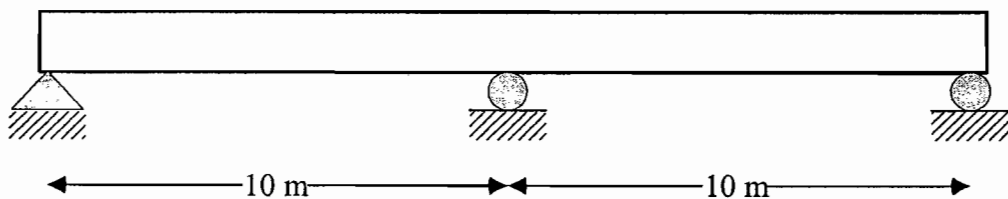


Fig. 2

SECTION C *Mechanics, Materials and Design*

*Answer not more than two questions from this section.*

6 A horizontal-axis wind turbine of swept area  $A$  is operating in undisturbed air of density  $\rho$  moving at a uniform speed  $V$ . The Betz limit for such a turbine is 59%.

- (a) Define the meaning of the Betz limit. [2]
- (b) Derive the Betz limit from first principles. Be sure to sketch a clear diagram showing the ideal flow around the wind turbine and on it identify the control volume for which you consider continuity, energy and momentum flux. [10]
- (c) Obtain an expression for the horizontal force acting on the turbine when it is operating at the Betz limit. You may assume that the speed of air passing through the turbine drops from  $V$  to  $V/3$ . [4]
- (d) Discuss briefly, with sketches, circumstances in which the measured efficiency might exceed the Betz limit. [4]

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7 (a) Discuss what factors need to be considered when designing turbine blades against fatigue failure. [7]

(b) Figure 3 gives a representative loading cycle for a critical part of a blade. The values of the stress at the peaks and troughs are indicated on the figure.

(i) Use the rainflow counting method to break down this cycle into a series of individual cycles of loading and tabulate the corresponding peak-to-peak stress ranges and mean stresses. An additional copy of Fig. 3 is provided and should be handed in with your answer. [5]

(ii) Hence estimate the life of this part of the blade, given that the peak-to-peak stress range  $S$  (in MPa) and the number of cycles to failure  $N$  for the material are related by

$$N = \left( \frac{300}{S} \right)^8$$

for cyclic loading with zero mean stress. You should include the effect of mean stress in your estimate and assume that the loading pattern given in Fig. 3 is repeated until the component fails. [5]

(iii) The load sequence in Fig. 3 is very short to make it amenable to hand calculation. In practice, how might you modify the design procedure for practical loading cycles to take advantage of computational power. [3]

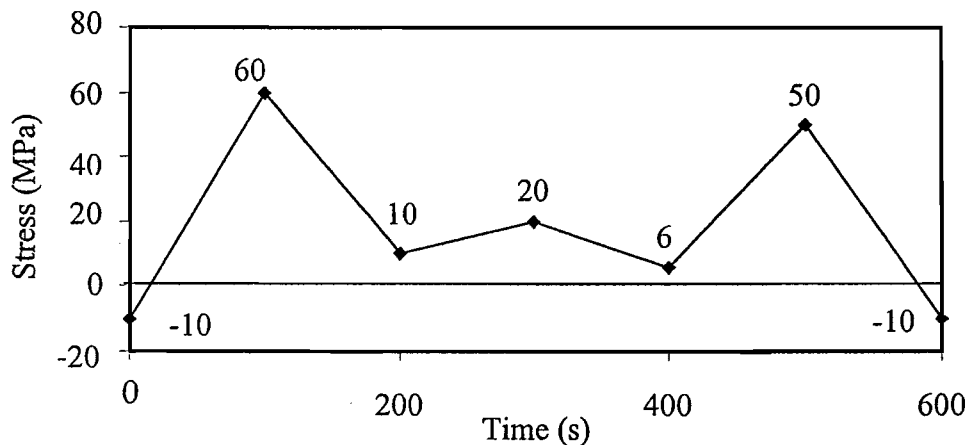


Fig. 3



8 (a) Compare wind turbines for electrical power generation in the UK with one other renewable energy source, highlighting specific advantages and disadvantages of each. [10]

(b) Explain what is meant by “streamlined life cycle analysis (LCA)” of an engineering product, and summarise its advantages over conventional LCA. Define the “energy-payback period” for a renewable-energy installation, illustrating your answer with respect to the output from a streamlined LCA of an onshore wind turbine. Discuss the likely changes in the relative importance within the overall life-cycle analysis of the different parts of a wind-turbine structure for the following changes in design:

- (i) increasing the machine power rating by 50%;
- (ii) using the original machine offshore.

[10]

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SECTION D *Aerothermal Engineering*

*Answer not more than two questions from this section.*

9 A new type of passenger aircraft is designed to cruise at a Mach number of 0.80 and has a wing area of  $750 \text{ m}^2$ . The aircraft total mass at the beginning of cruise is 166 tonnes and the variation of lift-to-drag ratio  $L/D$  at the design Mach number can be expressed as a function of lift coefficient  $C_L$  by the following relationship

$$L/D = -312C_L^2 + 178C_L$$

(a) Using the information in Table 1, find the initial cruising altitude required to give the maximum lift-to-drag ratio. [6]

(b) The distance,  $s$ , between start and end of cruise is given by Breguet's range equation, which can be written as

$$s = - \frac{V L/D}{g sfc} \ln \left( \frac{W_{end}}{W_{start}} \right)$$

where  $V$  is the flight speed,  $sfc$  is the thrust specific fuel consumption,  $g$  is the acceleration due to gravity and  $W_{start}$  and  $W_{end}$  are the aircraft weights at the beginning and end of cruise respectively. Show that the above equation can be rearranged to give the following equation for the fuel burn per payload-range:

$$\frac{W_f}{sW_p} = \frac{1}{s} \left( 1 + \frac{W_e}{W_p} \right) [\exp(s/H) - 1]$$

where  $H = \frac{V L/D}{g sfc}$ ,  $W_e$  is the empty aircraft weight (including reserve fuel),  $W_f$  is the fuel weight burned during cruise, and  $W_p$  is the payload weight. [4]

(cont.)

(c) The aircraft described above has an empty weight of 110 tonnes and a design range of 8000 km. If the engine *sfc* is  $0.016 \times 10^{-3} \text{ kg s}^{-1}\text{N}^{-1}$ , estimate the maximum number of passengers that can be carried without increasing the aircraft total mass at the beginning of cruise. What is the corresponding fuel burned per passenger-km?

It can be assumed that the average passenger mass is 100 kg (including baggage).

[6]

(d) Explain how the following changes would impact the fuel burn per passenger-km of the aircraft. Assume that the aircraft range and the number of passengers are kept constant.

- (i) Reducing the aircraft empty weight by extensive use of composite materials.
- (ii) Improving the engine design so that the *sfc* reduces by 5%.
- (iii) Limiting the aircraft cruising altitude to 39000 feet to satisfy air traffic control requirements.

[4]

Altitude (feet)	Temperature (K)	Pressure (kPa)	Density ( $\text{kg m}^{-3}$ )
31000	226.73	28.7	0.442
33000	222.82	26.0	0.410
35000	218.81	23.8	0.380
37000	216.65	21.4	0.344
39000	216.65	19.7	0.316
41000	216.65	17.9	0.287
43000	216.65	16.2	0.261

Table 1: ICAO Standard atmosphere

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10 (a) Using the station numbering shown in the schematic diagram in Fig. 4, sketch  $p$ - $v$  and  $T$ - $s$  diagrams for a simple gas turbine. Assume that the flow velocities are negligible throughout the cycle. Indicate on the  $T$ - $s$  diagram the impact of non-isentropic compression and expansion. [4]

(b) Show that the cycle efficiency  $\eta_{cycle}$  for the simple gas turbine can be expressed in terms of the cycle temperatures,  $T_2$ ,  $T_3$ ,  $T_4$ ,  $T_5$  as follows

$$\eta_{cycle} = \frac{W_{net}}{Q} = 1 - \frac{T_5/T_2 - 1}{T_4/T_2 - T_3/T_2}$$

where  $W_{net}$  is the net work output and  $Q$  is the heat input. With reference to the above expression and the  $T$ - $s$  diagram, explain how the following factors influence the cycle efficiency:

- (i) turbine entry temperature,  $T_4$ ;
- (ii) overall pressure ratio,  $p_3/p_2$ ;
- (iii) compressor isentropic efficiency,  $\eta_{comp}$ ;
- (iv) turbine isentropic efficiency,  $\eta_{turb}$ .

Identify what limits the values of each of these parameters for a modern gas turbine. [8]

(c) A turbojet engine is tested on a static test bed on two different days. The first test is performed during summer with an ambient temperature of 300 K and an atmospheric pressure of 98.50 kPa. The second test is during winter with an ambient temperature of 267 K and an atmospheric pressure of 102.44 kPa. In both tests the engine is operated at the same non-dimensional operating condition with the exhaust nozzle choked.

In the first test a thrust of 50 kN is measured.

- (i) Determine the thrust in the second test. Also calculate the ratio of mass flow rate of air through the engine in the summer test to the mass flow rate in the winter test. [4]

(cont.)

(ii) Write down two further non-dimensional expressions for the engine: one involving the rotational shaft speed  $N$  and one involving the turbine entry stagnation temperature  $T_{04}$ . Use these to identify which of the two tests would result in greater mechanical wear and possible damage to the engine components. [4]

You may assume that suitable non-dimensional expressions for the air mass flow rate through an engine and the gross thrust produced are given by

$$\frac{\dot{m}\sqrt{c_p T_{02}}}{p_{02} A_N} \quad \text{and} \quad \frac{F_G + p_a A_N}{p_{02} A_N}$$

where all the terms have their usual meaning, and the subscript  $N$  refers to the exhaust nozzle.

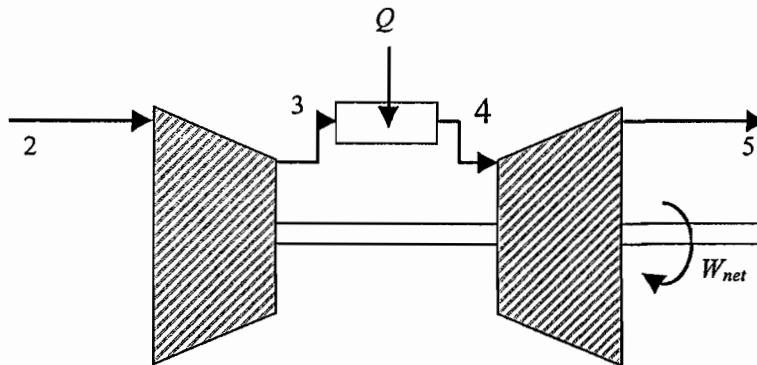


Fig. 4 Schematic of a simple gas turbine

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11 (a) Explain why modern jet engines used for passenger aircraft tend to have high bypass ratios. What factors limit the maximum bypass ratio that can be chosen for a turbofan engine design? [4]

(b) A turbofan engine generates 25 kN of net thrust whilst powering an aircraft cruising at a velocity  $V$  of  $250 \text{ m s}^{-1}$ . If the core and bypass jet velocities are both  $360 \text{ m s}^{-1}$ , and the engine bypass ratio is 12, determine the engine propulsive efficiency and find the mass flow rate through the core. [3]

(c) At cruise, the inlet stagnation temperature is 250 K and the inlet stagnation pressure is 38 kPa. Given that the engine fan has a hub-to-tip radius ratio of 0.3 and the mass flow per unit annulus area is 90% of that required for choking, calculate the fan diameter required.

Note that for choking, the non-dimensional group,  $\frac{\dot{m}\sqrt{c_p T_0}}{Ap_0} = 1.281$

where  $T_0$  is the stagnation temperature,  $p_0$  is the inlet stagnation pressure and  $c_p$  is the specific heat capacity of air at constant pressure. [3]

(d) The low-pressure turbine for the engine has an inlet stagnation temperature of 1200 K and an inlet stagnation pressure of 280 kPa. Calculate the work extracted from the turbine per kg of core mass flow. If the shaft rotational speed is 3000 rpm and the turbine mean radius is 0.55 m calculate the number of low-pressure turbine stages required for the work coefficient,  $\Delta h_0 / U^2$ , not to exceed 3.0 for any stage.

It can be assumed that the fan efficiency is given by  $\eta_{fan} = \frac{V_{jb}^2 - V^2}{2c_p \Delta T_0^{FAN}} = 0.90$

where  $\Delta T_0^{FAN}$  is the stagnation temperature rise across the fan and  $V_{jb}$  is the bypass jet velocity. [6]

(e) To reduce the engine noise, a modified design is proposed in which the fan diameter is increased by 10% but the mass flow rate through the core is unchanged. Calculate the new bypass ratio of the engine. If the thrust required at cruise is still 25 kN, estimate the new propulsive efficiency of the engine. Assume that both the hub-to-tip ratio of the fan and the mass flow per unit annulus area are unchanged. The core jet velocity is equal to the bypass jet velocity in both designs. [4]

SECTION E *Electrical Engineering*

*Answer not more than two questions from this section.*

*You may refer to the data sheet at the end of the examination paper.*

- 12 (a) With the aid of diagrams explain the operation of a passive matrix addressed liquid crystal display (LCD), including a description of the 3:1 addressing scheme commonly adopted. [7]
- (b) Compare the operation of the passive addressed LCD of (a) with that of an active matrix addressed LCD. [6]
- (c) Compare and contrast backlight technologies that can be used in LCDs including comments upon lifetime, power consumption and colour. Name one possible alternative flat screen technology which can compete with active matrix LCD for televisions of >40 inch diagonal. [7]
- 13 A MESFET with a transit time of 20 ps is to be designed using a 1  $\mu\text{m}$  thick layer of n-type silicon on an insulating substrate. The supply voltage is 2 V. The silicon has a conductivity of  $32 \Omega^{-1}\text{m}^{-1}$ , a scattering limited velocity of  $10^5 \text{ m s}^{-1}$  and a mobility of  $0.1 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ .
- (a) Determine the donor density in the silicon. [2]
- (b) Determine the electric field and source-drain field separation for the above transit time. [5]
- (c) The ratio of the width to length of the MESFET channel is 20. Determine the source current which flows for a gate-source voltage of zero. [6]
- (d) Determine the gate voltage required to turn off the transistor, assuming the dielectric constant of silicon is 12. [7]

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14 (a) Copper has a face-centred cubic lattice, with a lattice constant of  $3.61\text{\AA}$  and 4 atoms per unit cell. It has effectively 1 valence electron per atom and its conductivity is  $6 \times 10^7 \Omega^{-1} \text{ m}^{-1}$ . The electrons in a semiconductor have the same mobility as in copper. What is the conductivity of this semiconductor if it is doped to a donor concentration of  $4 \times 10^{21} \text{ m}^{-3}$  ? [6]

(b) Sketch the velocity-field diagrams of electrons in Si and GaAs. Explain the meaning and cause of the scattering limited velocity. If the scattering limited velocity of the doped semiconductor in (a) is  $5 \times 10^4 \text{ m s}^{-1}$ , what is the relevant phonon energy in electron volts? The free electron mass is  $9.1 \times 10^{-31} \text{ kg}$ . [6]

(c) If the doped semiconductor in (a) is used to make a FET operating at 2 V, with a channel with a width to length ratio of 50, at what length does the conduction become limited by the scattering limited velocity? What transit time does this correspond to? [8]



SECTION F *Information Engineering*

*Answer not more than two questions from this section.*

15 A grey level image,  $I(x, y)$ , is to be smoothed by an isotropic low-pass filter by convolution with a discrete approximation to the 2D Gaussian kernel,  $G_\sigma(x, y)$  of size  $(2n + 1) \times (2n + 1)$  pixels.

- (a) What is the effect on the image of filtering with a Gaussian low-pass filter and when would this be useful in image processing? [4]
- (b) Give an expression for computing the intensity of a smoothed pixel,  $S(x, y)$ . [4]
- (c) Show how the convolution can be performed by two discrete 1D convolutions along image rows and columns and comment on the computational saving this achieves. [4]
- (d) Determine the filter size and coefficients of the 1D Gaussian for  $\sigma = 1$ . State clearly any assumptions and approximations used. [4]
- (e) Show how an isotropic high-pass filter can be implemented efficiently from the low-pass filter described above. When are high-pass filters used in image processing? [4]

(TURN OVER

16 Local interest points in images and their descriptors are often used in matching image features and object detection.

(a) Describe an algorithm that localises an interest point in an image and is able to assign an appropriate scale. [10]

(b) The pixels in a  $16 \times 16$  patch extracted at an appropriate scale around an interest point can be used to describe the interest point for matching over different viewpoints and lighting. Describe the following descriptors and compare their advantages and disadvantages:

(i) the intensity patch with normalisation;

(ii) the response to a bank of filters tuned to different scales and orientations;

(iii) the histogram of gradient orientations;

(iv) the Scale-Invariant Feature Transform. [10]

17 Consider a set of  $N$  images  $S = \{\mathbf{x}_1, \dots, \mathbf{x}_N\}$  where each image is represented as a vector of  $M$  binary features, e.g.  $\mathbf{x}_n = (x_{n1}, \dots, x_{nM})$  and  $x_{nm} \in \{0,1\}$ .

(a) Given a new image  $\mathbf{x}_{N+1}$ , describe an algorithm and the relevant equations for finding the *nearest neighbour* to  $\mathbf{x}_{N+1}$  in the set  $S$ , that is, the closest image to  $\mathbf{x}_{N+1}$ . Make sure you specify the distance metric you propose to use. [6]

(b) Assume a probabilistic model of images where each feature is independent, and each feature  $x_{nm}$  has probability  $p_m$  of taking on the value 1,

$$P(x_{nm} = 1) = p_m \quad \text{for all } n, m.$$

Describe an algorithm and the relevant equations for finding the least probable image in  $S$  under this probabilistic model. Discuss how this can be used for outlier removal. [6]

(c) Assume you have two probabilistic models of images, one for images of "sunsets", and the other for images of "mountains", where

$$P(x_{nm} = 1 | \mathbf{x}_n \text{ is a "sunset"}) = p_m$$

and

$$P(x_{nm} = 1 | \mathbf{x}_n \text{ is a "mountain"}) = q_m .$$

Based on these two models, devise and describe a procedure for finding the top 10 features that best discriminate between mountain and sunset images. If the features encode colour and texture properties of images, what do you expect the most discriminative features will be? [8]

(TURN OVER)

SECTION G *Engineering for the Life Sciences*

*Answer not more than two questions from this section.*

18 (a) Define the following terms in the context of reinforcement learning:

- (i) reward vs. value function;
- (ii) policy;
- (iii) discount factor.

[4]

(b) Two subjects reach to a target twice. Subject A misses both times by 2 cm whereas subject B misses first by 1 cm and then by 3 cm.

(i) Which subject (if either) has the lower total loss if the loss function rises:

- A. quadratically with error;
- B. linearly with error;
- C. as the square root of error.

(ii) Why might making slower movements increase the accuracy of both subjects?

(iii) The human arm is highly redundant in that there are many ways in which we can reach a target. Why might having this redundancy allow more accurate movements?

[8]

(c) Answer the following in the context of human sensorimotor control.

(i) What is a forward dynamic model?

(ii) What is an inverse dynamic model?

(iii) Which of forward or inverse dynamic models is generally easier to learn and why?

(iv) What evidence is there that the brain has a forward model?

[8]

- 19 (a) Describe briefly the “modified random walk” strategy by which bacteria such as *Salmonella typhimurium* (*S. typhus*) and *Escherichia coli* (*E coli*) see nutrient molecules in their aqueous environment. [4]
- (b) Discuss the various circumstances in which a bacterial flagellar filament can switch between members of a family of twelve discrete helical waveforms. What is the structural basis of these different forms? [4]
- (c) Describe briefly how the protein devices known as bacterial *receptors* work. How may they be “re-engineered” to recognise different molecules, such as TNT (trinitrotoluene)? [4]
- (d) Biological organisms have a common scheme of growth by cell-division, in which each “daughter” cell contains the full DNA instructions for constructing the entire organism. Explain briefly how cells can “specialise” in order to build the different parts of multi-cellular organisms. [4]
- (e) Describe the principle of Darwin’s “tree of life”. What is the range of organisms included in this tree? How may the tree be mapped in detail? [4]

(TURN OVER

20 (a) A titanium implant, shown in Fig. 5, is bonded to cortical bone with a thin layer of cement and strained in the direction of the arrows.  $E_{\text{Titanium}} = 7E_{\text{bone}}$  and  $A_{\text{Titanium}} = A_{\text{bone}}$  where  $E$  is the elastic modulus of each material and  $A$  is the cross sectional area transverse to the loading direction.

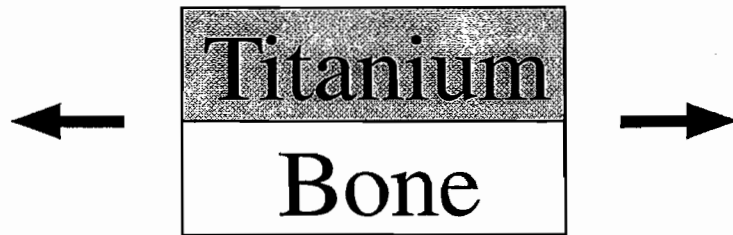


Fig. 5

- (i) What proportion of the force is carried in the titanium implant? [4]
- (ii) What long-term effect could this load sharing have on the bone? [1]

(b) Two bioresorbable polymers, both having an elastic modulus of 10 MPa, degrade at different volume loss rates, one slowly at  $10 \text{ cm}^3$  per day and one fast at  $100 \text{ cm}^3$  per day. A  $1000 \text{ cm}^3$  implant is made in each knee with the slower degrading material on one side and the faster degrading material on the other. As the implants degrade the implant volume is filled with replacement tissue at a rate of  $10 \text{ cm}^3$  per day. The elastic modulus of fully dense tissue is 5 MPa.

- (i) Sketch the variation of elastic modulus with of time for both polymer-tissue systems assuming that the two materials are loaded in parallel. [3]
- (ii) Which polymer would you choose for a load-bearing joint prosthesis and why? [2]

(cont.)

(c) Geckos are the largest animals capable of running upside down on a smooth substrate and they possess the finest known adhesive hair endings. It has been proposed that the strength of adhesion increases for finer adhesive hairs.

(i) The adhesive force  $f$  of a flat-ended cylindrical seta (radius  $r$ ) was found to be proportional to  $r^{1.5}$ . It is assumed that setae occupy a constant proportion of the pad area and contribute equally to the total force of an adhesive pad. If setae are constructed so that an animal can sustain its own body weight, what is the expected scaling of hair density  $N_A$  with body mass  $m$ ? [7]

(ii) What other factors may explain the fact that geckos have very fine adhesive hairs? [3]

(TURN OVER

SECTION H *Manufacturing, Management and Design*

*Answer not more than two questions from this section*

21 (a) You have created a novel technology that enables pictures and posters to be attached to a wall, without leaving any marks on the wall. Describe two different ways in which you could define and segment the potential market for this technology. In each case, identify the types of potential customers for this technology. [10]

(b) Before doing further product development, you wish to understand better the customer needs for this new product. Outline a plan for surveying customer needs, focusing specifically on from whom you would gather data and the methods you would use. Explain why these methods are appropriate. [6]

(c) “Personas” can be useful in the design process. Explain this concept, and comment on the strengths and weaknesses of this design tool. [4]

22 (a) Describe the differences between debt funding and equity funding. Your answer should include reference to the uses and sources of each type of funding. What are the challenges for a start-up company seeking to access each type of funding? [8]

(b) Instead of trying to raise funding, a start-up company may choose to form partnerships with other companies to access the resources needed to grow its business.

(i) List the main types of organizational structures that could be used for such partnerships.

(ii) Describe the stages of a typical partnership process.

(iii) Give three examples of start-up firms that have attempted to build partnerships with large firms. Describe the motives that each one had for forming the partnership and give examples of the challenges they faced.

[12]



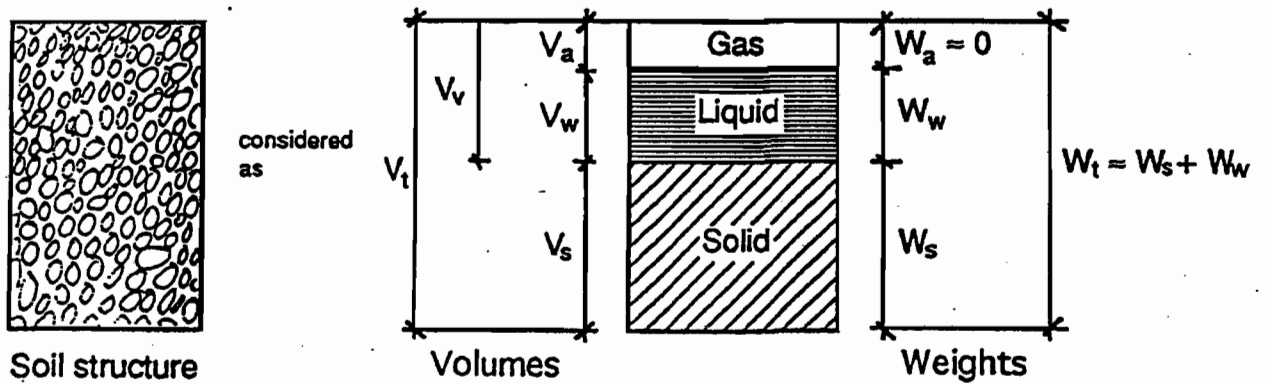
23 (a) Describe the structure of a typical UK patent. State and explain the four tests which an invention must satisfy in order to be patentable. Discuss the advantages and disadvantages of protecting the details of a manufacturing process by a patent. [12]

(b) An important process in manufacturing semiconductor chips involves polishing the surface with a suspension of very small abrasive particles in a water-based slurry. Your company has developed a new polishing slurry, which you hope will make a major impact in this international market. The slurry contains iron oxide particles, which are produced by a new process which you have developed, and a further four chemicals, all mixed with water. In making the slurry it is essential that the mixture is heated and agitated in a special way while certain of the chemicals are added. Discuss the ways in which your competitors might be prevented from copying your slurry. [8]

**END OF PAPER**

Data sheet – Soil Mechanics

General definitions



Specific gravity of solid	$G_s$
Voids ratio	$e = V_v/V_s$
Specific volume	$v = V_t/V_s = 1 + e$
Water content	$w = (W_w/W_s)$
Degree of saturation	$S_r = V_w/V_v = (w G_s/e)$
Unit weight of water	$\gamma_w = 9.81 \text{ kN/m}^3$ (although we assume $10 \text{ kN/m}^3$ )
Unit weight of soil	$\gamma = W_t/V_t = \left( \frac{G_s + S_r e}{1 + e} \right) \gamma_w$
Buoyant (effective or submerged) unit weight	$\gamma' = \gamma - \gamma_w = \left( \frac{G_s - 1}{1 + e} \right) \gamma_w$ (soil saturated)
Unit weight of dry soil	$\gamma_d = W_s/V_t = \left( \frac{G_s}{1 + e} \right) \gamma_w$
Relative density	$I_d = \frac{(e_{\max} - e)}{(e_{\max} - e_{\min})}$

where  $e_{\max}$  is the maximum voids ratio achievable in the quick tilt test (for sands), and  $e_{\min}$  is the minimum voids ratio achievable by vibratory compaction (for sands).

### Classification of particle sizes

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D equivalent diameter of soil particle

$D_{10}, D_{60}$  etc particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

### Stress components

Principle of effective stress (saturated soil):

$$\begin{aligned} \text{total stress } \sigma &= \text{effective stress } \sigma' + \text{pore water pressure } u \\ \tau &= \tau' + 0 \end{aligned}$$

and

$$\sigma_v = \text{vertical stress}$$

$$\sigma_h = \text{horizontal stresses}$$

$$\tau = \text{shear stress}$$

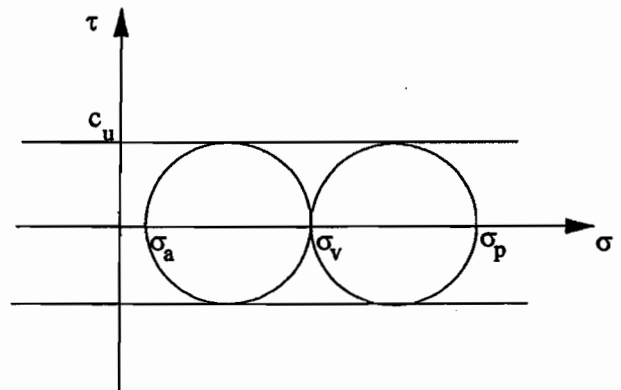
### Strength of clays (undrained behaviour only)

Under constant volume (undrained) conditions only, the strength of clays can be characterised by the *undrained shear strength*  $c_u$  which is mobilized when the shear stress  $\tau = c_u$ . This conforms to Tresca's criterion, and the active and passive total horizontal stresses,  $\sigma_a$  and  $\sigma_p$  respectively, are given by

$$\sigma_a = \sigma_v - 2 c_u$$

$$\sigma_p = \sigma_v + 2 c_u$$

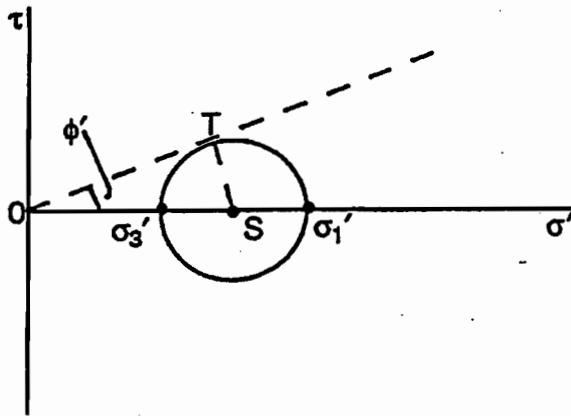
where  $\sigma_v$  is the total vertical stress.



## Strength of sands

Mobilised angle of shearing  $\phi'$

where  $\tau = \sigma' \tan \phi'$



$$\sin \phi' = TS/OS$$

$$= \frac{(\sigma'_1 - \sigma'_3)/2}{(\sigma'_1 + \sigma'_3)/2}$$

$$\therefore \phi' = \sin^{-1} \left[ \frac{\left( \frac{\sigma'_1}{\sigma'_3} \right) - 1}{\left( \frac{\sigma'_1}{\sigma'_3} \right) + 1} \right]$$

Earth pressure coefficient K:

$$\sigma'_h = K\sigma'_v$$

Active pressure:  $\sigma'_v > \sigma'_h$

$$\therefore \sigma'_1 = \sigma'_v$$

$$\sigma'_3 = \sigma'_h$$

$$K_a = (1 - \sin \phi') / (1 + \sin \phi')$$

Passive pressure:  $\sigma'_h > \sigma'_v$

$$\therefore \sigma'_1 = \sigma'_h$$

[We assume principal stresses

$$\sigma'_3 = \sigma'_v$$

are horizontal and vertical]

$$K_p = (1 + \sin \phi') / (1 - \sin \phi') = \frac{1}{K_a}$$

Angle of shearing resistance:

at peak strength  $\phi'_{\max}$  at  $\left( \frac{\sigma'_1}{\sigma'_3} \right)_{\max}$

at critical state  $\phi'_{\text{crit}}$  after large strains.

## Sand strength data: friction hypothesis

In any shear test on soil, failure occurs when  $\phi' = \phi'_{\max}$  and

$$\phi'_{\max} = \phi'_{\text{crit}} + \phi'_{\text{dilatancy}}$$

where  $\phi'_{\text{crit}}$  is the ultimate angle of shearing resistance of a random aggregate which deforms at constant volume, so the dilatancy, which indicates an increase in volume during shearing, approaches zero ( $\phi'_{\text{dilatancy}} \rightarrow 0$ ) as  $\phi'_{\max} \rightarrow \phi'_{\text{crit}}$ .

$\phi'_{crit}$  is a function of the mineralogy, size, shape and distribution of particles. For a particular soil it is almost independent of initial conditions. Typical values ( $\pm 2^\circ$ ):

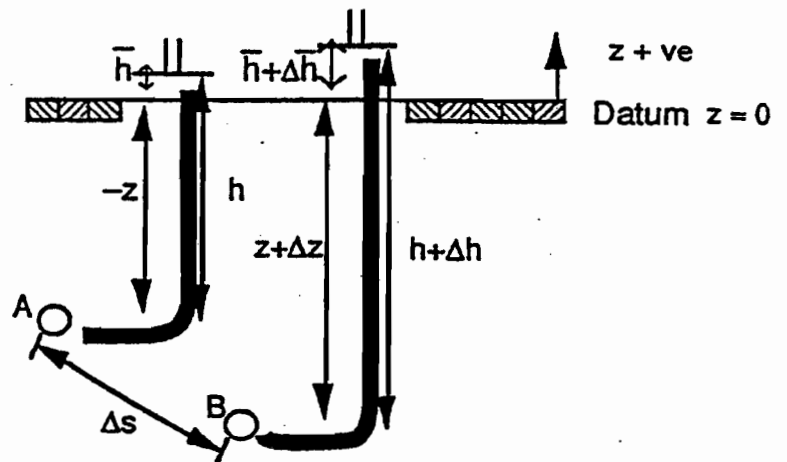
	$\phi'_{crit}$	$\phi'_{max}$	
feldspar	40°		
quartz	33°	53°	( $I_d = 1$ , and mean effective stress OS < 150 kPa)
mica	25°		

## Seepage

Excess pore water pressure

Head  $h = u/\gamma_w$

Potential  $\bar{h} = h + z$



Total pore water pressure head at A:  $u = \gamma_w h = \gamma_w (\bar{h} - (-z))$

B:  $u + \Delta u = \gamma_w (h + \Delta h) = \gamma_w (\bar{h} + z + \Delta \bar{h} + \Delta z)$

[Excess pore water pressure at A:  $\bar{u} = \gamma_w \bar{h}$

B:  $\bar{u} + \Delta \bar{u} = \gamma_w (\bar{h} + \Delta \bar{h})]$

Hydraulic gradient A-B  $i = -\frac{\Delta \bar{h}}{\Delta s} = -\frac{\Delta \bar{u}}{\gamma_w \Delta s}$

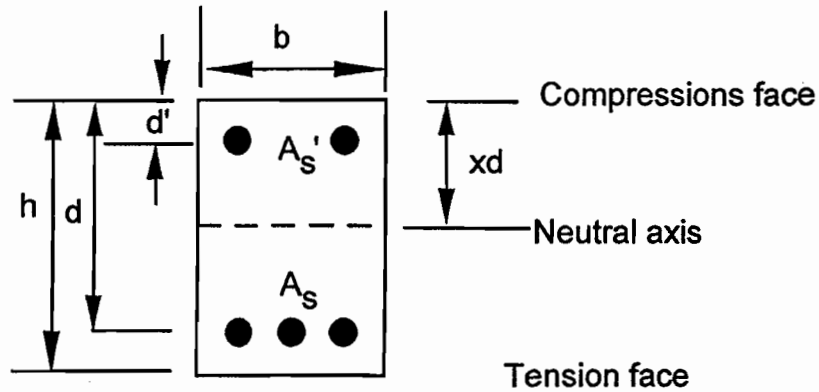
Darcy's law  $v = ki$   
 $v$  = average or superficial seepage velocity  
 $k$  = coefficient of permeability

### Typical permeabilities

$D_{10} > 10\text{mm}$	:	non-laminar flow
$10\text{ mm} > D_{10} > 1\mu\text{m}$	:	$k \cong 0.01(D_{10}\text{ in mm})^2\text{ m/s}$
clays	:	$k \cong 10^{-9}\text{ to }10^{-11}\text{ m/s}$

## Design of reinforced concrete

Data sheet for use in Part IB Civil Engineering Elective Course.



### Design Stresses

Cube strength for concrete  $f_{cu}$ . At failure in bending, stress in concrete =  $0.4f_{cu}$  over whole area of concrete in compression.

Tensile yield stress of steel  $f_y$ . At failure in bending, stress in bars in tension =  $0.87f_y$ , stress in bars in compression =  $0.75f_y$ .

### Design Equations

Moment capacity of singly reinforced beam

$$M \leq 0.15 f_{cu} b d^2$$

$$M = 0.87 f_y A_s d (1 - x/2)$$

$$x = 2.175 (f_y / f_{cu}) (A_s / b d) \quad (\leq 0.5 \text{ to avoid over reinforcement})$$

Moment capacity of doubly reinforced beam

$$M = 0.15 f_{cu} b d^2 + 0.75 f_y A_s' (d - d')$$

$$0.87 f_y A_s = 0.75 f_y A_s' + 0.2 f_{cu} b d$$

**Shear capacity of all beams**

$$\text{Total shear capacity } V = (v_c + v_s)bd$$

$$\text{Where, } v_c = 0.68(100 A_s/bd)^{0.33} (400/d)^{0.25} \quad (\text{N/mm}^2)$$

$$\text{and } v_s = 0.87f_y A_{sq}/(bs)$$

in which  $s$  = shear link spacing,  $A_{sq}$  is total area of all shear bars in a link and  $A_s$  is the total area of effective longitudinal *tension* steel at the section.

**Standard bar sizes**

Diameter (mm)	6	8	10	12	16	20	25	32	40	50
Area (mm <sup>2</sup> )	28	50	78	113	201	314	491	804	1256	1963

**Available steel types**

Deformed high yield steel  $f_y = 460 \text{ N/mm}^2$

Plain mild steel  $f_y = 250 \text{ N/mm}^2$

**Lap and anchorage lengths** 40 bar diameters

**Density of reinforced concrete:**  $24 \text{ kN/m}^3$

**Reinforcement areas per metre width**

	Spacing of bars (mm)									
	75	100	125	150	175	200	225	250	275	300
Bar Dia. (mm)										
6	377	283	226	189	162	142	126	113	103	94.3
8	671	503	402	335	287	252	224	201	183	168
10	1050	785	628	523	449	393	349	314	285	262
12	1510	1130	905	754	646	566	503	452	411	377
16	2680	2010	1610	1340	1150	1010	894	804	731	670
20	4190	3140	2510	2090	1800	1570	1400	1260	1140	1050
25	6550	4910	3930	3270	2810	2450	2180	1960	1790	1640
32	10700	8040	6430	5360	4600	4020	3570	3220	2920	2680
40	16800	12600	10100	8380	7180	6280	5580	5030	4570	4190
50	26200	19600	15700	13100	11200	9820	8730	7850	7140	6540
Areas calculated to 3 significant figures according to B.S.I recommendations										

# Part IB Data Sheet: Electrical Engineering Elective Transistor Design Summary Sheet

## Gauss's Theorem

$\epsilon_0 \epsilon_r E_1 - \epsilon_0 \epsilon_r E_2 = \text{charge per unit area enclosed between upper surface 1 and lower surface 2.}$

## FET Design Summary

- $\tau_t \rightarrow$  switching time as 1st approx. (scattering limited transit time).
- $\tau_{\text{eff}} = \tau_t + R_{\text{load}} C_{\text{eff(ouput)}} \rightarrow$  switching time as 2nd approx.
- $L = v_s \tau_t$  (source-drain spacing).
- $I_{\text{sat}} = e N v_s W d_s = e N W L d_s / \tau_t$
- Aspect ratio  $W/L$  (technology?).
- $(1/2)eN(d_s)^2 / \epsilon_0 \epsilon_r = (\text{Max Gate Voltage})$
- $E'_{\text{peak}} = eN d_s / \epsilon_0 \epsilon_r < E_{\text{breakdown}}$
- Minimum Drain Source Voltage  $\sim E_s L$  ( $E_s$  is the field required to reach limiting velocities).

## Mutual Conductance

$$g_{\text{mo}} \sim I_{\text{sat}} / V_{\text{gate(max)}}$$

Mutual conductance reduces with frequency as  $g_m(\omega) \approx g_{\text{mo}} / (1 + j\omega \tau_t)$ ;

$$v_{\text{out}} = g_m(\omega) R (1 + j\omega R C_{\text{eff(ouput)}}) \approx g_{\text{mo}} R / [1 + j\omega(\tau_t + R C_{\text{eff(ouput)}})] \\ = g_{\text{mo}} R / [1 + j\omega \tau_{\text{eff}}]$$

## Capacitances for FET

Parallel Plate Capacitance:  $\epsilon_0 \epsilon_r \text{Area/spacing}$

Used for rough estimates of parasitic capacitance.

Effective Capacitances for FET

$$C_{\text{eff(ouput)}} \rightarrow C_{\text{gate/drain}} + C_{\text{drain/source}} + C_{\text{load}};$$

$$C_{\text{eff(in)}} \rightarrow M C_{\text{gate/drain}} + C_{\text{gate/source (proximity)}} + C_{\text{gate/source (electronic)}};$$

$$C_{\text{electronic}} = g_{\text{mo}} \tau_t \quad ; \quad M = (1 + g_{\text{mo}} R_{\text{load}}).$$

## Time Constants for FET

$\mu = e\tau / m^*$  relates mean free time  $\tau$  and mobility.

Transit time  $\tau_t$  over distance  $L$  and scattering limited velocity  $v_s$  are related

by  $\tau_t = L / v_s$ .

$$v_{\text{out}} \approx g_{\text{mo}} R / [1 + j\omega \tau_{\text{eff}}] = g_{\text{mo}} R / [1 + j\omega / (2\pi f_t)]$$

$$1 / (2\pi f_t) = \tau_t + R C_{\text{eff(ouput)}} = \tau_{\text{eff}} \quad \text{The transition frequency is } f_t.$$

$$10\% \text{ to } 90\% \text{ rise time is } T = 2.2 \tau_{\text{eff}} = (2.2/2\pi)(1/f_t) = 0.35/f_t.$$



# Electrical Engineering Elective: Tunnel Barrier Design Summary Sheet

## Schrodinger's Equation

Complex Wave  $\Psi = A \exp(-j2\pi f t + j2\pi x/\lambda) = A \exp(-j\omega t) \exp(jkx)$ ;  
<momentum>  $\Psi = p\Psi = (\hbar/\lambda)\Psi = -j(\hbar/2\pi)\partial\Psi/\partial x$ ;

<Total energy>  $\Psi = E\Psi = hf\Psi = j(\hbar/2\pi)\partial\Psi/\partial t$   
 $(\hbar/2\pi) \rightarrow \hbar$ ;  $h = 6.625 \times 10^{-34}$  J/s.

Schrodinger's equation:-

$$E\Psi = (1/2m)[-\hbar^2\partial^2/\partial x^2]\Psi + e\phi\Psi$$

## Tunneling (Rectangular barrier $e\phi$ )

Propagating waves outside barrier with incident kinetic energy  $U_{\text{incident}} = (\hbar k_i)^2 / 2m^*$

Evanescent waves inside barrier:  $-(\hbar k)^2 / 2m^* = (\hbar k_i)^2 / 2m^* - e\phi$

## Technology Design Summary

### Diffusion

Constant Surface Concentration:

$$C(x,t) = C_s \operatorname{erfc}\left[\frac{x}{2(Dt)^{1/2}}\right]$$

Constant Total Dopant:

$$C(x,t) = \frac{S}{(\pi Dt)^{1/2}} \exp\left[\frac{-x^2}{4Dt}\right]$$

ENGINEERING TRIPOS PART IB

Friday 8 June 2007 2 to 4.30

Paper 8

Selected Topics

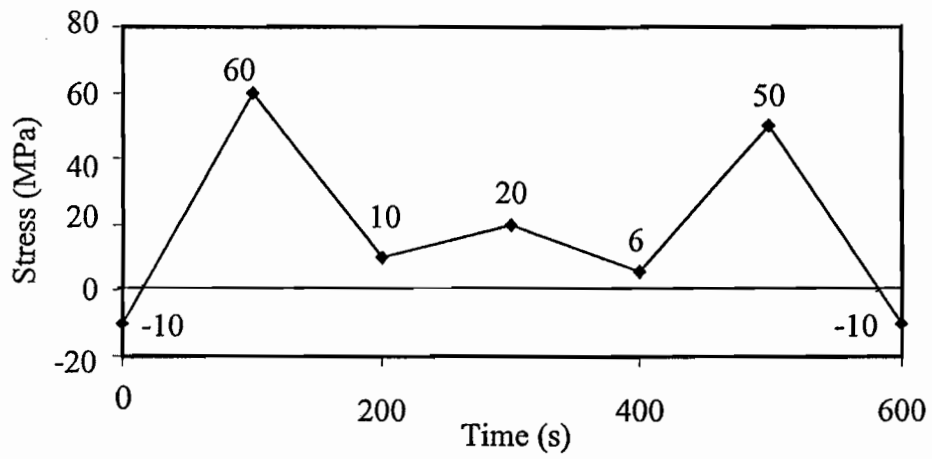


Fig. 3

Working sheet for Q 7  
(to be handed in with your script)