P2 QU1 2007/08

1a) $s - m = b + r - Dj$ (Maxwell's equ)

$= 3 + 3\times 2 - 2\times 4 = 1$; no mechanism $\Rightarrow$ $m = 0$ and $s = 1$

$\therefore$ 1 redundancy [or: remove 1 bar → stat. det. 3 pinned arch]

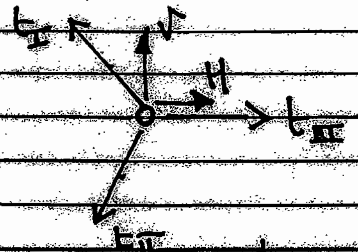
1b)

Set $t_{III} = 1$: $R\uparrow \Rightarrow t_I = t_{II}$

$R\rightarrow$: $t_{III} - 2t_I\cos 60 \Rightarrow t_I = t_{III}$

$\underset{\sim}{s} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ self-stress vector.

1c)



$V = F\sin\theta$, $H = F\cos\theta$

Set t_{III} as redundant bar: general solution for $t_{III} = 0$ has

$R\rightarrow$ $H - t_I \cdot \frac{1}{2} - t_{II} \cdot \frac{1}{2} = 0$; $R\uparrow$ $V + t_I \cdot \frac{\sqrt{3}}{2} - t_{II} \cdot \frac{\sqrt{3}}{2} = 0$ (= sin 60°)

$\Rightarrow t_I + t_{II} = 2H$; $t_I - t_{II} = -\frac{2V}{\sqrt{3}}$ $\Rightarrow$ $t_I = H - V/\sqrt{3}$, $t_{II} = H + V/\sqrt{3}$

$\Rightarrow$ $\underset{\sim}{t_0} = [t_I \; t_{II} \; t_{III}]^T = [H - V/\sqrt{3}, H + V/\sqrt{3}, 0]^T$

Bar forces $\underset{\sim}{t} = \underset{\sim}{t_0} + x\underset{\sim}{s}$ (self-stress) ; x given by compatibility conditions.

Flexibility matrix $\underset{\sim}{F} = \frac{L}{AE}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$; initial extension $\underset{\sim}{e_0} = [0 \; 0 \; -e_0]^T$ (bar III too short)

$\Rightarrow$ $\underset{\sim}{e} = \underset{\sim}{F}\underset{\sim}{t} + \underset{\sim}{e_0} = \frac{L}{AE}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \cdot \begin{bmatrix} H - V/\sqrt{3} + x \\ H + V/\sqrt{3} + x \\ 0 + x \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -e_0 \end{bmatrix}$

bar extensions

Value of x given by satisfaction of compatibility constraint

$\underset{\sim}{s}^t \underset{\sim}{e} = 0$ $\Rightarrow$ $[1 \; 1 \; 1] \cdot \frac{L}{AE}\begin{bmatrix} H - V/\sqrt{3} + x \\ H + V/\sqrt{3} + x \\ x - AEe_0/L \end{bmatrix} = 0$

$\Rightarrow H - V/\sqrt{3} + x + H + V/\sqrt{3} + x + x - AEe_0/L = 0$

$3x = AEe_0/L - 2H \Rightarrow$ $\underline{x = AEe_0/3L - 2H/3}$

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$$\underset{\sim}{t} = \underset{\sim}{t}_e + x\underset{\sim}{s} = \begin{bmatrix} H - V/\sqrt{3} + x \\ H + V/\sqrt{3} + x \\ x \end{bmatrix} = \begin{bmatrix} H/3 - V/\sqrt{3} + AEe_0/3L \\ H/3 + V/\sqrt{3} + AEe_0/3L \\ -2H/3 + AEe_0/3L \end{bmatrix}$$

[Check: when $H = V = 0$, $t_{III} = AEe_0/3L > 0$ i.e. bar in tension because it is too short]

1d) When $\theta = 90°$, $H = 0$; use Virtual Work to compute deflection components.

$$\delta_V = \underset{\sim}{t}^* \cdot \underset{\sim}{e} \quad \text{real extn}$$

$+1$ in vertical direction.

$$\underset{\sim}{e}\Big|_{H=0} = \frac{L}{AE}\begin{bmatrix} -V/\sqrt{3} \\ V/\sqrt{3} \\ 0 \end{bmatrix} + \frac{e_0}{3}\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \qquad \underset{\sim}{t}^* = 1 \text{ at pin} \uparrow \text{ set } t_{III} = 0$$

$$\Rightarrow t_I^* = -t_{II}^* = -1/\sqrt{3}$$

$$\underset{\sim}{t}^* \cdot \underset{\sim}{e} = \left[-1/\sqrt{3} \;\; 1/\sqrt{3} \;\; 0\right] \cdot \left[\frac{L}{AE}\begin{bmatrix} -V/\sqrt{3} \\ V/\sqrt{3} \\ 0 \end{bmatrix} + \frac{e_0}{3}\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}\right]$$

$$= -1/\sqrt{3}\left[L/AE \cdot \frac{-V}{\sqrt{3}} + \frac{e_0}{3}\right] + 1/\sqrt{3}\left[L/AE \cdot V/\sqrt{3} + \frac{e_0}{3}\right] + 0 \cdot (-)$$

$$= 2VL/AE \quad \Rightarrow \quad \delta_V = \frac{2VL}{AE} \uparrow \text{ (vertical)}$$

Horizontal: $\underset{\sim}{t}^* = [1 \;\; 1 \;\; 0]$ (after setting $H^* = 1$, $V^* = 0$ at centre)

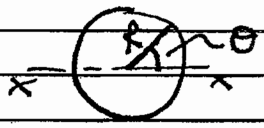
$$\Rightarrow \underset{\sim}{t}^* \cdot \underset{\sim}{e}\Big|_{H=0} = [1 \;\; 1 \;\; 0] \cdot \left[\frac{L}{AE}\begin{bmatrix} -V/\sqrt{3} \\ V/\sqrt{3} \\ 0 \end{bmatrix} + \frac{e_0}{3}\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}\right] = \frac{2e_0}{3} \Rightarrow \delta_H = \frac{2e_0}{3} \rightarrow$$

Student errors: Setting 3rd element in $\underset{\sim}{t}$ as $L - e_0$ instead of L (which complicates expression for $\underset{\sim}{e}$); forgetting about e_0 or getting its sign wrong; determining x incorrectly; over-working (i.e. not using virtual work efficiently) the displacement calculation.

Generally very well answered. Students should avoid trying to do too much mental arithmetic or anticipate the signs of bar forces.

P2 Qu2 2007/08

2a) $I_{xx} = \int y^2 dA = 4\int_{\theta=0}^{\theta=\pi/2} (\underbrace{R\sin\theta}_{y})^2 \underbrace{t\cdot R\,d\theta}_{dA} = \underline{\pi R^3 t}$



$J = I_{xx} + I_{yy} = \underline{2\pi R^3 t}$ $[I_{xx} = I_{yy}]$

2bi) Use databook expression $\underline{\varepsilon_{aa} = \varepsilon_{xx}\cdot\cos^2\theta + \varepsilon_{yy}\cdot\sin^2\theta + \gamma_{xy}\sin\theta\cos\theta}$

$\theta = 60^\circ \Rightarrow \varepsilon_{60} = \varepsilon_{xx}\cdot 1/4 + \varepsilon_{yy}\cdot 3/4 + \gamma_{xy}\cdot\sqrt{3}/4$ — ①

$\theta = 120^\circ \Rightarrow \varepsilon_{120} = \varepsilon_{xx}\cdot 1/4 + \varepsilon_{yy}\cdot 3/4 - \gamma_{xy}\cdot\sqrt{3}/4$ — ②

$\Rightarrow$ ① + ② $\varepsilon_{60} + \varepsilon_{120} = + 1/2\cdot\varepsilon_{xx} + 3/2\cdot\varepsilon_{yy}$; $\varepsilon_{xx} = \varepsilon_0$

$\Rightarrow \underline{\varepsilon_{yy} = \{2(\varepsilon_{60} + \varepsilon_{120}) - \varepsilon_0\}/3}$

① − ② $\Rightarrow \gamma_{xy}\cdot\sqrt{3}/2 = \varepsilon_{60} - \varepsilon_{120} \Rightarrow \underline{\gamma_{xy} = 2(\varepsilon_{60} - \varepsilon_{120})/\sqrt{3}}$

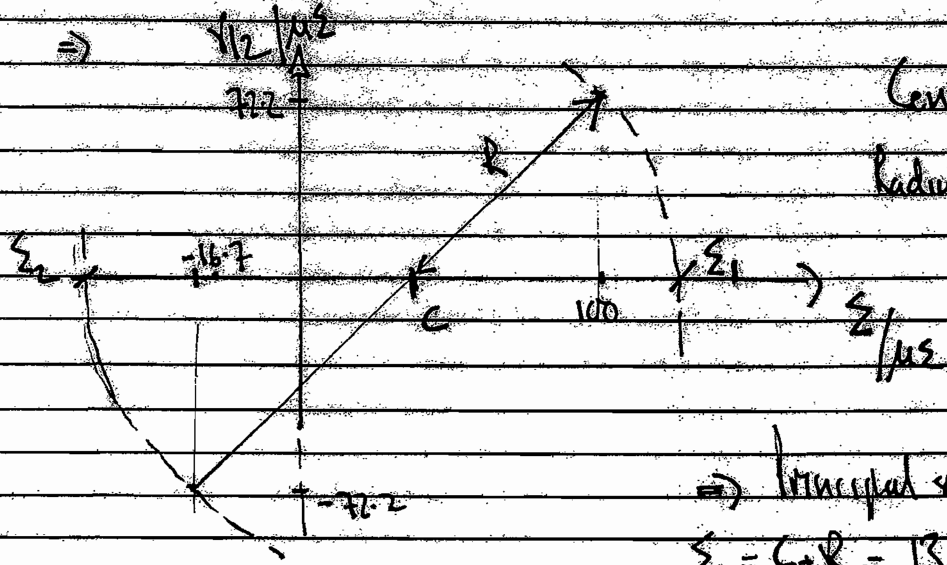
2b ii) $\varepsilon_0 = 100\mu\varepsilon$; $\varepsilon_{60} = -50\mu\varepsilon$, $\varepsilon_{120} = 75\mu\varepsilon$

$\Rightarrow \varepsilon_{xx} = 100\mu\varepsilon$, $\varepsilon_{yy} = (-100-100+150)/3 = -\frac{50}{3}\mu\varepsilon$, $\gamma_{xy} = \frac{-100-150}{\sqrt{3}} = -\frac{250}{\sqrt{3}}\mu\varepsilon$

Plot $(\varepsilon_{xx}, -\gamma_{xy}/2)$ vs $(\varepsilon_{yy}, \gamma_{xy}/2)$ on Mohr's circle

$100\mu\varepsilon, +72.2\mu\varepsilon$ $\quad -16.7\mu\varepsilon, -72.2\mu\varepsilon$

$\Rightarrow$



Centre $= \frac{100-16.7}{2} = 41.7\mu\varepsilon$

Radius $= \left[(100-41.7)^2 + 72.2^2\right]^{1/2} = 92.8\mu\varepsilon$

$\Rightarrow$ Principal strains

$\varepsilon_1 = C + R = 134.5\mu\varepsilon$

$\varepsilon_2 = C - R = -51.2\mu\varepsilon$

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Use databook to compute principal stresses

$$\sigma_1 = \frac{E}{1-\nu^2}\left[\varepsilon_1 + \nu\varepsilon_2\right]; \quad \sigma_2 = \frac{E}{1-\nu^2}\left[\varepsilon_2 + \nu\varepsilon_1\right] \qquad \left.\begin{matrix} E = 210\,\text{GPa} \\ \nu = 0.3 \end{matrix}\right\} \text{steel}$$

$$\sigma_1 = \frac{210\times10^9}{1-0.3^2}\left[134.5 + 0.3\times -51.2\right]\times10^{-6}; \quad \sigma_2 = \frac{210\times10^9}{1-0.3^2}\left[-51.2 + 0.3\times134.5\right]\times10^{-6}$$

$$\Rightarrow \underline{\sigma_1 = 27.5\,\text{MPa}},\ \underline{\sigma_2 = -2.5\,\text{MPa}} \quad \left[\sigma_3 = 0,\ \text{thin-walled}\right]$$

2biii) Let factor = λ on stresses $\Rightarrow \lambda^2\underbrace{\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right] = 2Y^2}_{\text{von-Mises.}}$

$Y = 275\,\text{MPa}$ (uniaxial yield stress) $\Rightarrow \lambda^2\left[30^2 + 2.5^2 + 27.5^2\right] = 2\cdot275^2$

$$\Rightarrow \underline{\underline{\lambda = 9.54}}$$

Comments: generally well done by all. Common mistakes were typo numerical errors. Some students inadvertently used $\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy}$ to find $\sigma_x, \sigma_y, \tau_{xy}$ (stresses) before constructing a Mohr's circle of stresses: this approach is equally valid but only works for isotropic materials (such as steel), since principal strains' directions are the same as principal stresses' directions. A few did not know the meaning of polar moment of area (J).

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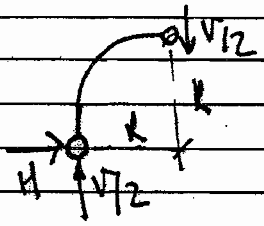
3a)



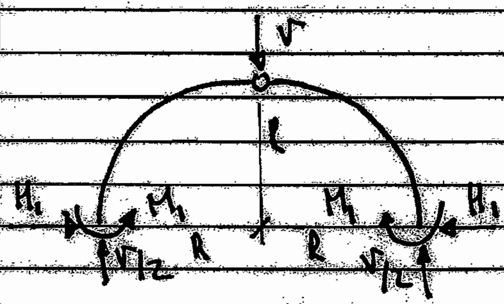
moments about apex $\quad R.V/2 - RH \Rightarrow \underline{H = V/2}$

Reaction = $V/2 \uparrow$ $V/2 \rightarrow$ (l.h.s.)

$V/2 \leftarrow$ (r.h.s.)

Statically determinate; no change in reaction when temperature increases (provided thermal strains are very small).

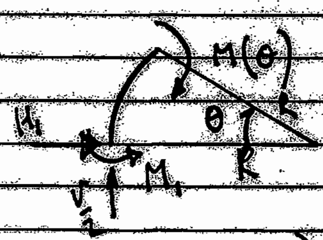
3bi)



Two redundancies M_1, H_1 (symmetrical)

Moments about apex for half of arch ⇒

$$\underline{-M_1 + \frac{V}{2}R - H_1 R = 0} \quad \text{— (A)}$$



$$M(\theta) - M_1 + \frac{V}{2}R(1-\cos\theta) - H_1 R\sin\theta$$

$$\underset{\text{real}}{=} EI\kappa(\theta)$$

$$\Rightarrow \text{curvature } \kappa(\theta) = \frac{1}{EI}\left(M_1 - \frac{V}{2}R(1-\cos\theta) + H_1 R\sin\theta\right)$$

$$\Rightarrow \kappa(\theta) = \frac{1}{EI}\left[\frac{V}{2}R - \frac{V}{2}R(1-\cos\theta) + H_1 R\sin\theta - H_1 R\right]$$

$$\Rightarrow \underline{\kappa(\theta) = \frac{1}{EI}\left[\frac{VR}{2}\cos\theta + H_1 R(\sin\theta - 1)\right]} \quad \text{— (B)}$$

Rotation at either foot is zero (built-in), so consider "released" structure with pins at base and ~~set~~ reintroduce unit moment as reactant (self-stress) in equilibrium with no loads (i.e. $V=0$) applied; this is the virtual set.

N.B. $1/R$ is the horizontal virtual force, which is found by substituting $V=0$ and $M_1=1$ into (A).

$$\Rightarrow m^*(\theta) - 1 + \frac{1}{R}\cdot R(\sin\theta) \Rightarrow \underline{m^* = 1 - \sin\theta} \quad \text{— (C)}$$

Apply virtual work equation for overall structure

$$\text{eg.} \quad \overbrace{\underbrace{\Sigma F \cdot \delta + \Sigma M \cdot \phi = \int m^* \kappa \, ds}_{\text{eqlm set}}}^{\text{compatible}}$$

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$$\Rightarrow \underbrace{2}_{\text{both halves of arch}}\left\{ \overset{H_1^*}{\frac{1}{R}}\cdot \underbrace{0}_{\text{clamped support}} + \overset{M_1^*}{1}\cdot \underbrace{0}_{\text{rotation = zero}} = \int_0^{\pi/2} \underset{(C)}{M^*} \cdot \overset{(B)}{K} \cdot \underbrace{R\,d\theta}_{ds} \right\}$$

$$\Rightarrow \int_0^{\pi/2} (1-\sin\theta)\left[\frac{VR}{2}\cos\theta + H_1R(\sin\theta-1)\right]d\theta = 0$$

$\int_0^{\pi/2} d\theta = \pi/2$; $\int_0^{\pi/2}\sin\theta\,d\theta = \int_0^{\pi/2}\cos\theta\,d\theta = 1$; $\int_0^{\pi/2}\sin\theta\cos\theta\,d\theta = 1/2$; $\int_0^{\pi/2}\sin^2\theta\,d\theta = \frac{\pi}{4}$

$$\Rightarrow \int_0^{\pi/2} \frac{VR}{2}\left[\cos\theta - \sin\theta\cos\theta\right] + H_1R\left[-1-\sin^2\theta + 2\sin\theta\right]d\theta = 0$$

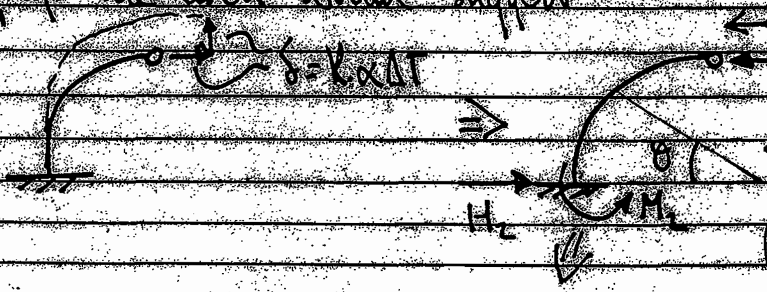
$$\Rightarrow \frac{VR}{2}\left[1-1/2\right] + H_1R\left[-\pi/2 - \pi/4 + 2\right] = 0$$

$$V/4 = H_1\left[3\pi/4 - 2\right] \Rightarrow \underline{H = V/(3\pi-8)}$$

$$M_1 = \frac{V}{2}R - H_1R = VR\left[\frac{1}{2} - \frac{1}{3\pi-8}\right] = VR\left[\frac{3\pi-8-2}{2(3\pi-8)}\right] = \underline{\frac{VR}{2}\left[\frac{3\pi-10}{3\pi-8}\right]}$$

[Need both H_1 & M_1 to completely define reaction, along with $V/2$]

3b(ii) If the temperature rise were unconstrained, then the expansion of one half of the arch would suggest

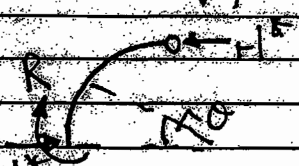


This is the self-stress state due to ΔT: find H_2 and then superpose with soln from (bi).

$$\underline{H_2\cdot R + M_2 = 0}$$

$$K(\theta) = \frac{1}{EI}\cdot\Big[\underbrace{M_2 + H_2R\sin\theta}_{M(\theta)}\Big] = \frac{H_2R}{EI}\left[\sin\theta - 1\right]$$

Apply a virtual force in the sense of δ



$$M^*(\theta) = -R(1-\sin\theta)$$

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Apply virtual work eqn $\Rightarrow$ $1^{*}.\overset{\alpha\Delta T.R}{\delta} = \int m^{*} K\, R\,d\theta$

$$\Rightarrow 1\cdot\alpha\Delta T R = \int_0^{\pi/2} R(\sin\theta-1)\cdot\frac{H_2 R}{EI}(\sin\theta-1)\,R\,d\theta$$

$$= \frac{H_2 R^3}{EI}\int_0^{\pi/2}\sin^2\theta - 2\sin\theta + 1\,d\theta$$

$$= H_2R^3/EI\cdot\left[\pi/4 - 2 + \pi/2\right] = H_2R^3/EI\left[\frac{3\pi}{4}-2\right]$$

$$\Rightarrow H_2 = \frac{\alpha\Delta T.\cancel{R}.EI}{R^{\cancel{3}2}}\cdot\frac{4}{3\pi-8}$$

For a purely vertical reaction (not considering moment)

$$\Rightarrow H_1 + H_2 = 0 \Rightarrow \frac{\alpha\Delta T EI.4}{R^2[\cancel{3\pi-8}]} + \frac{V}{\cancel{3\pi-8}} = 0$$

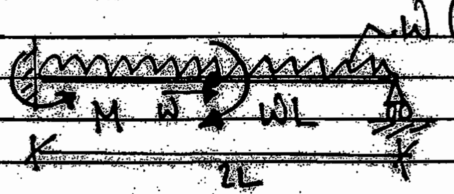
$$\Rightarrow \Delta T = -VR^2/4EI.\alpha$$ [cooling results in no horizontal force component].

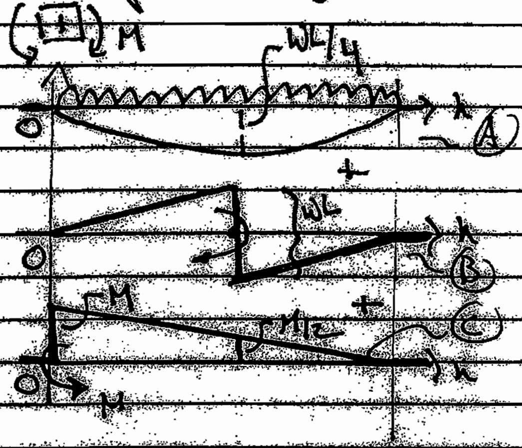
Comments: only *one* student achieved a correct solution: the rest could not properly construct the virtual work equation; in particular the need for unit moment was not applied in most cases. This example, excluding the temperature part, is very similar to a lectured example – it's therefore a surprise that so many (~ half all students who did this) failed to ~~even~~ set up the basic solution framework.

P2 Qu 4 2007/08

4a) $Z_p = \underbrace{\sum A_r \cdot d_r}_{\text{equal area axis}} = bt \cdot \frac{t}{2} + bt \cdot \frac{b}{2} = \frac{b^2 t}{2}\left[1 + \underset{\ll 1}{t/b}\right] \simeq \frac{b^2 t}{2}$

4b) Only need to consider horizontal span (column is rigid). There is one redundancy → choose M at root. Also, ignore horizontal tension due to W →





Apply lower bound to ensure that salient values of b.m. $< M_p$

End of beam: $M \leq M_p$ — (1) set $M = M_p$

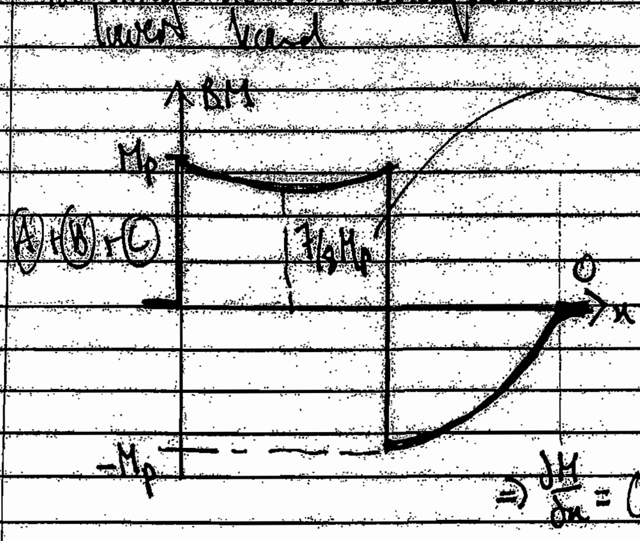
In middle: $\left| \underbrace{\frac{WL}{4}}_{\text{dist load}} \underbrace{\pm \frac{WL}{2}}_{\pm \text{ discontinuity}} - M/2 \right| < M_p$ — (2)

For $M = M_p$ (2)$^+$ ⇒ $\frac{WL}{4} + \frac{WL}{2} - M_p/2 < M_p$ ⇒ $W < \frac{2M_p}{L}$

(2)$^-$ ⇒ $\frac{WL}{2} - \frac{WL}{4} + \frac{M_p}{2} < M_p$ ⇒ $W < 2M_p/L$

Safest lower bound is $W = 2M_p/L$

Bending moment diagram at lower load



check: in the first half of beam

$BM|_A = \frac{W}{2}\left(\frac{x^2}{2L} - x\right) \quad 0 < x < L$

$BM|_B = Wx/2 \; ; \; B.M|_C = M\left(1 - \frac{x}{2L}\right)$

Total b.M. for $W = 2M_p/L$ = A+B+C $= M_p\left[1 - \frac{x}{2L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right]$

⇒ $\frac{dM}{dx} = 0$ ⇒ $x = L/2$, $M_{min} = \frac{7}{8}M_p$, as shown

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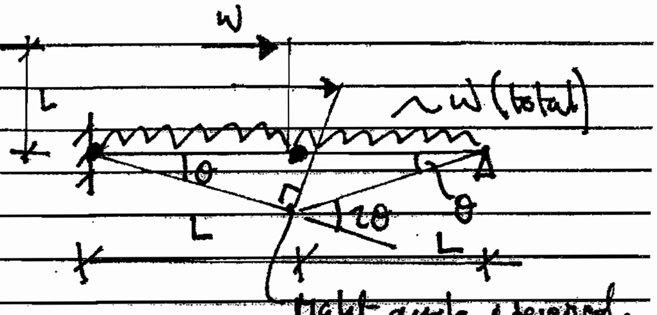


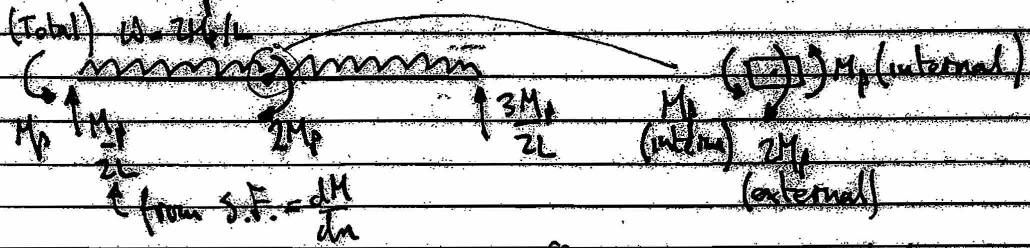
4c) Collapse mechanisms

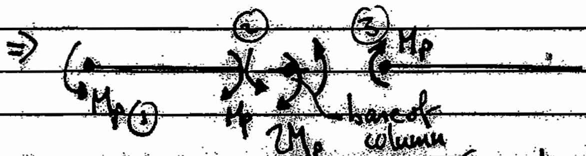
$$\underbrace{W \cdot L\theta}_{\rightarrow} + \underbrace{w \cdot \left(\frac{\theta \cdot L}{2}\right)}_{\text{avg movement of } w} \downarrow = M_p(\underset{\text{root}}{\theta} + \underset{\text{middle}}{2\theta})$$

$$\Rightarrow \frac{3wL\theta}{2} = 3M_p\theta \Rightarrow \underline{\underline{W_{coll} = \frac{2M_p}{L}}}$$

Matches lower bound ∴ correct mechanism.

[Statical check → not required, but is useful in terms of thinking about collapse mechanism]



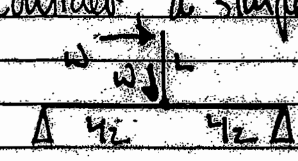


①: M_p resisting rotation here ⇒ hinge forms.

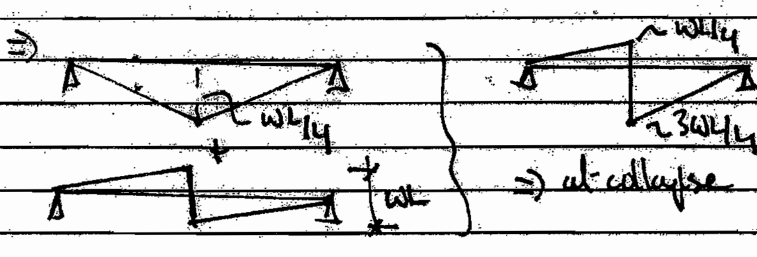
③: M_p also resisting rotation ⇒ hinge forms

②: bending hinge moment in same sense as rotation, so no hinge forms ⇒ hinge forms to the right hand side of base of column

Consider a simpler problem for visualising how hinge forms.

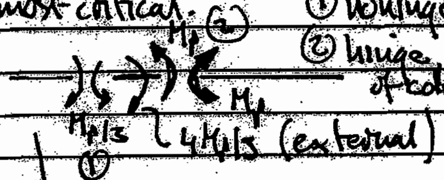


Vertical point load W: horizontal load, W, applied via column, L.



⇒ at collapse

Lower bound ⇒ $\frac{3WL}{4} < M_p \Rightarrow W = \frac{4M_p}{3L}$

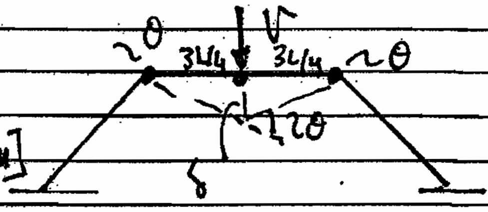


① no hinge ($< M_p$)
② hinge to right of column base

Comments: popular but poorly answered in 4(b) - the bending moment diagram eluded a high proportion of candidates. Many did not see the subtlety in 4(c) with the collapse mechanism [the column base remaining at right angles]

M Q05 2007/08

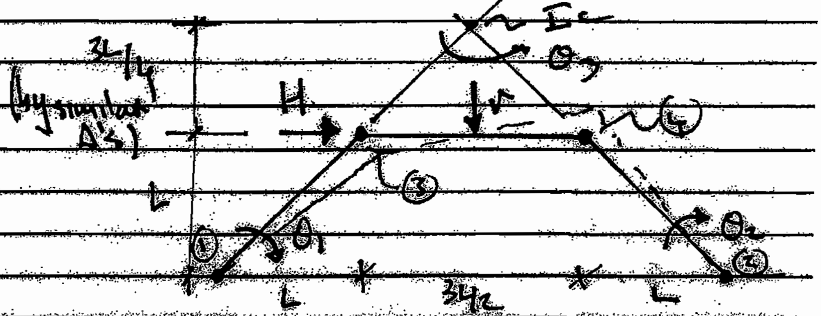
5a) Beam mechanism



$V \cdot \frac{3L}{4} \cdot \theta = M_p[\theta + \theta + 2\theta]$ [WD int = ext. WD]

⇒ $V = 16M_p/3L$ — (A)

5b) Sway mechanism.



$\theta_1 = \theta_2 = \theta$:

horizontal displacement of H ⇒ $L \cdot \theta_1 = \theta_3 \cdot \frac{3L}{4}$

⇒ $\theta_3 = 4/3\,\theta_1$ for compatibility

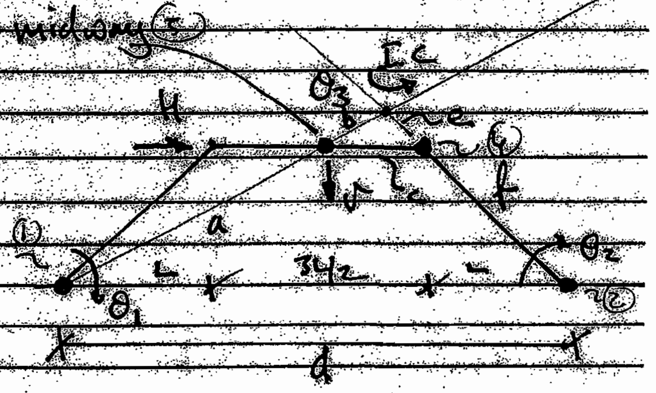
⇒ $\cancel{4/3\,\theta_1}$ $H \cdot L\theta_1 + \underbrace{V \cdot 0}_{\text{centre of bm does not displace}} = M_p[\underbrace{\theta_1}_{(1)} + \underbrace{\theta_2}_{(2)} + \underbrace{(\theta_1+\theta_3)}_{(3)} + \underbrace{(\theta_2+\theta_3)}_{(4)}]$

⇒ $H \cdot L \cdot \theta = M_p[\theta + \theta + \theta + \frac{4}{3}\theta + \theta + \frac{4}{3}\theta] = M_p[4\theta + \frac{8}{3}\theta]$

⇒ $H = 20M_p/3L$ — (B)

5c) Combined mechanism

Assume hinges as shown: use similar triangles to calculate key geometrical distances, as follows.



$b/(a+b) = c/d$: $c = \frac{3L}{4}$; $d = 7L/2$

⇒ $c/d = 3/14$

$b/(a+b) = 3/14$ ⇒ $14b = 3a + 3b$ ⇒ $a/b = 11/3$: also $e/(e+f) = c/d$ ⇒ $e/f = 3/11$

Compatibility of displacements at points ⇒ $a \cdot \theta_1 = b \cdot \theta_3$ ⇒ $\theta_3 = a/b \cdot \theta_1 = 11/3\,\theta_1$

$f \cdot \theta_2 = e \cdot \theta_3$ ⇒ $\theta_2 = e/f \cdot \theta_3 = 3/11 \cdot \frac{11}{3}\theta_1 = \theta_1$

⇒ $\theta_1 = \theta_2 = \theta$, $\theta_3 = 11/3\,\theta$

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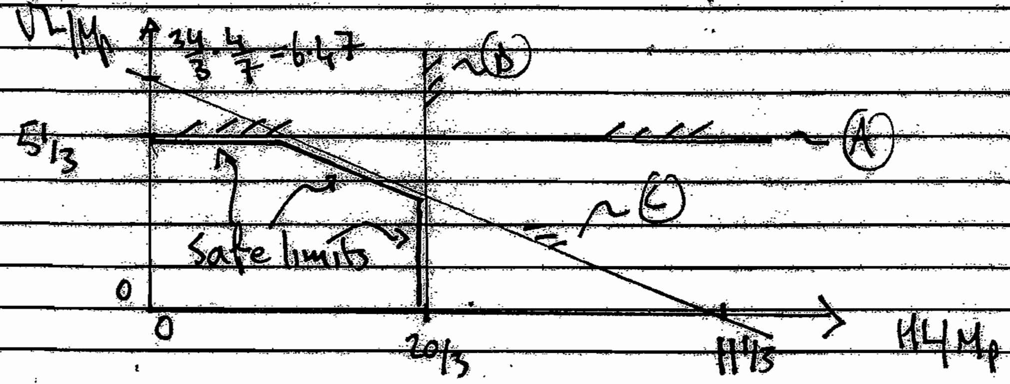
Work equation $H\cdot L\cdot\theta_1 + V\cdot\left[L+\frac{3L}{4}\right]\cdot\theta_1 = M_p\cdot\left[\overset{①}{\theta_1}+\overset{②}{\theta_2}+\overset{③}{(\theta_1+\theta_3)}+\overset{④}{(\theta_2+\theta_3)}\right]$

$\Rightarrow HL\theta + V\cdot\frac{7L}{4}\theta = M_p\left[\theta+\theta+(\theta+{}^{11}/_{3}\theta)+(\theta+{}^{11}/_{3}\theta)\right]$

$H + 7V/4 = M_p/L\left[4+\frac{22}{3}\right] = \frac{34}{3}\cdot M_p/L$

$\Rightarrow$ ~~Zp~~ $\underline{H + 7V/4 = \frac{34M_p}{3L}}$ — (C)

Interaction diagram: plot VL/M_p vs HL/M_p



Comments: well answered by most who took this question. Some candidates failed to notice that the sway mechanism in (b) obviated a rotation of the top beam, thereby resulting in no work of V. The geometry associated with the location of instantaneous centres also frustrated many candidates — when a simple exercise in similar triangles simplifies the process.

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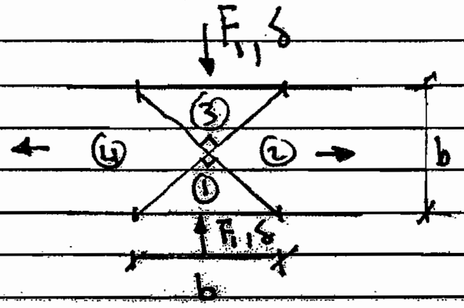
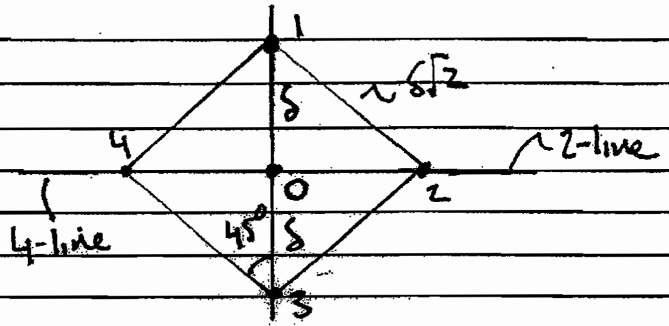
6a)





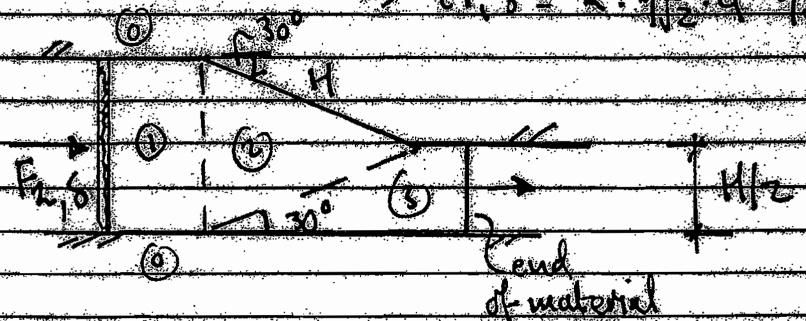
Work equation: (per unit depth into page)

$$(F_1\cdot\delta)2 = \underbrace{k}_{\text{shear yield}}\left[\delta_{12}\cdot\underbrace{b/\sqrt{2}}_{\text{length of slip plane 12}} + \delta_{14}\cdot\frac{b}{\sqrt{2}} + \delta_{32}\frac{b}{\sqrt{2}} + \delta_{34}\frac{b}{\sqrt{2}}\right]$$

all δ's $= \delta/\sqrt{2}$

$$\Rightarrow 2F_1\delta = k\cdot b/\sqrt{2}\cdot 4\cdot \delta/\sqrt{2} \Rightarrow \underline{\underline{F_1 = 2bk}}$$

6b)



$\underline{\delta_{12} = \delta/\sqrt{3}}$

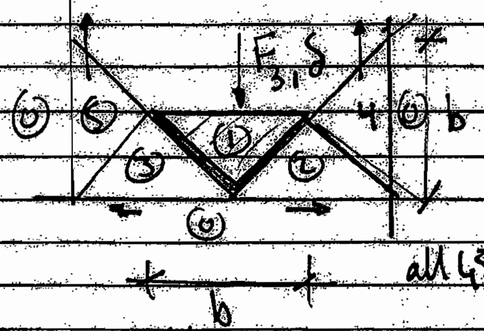
$\underline{\delta_{02} = \delta_{23} = 2\delta/\sqrt{3}}$

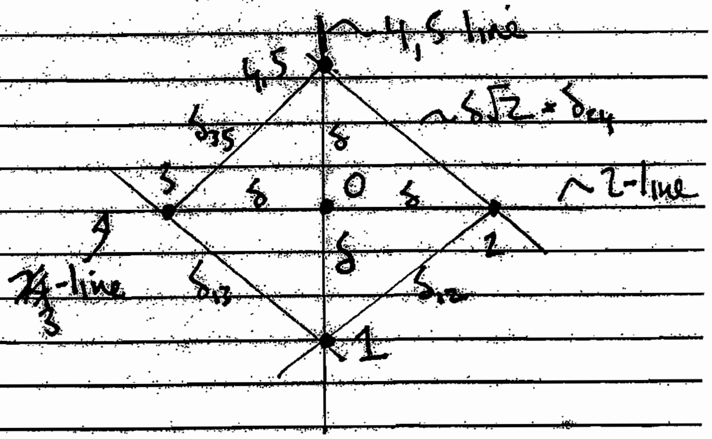
$$\Rightarrow \text{Work eqn: } F_2\cdot\delta = k\left[\underbrace{H\cdot\delta_{12} + H\cdot\delta_{23}}_{\text{both slip planes of length } H}\right]$$

no other term for walls are smooth

$$\Rightarrow F_2\delta = k\cdot H\left[\delta/\sqrt{3} + 2\delta/\sqrt{3}\right] \Rightarrow \underline{\underline{F_2 = \sqrt{3}kH}}$$

6c)





one possible collapse mechanism: there are others.

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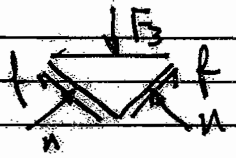
Work equ :

$$F_3 \cdot \delta = k\Big[\underbrace{\delta_{20}b + \delta_{30}b}_{2\cdot\delta\cdot b} + \underbrace{\delta_{04}b + \delta_{05}b}_{2\cdot\delta\cdot b} + \underbrace{\delta_{24}\cdot\tfrac{b}{\sqrt{2}} + \delta_{35}\cdot\tfrac{b}{\sqrt{2}}}_{2\cdot\delta\sqrt{2}\cdot b/\sqrt{2}}$$

$\Rightarrow$ $\underline{\underline{F_3 = 6kb}}$ (smooth – no terms due to δ_{12}, δ_{13})

If friction acts at material/billet interface, then the set of forces on the billet, assuming slip, is:



(all 45°) f = friction force ; n = normal reaction

$R\uparrow$: $F_3 = 2\left[f/\sqrt{2} + n/\sqrt{2}\right]$

If slipping $\Rightarrow$ $f = \mu n$ $\Rightarrow$ $n = f/\mu$ $\Rightarrow$ $F_3 = 2f\left[1/\sqrt{2} + 1/(\sqrt{2}\cdot\mu)\right]$

$\Rightarrow$ $f = F_3 \cdot \mu / \sqrt{2}(1+\mu)$

If friction force moves δ_{12} then WD $= \dfrac{F_3\mu}{\sqrt{2}(1+\mu)} \times \overset{\text{both sides}}{2} \times \underbrace{\sqrt{2}\delta}_{\delta_{12}} = \dfrac{2F_3\cdot\delta\mu}{1+\mu}$

Work equ

$$\underbrace{F_3\cdot\delta = 6bk}_{\text{before}} + \frac{2F_3\cdot\delta\mu}{1+\mu} \Rightarrow F_3\left[1 - \frac{2\mu}{1+\mu}\right] = 6kb$$

$$\Rightarrow \underline{\underline{F_3 = 6bk\left[\frac{1+\mu}{1-\mu}\right]}}$$

Comments : The least popular question. Many candidates could not quantify the external force in (b) being confounded by the necessary displacement diagram. Surprisingly (c) was answered better except for the final part where friction had to be introduced – some introduced it but no-one furnished a correct solution.