

**CRIBS****Question 1**

All calculations are based on assuming equilibrium conditions and therefore the lever rule is applicable.

(a)

- (i) 28 wt% Al, 72 wt% Ag;
- (ii) (Al) and  $\delta$ ;
- (iii) 566°C.

(b)

*Above 660°C:* Liquid is stable (no phase reactions).

*From 660°C to 566°C:* At 660°C, regions of primary  $\delta$  phase begin to form and grow as the temperature falls to 566°C. The composition of the liquid and  $\delta$  phases becomes richer in Al following the liquidus and solidus lines respectively. Just above the eutectic temperature (566°C), the composition is about 62% primary  $\delta$  (15 wt% Al) and 38% liquid of eutectic composition (28 wt% Al).

weight fraction of  $\delta = \frac{28-20}{28-15} = 0.62$ , so (Al)=0.38

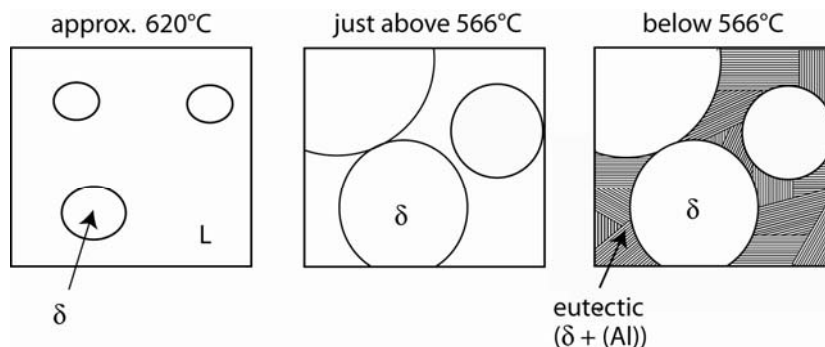
*At 566°C:* The eutectic reaction takes place, converting the remaining liquid into a two-phase mixture of  $\delta$  (15 wt% Al) and Al-rich solid solution (44.4% wt Al).

*From 566°C to room temperature:* The microstructure does not change significantly. Owing to the increasing solubility of Al in (Al), the weight fraction of (Al) increases. (At room temperature (Al) is pure Al.)

For instance, at 500°C, weight fraction of  $\delta$  in eutectic =  $\frac{72-20}{72-15} = 0.91$ , so (Al)=0.09.

At room temperature, weight fraction of  $\delta$  in eutectic =  $\frac{99-20}{99-15} = 0.94$ , so (Al)=0.06.

As  $\delta$  shows no change in solubility with temperature there is no precipitation taking place within the primary  $\delta$  grains.



(c) During solution heat treatment, the alloy acquires a homogeneous single phase structure - all the solute atoms dissolve in the matrix phase and become uniformly distributed.

Alloy A:  $\sim 560^\circ\text{C}$  (temperature  $T$  must be within the (Al) region)

Alloy B:  $\sim 400^\circ\text{C}$  ( $T$  must be within the (Al) region)

Alloy C:  $\sim 500^\circ\text{C}$  ( $T$  must be within the  $\alpha$  region)

From Table I, several points can be made:

During slow cooling to room temperature, the precipitates have time to nucleate and grow, since the required diffusional processes occur more quickly at higher temperatures. This generates coarse precipitates which would not impede dislocation motion significantly and therefore would not strengthen the alloys effectively. As illustrated in Table I, quenching and reheating at intermediate temperatures (in this case 200°C) has the capacity to harden and strengthen an alloy to a greater degree.

The difference between B & C alloys is that alloy B is non “heat treatable” whereas alloy C is. Al-Cu alloy, containing up to about 5% wt Cu, can be hardened and strengthened by the creation of a fine array of Al-Cu precipitates. Firstly, the alloy must undergo a solution treatment to allow Cu to enter into solid solution. Solution heat treatment is followed by quenching. This involves quenching into a bath of oil or water depending on the required cooling rate) Quenching generates a super-saturated solid solution, in which the solute (Cu in this case) remains in solution, despite the fact that, according to the phase diagram, there should be a mixture of (Al) and  $\theta$  phases. The growth of precipitates from supersaturated solid solution is carefully controlled by subsequent ageing treatments. These are carried out at intermediate temperatures, which are low enough to provide a driving force for precipitation, but high enough to ensure that the desired degree of precipitation occurs fairly quickly. Alloy A is also an age hardening alloy, i.e. Al can be hardened by alloying with Ag.

Alloy A has the highest yield strength after slow cooling to room temperature because it contains a higher fraction of solute (solid solution hardening) compared to alloys B and C.

Alloy A has higher yield strength, after quenching and reheating, than alloy C because it contains a higher volume fraction of precipitates.

For the Al-50 wt% Ag alloy, in principle, the weight fraction of precipitates will be

$$\frac{99 - 50}{99 - 15} = 0.58$$

For the Al-4 wt% Cu alloy, this will be

$$\frac{4}{54 - 4} = 0.08$$

It's also worth noting that heat treatment at 200°C for 5 hours may result in “over-ageing” (softening) of the alloys. Over-aging occurs if the aging period is prolonged or the aging temperature is too high. There is then a driving force for the precipitate dispersion to become progressively coarser making age hardening less effective.

Examiner's comments:

*This was one of the most popular questions on the paper. Parts (a) and (b) were generally answered well, while part (c) was answered less well. There was confusion about solid solution temperatures. A large proportion of the candidates failed to understand that, during solution heat treatment, the alloy acquires a single phase structure and suggested solution treatment temperatures above the liquidus. Many students were unable to account for the variations in the yield strength values between the different alloys.*

**Question 2**

(a) The difference between diffusive and displacive phase transformations is that the latter does not involve any diffusion. In displacive transformation, the atoms move in an organised manner relative to their neighbours. This homogeneous shearing creates a new crystal structure, without any compositional change (no diffusion). An example of displacive phase transformation includes formation of martensite from austenite, in which case the Fe lattice changes from fcc to bcc, with each Fe atom being displaced by a fraction of an atom spacing. An example of diffusive transformation is transformation of austenite to pearlite, which is controlled by diffusion of C to form  $\alpha + \text{Fe}_3\text{C}$ .

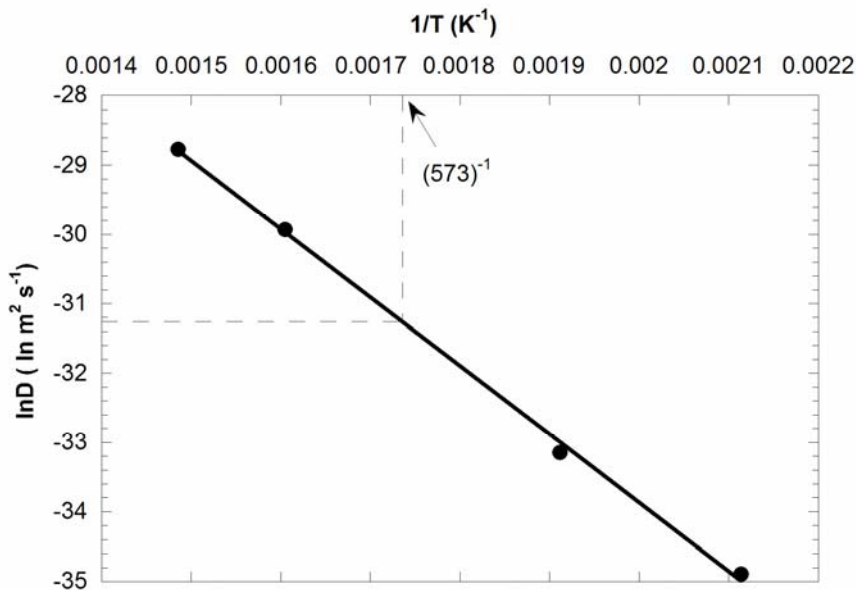
(b) Note that these are plain carbon steels ( $<2.1\% \text{ C}$ )

Figure 3a: This microstructure is almost entirely pearlitic (alternate plates of  $\alpha$  and  $\text{Fe}_3\text{C}$ ). The carbon content is very close to the eutectoid composition (0.8 wt% C).

Figure 3b: This is a hypo-eutectoid steel (note the low hardness value). It contains about 50% pro-eutectoid ferrite (white regions) and 50% pearlite (dark regions). Using the Fe-C equilibrium diagram and the lever rule, it can be found that the carbon content is about 0.4 wt%.

Figure 3c: From its hardness value and the microstructure (needle-like structure), it can be concluded that the microstructure is martensitic.

(c)



From the plot:

At  $300^\circ\text{C}$  (573 K)

$\ln D = -31.3$

$D_{300^\circ\text{C}} = 2.55 \times 10^{-14} \text{ m}^2 \text{ s}^{-1}$

From  $x \sim \sqrt{Dt}$ , where  $D_{300^\circ\text{C}} = 2.55 \times 10^{-14} \text{ m}^2 \text{ s}^{-1}$ ,  $t = 1 \text{ minute} = 60 \text{ s}$

$x \sim 1.24 \text{ } \mu\text{m}$

The ratio between the diffusion times needed to diffuse carbon the same distance at  $200^\circ\text{C}$  (473 K) and  $350^\circ\text{C}$  (623 K) is

$$\frac{t_{200^\circ\text{C}}}{t_{350^\circ\text{C}}} = \frac{D_{350^\circ\text{C}}}{D_{200^\circ\text{C}}} \Rightarrow \frac{t_{200^\circ\text{C}}}{t_{350^\circ\text{C}}} = \frac{1 \times 10^{-13}}{7 \times 10^{-16}} \sim 143$$

(d)  $D$  is given from

$$D = D_o \exp\left(-\frac{Q}{RT}\right), \text{ where } D_o = 2 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}, Q = 84 \text{ kJ mol}^{-1}, R = 8.3143 \text{ kJ kmol}^{-1} \text{ K}^{-1}, \\ T = 1223 \text{ K}$$

Hence  $D = 5.168 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$

$$\frac{C(x,t) - C_o}{C_s - C_o} = 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right) \Rightarrow \frac{0.8 - 0.15}{1.3 - 0.15} = 1 - \operatorname{erf}\left(\frac{1 \times 10^{-3}}{2\sqrt{5.168 \times 10^{-10} \cdot t}}\right) \\ \operatorname{erf}\left(\frac{1 \times 10^{-3}}{2\sqrt{5.168 \times 10^{-10} \cdot t}}\right) \approx 0.43$$

From Page 8 of the Materials Data Book

$$\left(\frac{1 \times 10^{-3}}{2\sqrt{5.168 \times 10^{-10} \cdot t}}\right) = 0.4 \Rightarrow \\ t \sim 3023 \text{ s } (\sim 50 \text{ min})$$

Examiner's comments:

*This was the third most popular question on the paper. There was confusion with estimating the carbon content of the steels in part (b). Most candidates failed to identify that Fig. 3(b) showed a hypoeutectoid steel. While most candidates spotted that there were two phases present (ratio 1:1), only a few noticed the low hardness value and used the Fe-C equilibrium diagram (and the lever rule) to find the carbon content. In part (c), some candidates failed to construct an Arrhenius plot of the data. This was surprising, given that they've covered this in lectures and in the examples papers. Part (d) was answered very well.*

**Question 3**

(a) The Gibbs free energy  $G$  is defined as

$$G = H - TS = U + pV - TS \text{ using that } H = U + pV$$

where  $H$  is the enthalpy,  $T$  is the absolute temperature,  $S$  is the entropy,  $U$  is the internal energy and  $p$  is the pressure. It can be thought of as the energy available to do useful work.

At constant  $T$  and  $p$ , equilibrium is attained when  $G$  tends to a minimum ( $dG=0$ ). For changes towards equilibrium at constant  $T$  and  $p$ ,  $dG < 0$ , so that  $G$  always decreases. The reduction in free energy provides the driving force for a transformation to take place and this why  $G$  is a useful function for determining the direction of reactions in phase transformations.

(b) By assuming that solid phase nucleates spheres of radius  $r$ , the net free energy change for a single nucleus is given by

$$\Delta G_{tot} = \frac{4}{3}\pi r^3 \Delta G + 4\pi r^2 \gamma$$

where  $\Delta G$  is the free energy change per unit volume associated with the transformation and  $\gamma$  is the solid-liquid interfacial energy per unit area. The critical radius  $r^*$  at which  $\Delta G_{tot}$  is maximum ( $\partial(\Delta G_{tot})/\partial r = 0$ ) is given by

$$r^* = \frac{-2\gamma}{\Delta G}$$

Recall that

$$\Delta G = \Delta H_v - T\Delta S$$

where  $\Delta H_v$  is the enthalpy change and  $\Delta S$  is the entropy change for the transformation.

$$\text{At } T_E \quad \Delta G = 0$$

Therefore

$$\Delta H_v = T_E \Delta S \Rightarrow \Delta S = \frac{\Delta H_v}{T_E}$$

Assuming that  $\Delta H_v$  and  $\Delta S$  are independent of temperature, i.e.  $\Delta S \approx (\Delta S)_{T_E}$  and  $\Delta H_v \approx (\Delta H_v)_{T_E}$

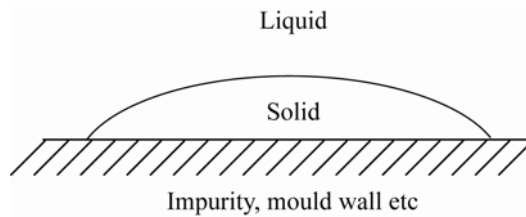
At any temperature  $T$ , you can write

$$\Delta G(\text{at } T) = \Delta H_v - T \frac{\Delta H_v}{T_E} = \frac{\Delta H_v(T_E - T)}{T_E}$$

Substituting for  $\Delta G = \frac{\Delta H_v(T_E - T)}{T_E}$  gives

$$r^* = -\frac{2\gamma T_E}{\Delta H_v(T_E - T)}$$

The homogenous nucleation of spherical nuclei during solidification of a liquid represents the simplest nucleation event, as the only atoms involved are those of the material itself. Heterogeneous nucleation introduces the idea that nuclei can form at preferential sites (eg mould wall, impurities etc). In this case, a different surface energy balance applies including extra interfacial energy terms (solid-mould, liquid-mould). The nucleus is no longer spherical (see schematic below) and achieves the critical radius  $r^*$  with fewer atoms. A smaller undercooling  $\Delta T = T_E - T$  is therefore needed to get heterogeneous nucleation.



Examples of heterogeneous nucleation are:

- The addition of Ti in steel castings to promote heterogeneous nucleation. This leads to a fine grain size and minimises segregation.
- In steels, nucleation of new phases at austenite grain boundaries.

(c)

(i) The grains may start off as spheres, but once they begin to impinge on each other they'll move towards cubes. For a final grain size of  $45\ \mu\text{m}$  (linear dimension), assuming that the grains are arranged in a regular cubic array, the number of grains present will be about

$$\frac{1}{\text{vol. of grain}} = \frac{1}{(45 \times 10^{-6})^3} = 1.1 \times 10^{13}\ \text{m}^{-3}$$

(ii) The area of the shaded rectangle (Fig. 4) must contain  $1.1 \times 10^{13}\ \text{m}^{-3}$  nuclei (because these are the ones that will grow into grains in the final product) while the shaded and unshaded rectangles together contain  $3 \times 10^{13}\ \text{m}^{-3}$  nuclei. Thus

$$r^* = 5 - \frac{1.1 \times 10^{13}}{3.0 \times 10^{13}} \times 3 = 3.9\ \text{nm}$$

(iii) Rearranging the equation in Question b gives

$$T_E - T = -\frac{2\gamma T_E}{\Delta H_V r^*} = \frac{2 \times 0.18 \times (850 + 273)}{1.5 \times 10^9 \cdot 3.9 \times 10^{-9}} = 69$$

$$\therefore T = 1054\ \text{K}$$

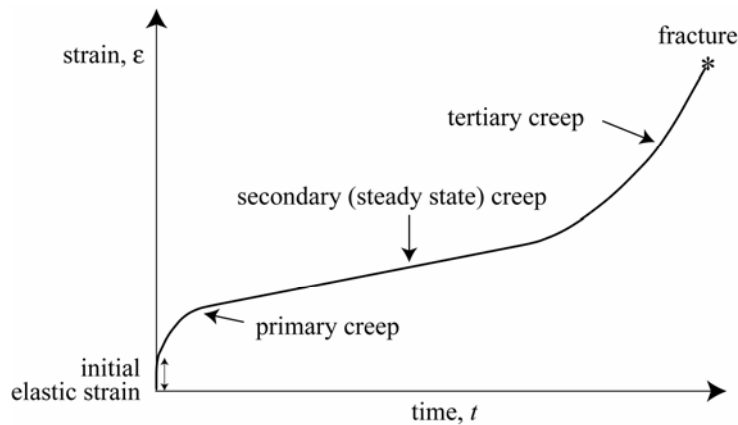
Thus an annealing temperature of approximately 1054 K must be used.

Examiner's comments:

Many candidates failed to define the Gibbs free energy correctly in part (a). Also, many resorted to various artifices in order to obtain the required answer for the critical radius in part (b). In addition, only a handful of candidates were able to show that  $\Delta G = \Delta H_V(T_E - T)/T_E$ . Part (c) was answered well. A lot of candidates used a spherical grain shape in order to obtain the number of grains, but the grains must tessellate (fill space) so clearly a cubic shape would be more sensible.

**Question 4**

(a) A schematic showing the three stages in a typical creep curve is shown below:



Three stages to the creep curve may be identified:

Stage I: Primary creep in which the creep resistance increases with strain leading to a decreasing creep strain rate.

Stage II: Secondary (Steady State) creep in which strain increases steadily with time (i.e. constant creep rate).

Stage III: Tertiary creep in which there is an accelerating creep rate due to the accumulating damage, which leads to creep rupture.

(b) Dislocation creep involves climb of dislocations to enable plastic strain. Dislocation motion is resisted by obstacles such as solute atoms, precipitates and grain boundaries. Dislocations can get unlocked from the obstacles by core or bulk diffusion. Core diffusion dominates at lower temperatures and involves atoms diffusing along the core of the dislocation. Bulk diffusion involves atoms diffusing through the bulk. In this case, dislocations diffuse away from obstacles by climbing, allowing further slip. This mechanism dominates at higher temperatures.

Diffusion creep occurs by the redistribution of material by diffusion within the bulk and along the grain boundaries, enabling deformation in the direction of applied stress. Grain boundary diffusion dominates at lower temperatures and bulk diffusion at higher temperatures.

(c)

(i) The acceleration at a position  $r$  from the centre of the turbine is given by  $\omega^2 r$ . It is greatest at the tip of the blade.

(ii) Consider an element of width  $dr$  at a radius  $r$ , the force for the element will be given by  $m \omega^2 r$  where  $m = \rho \times \text{volume} = \rho \times A \times dr$

Assuming that the cross-sectional area  $A$  of the blade is constant along the blade's length, the total force is obtained by integrating over radius between  $r_r$  and  $r_t$

$$F = \int_{r_r}^{r_t} \rho A \omega^2 r \, dr = \frac{\rho A \omega^2}{2} (r_t^2 - r_r^2)$$

The stress  $\sigma$  is then given by

$$\sigma = \frac{F}{A} = \frac{\rho \omega^2}{2} (r_t^2 - r_r^2)$$

(iii) Substitute for stress in

$$\dot{\epsilon} = B\sigma \exp\left[-\frac{Q}{RT}\right]$$

The strain rate at position  $r$  is then given by

$$\dot{\epsilon} = B \frac{\rho\omega^2}{2} \exp\left[-\frac{Q}{RT}\right] (r_i^2 - r^2)$$

The rate of change of length of an element is given by  $\dot{\epsilon} dr$ . So the rate of length change of the whole blade can be found by integrating this along the length

$$\frac{dl}{dt} = \int_{r_r}^{r_i} \dot{\epsilon} dr = B \frac{\rho\omega^2}{2} \exp\left[-\frac{Q}{RT}\right] \int_{r_r}^{r_i} (r_i^2 - r^2) dr$$

Integrating and substituting the numbers

$$\begin{aligned} \frac{dl}{dt} &= B \frac{\rho\omega^2}{2} \exp\left[-\frac{Q}{RT}\right] \int_{r_r}^{r_i} (r_i^2 - r^2) dr = B \frac{\rho\omega^2}{2} \exp\left[-\frac{Q}{RT}\right] \left[ r r_i^2 - \frac{r^3}{3} \right]_{r_r}^{r_i} \\ &= 4.0 \times 10^{-11} \times \frac{8900}{2} \times \left( \frac{2\pi \times 12000}{60} \right)^2 \exp\left[-\frac{135000}{8.3143 \times 1173}\right] \left( \left( 0.3^3 - \frac{0.3^3}{3} \right) - \left( 0.2 \times 0.3^2 - \frac{0.2^3}{3} \right) \right) \\ &= 7.3 \times 10^{-10} \text{ m s}^{-1} \end{aligned}$$

In 1000 hours, the blade will extend

$$7.3 \times 10^{-10} \times 1000 \times 3600 \sim 0.0026 \text{ m (2.6 mm)}$$

(iv) Larger grain size, so that diffusion distances are long.  
Removal of grain boundaries to produce a single crystal.

Assessor's comments:

*This was the most popular question on the paper and was generally well answered. Many candidates failed to draw a correct creep curve, with a significant number not attempting the question. Of those that made an incorrect attempt, a significant proportion did not label the axes correctly, labeling the vertical axis as the strain rate rather than the strain. In computing the creep elongation of the blade, some candidates did not convert  $Q$  into SI units which led to an incorrect answer.*



**Question 5**

(a)

(i) The relative density of a 2D foam with a square unit cell of side length  $L$  and wall thickness  $t$  is given by:

$$\frac{\rho^*}{\rho_s} = \frac{\text{perimeter} \times \text{thickness}/2}{\text{area}} = \frac{2Lt}{L^2} = \frac{2t}{L}$$

If  $t \ll L$ , deformation is dominated by bending of the horizontal struts, with any dimensional changes induced directly by the applied load being negligible in comparison.

Assuming that the end-clamped condition applies, using the Structures Databook, it can be found that the central deflection of an end-clamped beam is given by

$$\delta = \frac{WL^3}{48E_s I} - \frac{WL(L/2)^2}{8 \cdot 2E_s I} = \frac{WL^3}{192E_s I}$$

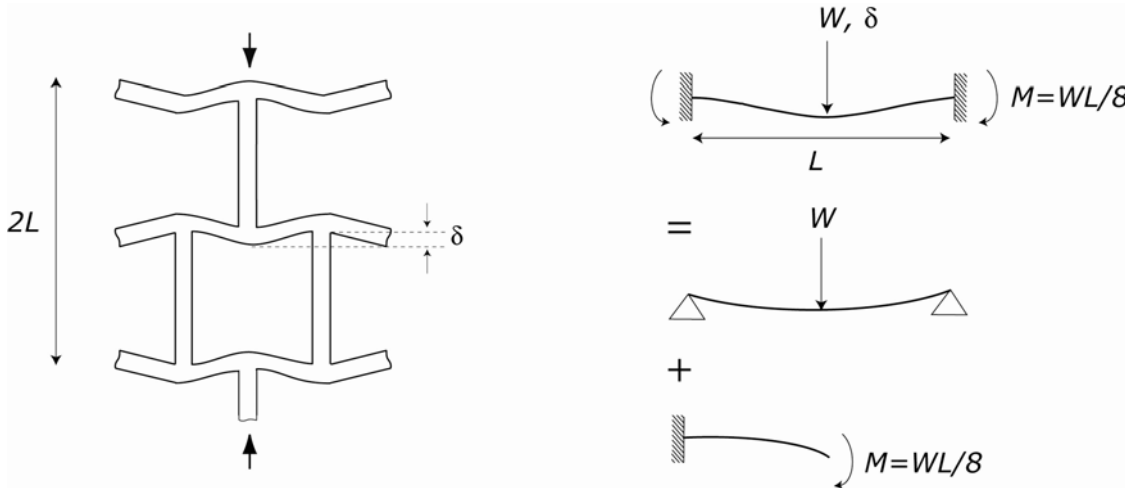
Furthermore

$$I = \frac{t^3}{12}, \quad \sigma = \frac{W}{L} \quad \text{and} \quad \varepsilon = \frac{2\delta}{2L} = \frac{\delta}{L}$$

Combining the above gives

$$E^* = \frac{\sigma}{\varepsilon} = \frac{\frac{W}{L}}{\frac{\delta}{L}} = \frac{192 W E_s I}{WL^3} = \frac{192 E_s (t^3/12)}{L^3} = 16 E_s \left( \frac{t^3}{L^3} \right)$$

$$\therefore \frac{E^*}{E_s} = 16 \left( \frac{t}{L} \right)^3 = 2 \left( \frac{\rho^*}{\rho_s} \right)^3$$



(b)

(i)

Since the two foams have the same relative density, we can write

$$\frac{\rho^*}{\rho_s} = \frac{2t}{L} = \frac{5L_1 t}{4L^2}$$

$$\therefore L_1 = \frac{5L}{8}$$

(ii) For the anisotropic foam:

$$I = \frac{t^3}{12}, \sigma = \frac{W}{4L_1} = \frac{W}{5L/2} \text{ and } \varepsilon = \frac{2\delta}{2 \times L_1} = \frac{8\delta}{5L}$$

where

$$\begin{aligned} \delta &= \frac{W(4L_1)^3}{48E_s I} - \frac{W(4L_1)(4L_1/2)^2}{8 \cdot 2E_s I} = \frac{W(5L/2)^3}{48E_s I} - \frac{W(5L/2)(5L/4)^2}{8 \cdot 2E_s I} \\ &= \frac{125 WL^3}{384E_s I} - \frac{125 WL^3}{512E_s I} = \frac{125 WL^3}{128 E_s I} \left( \frac{1}{3} - \frac{1}{4} \right) = \frac{125 WL^3}{1536 E_s I} \end{aligned}$$

Combining the above, the stiffness of the anisotropic foam can be written as

$$E^*_{anisotropic} = \frac{\sigma}{\varepsilon} = \frac{\frac{W}{5L/2}}{\frac{8\delta}{5L}} = \frac{1536 WE_s I}{500 WL^3} = \frac{1536 E_s (t^3/12)}{500L^3} = 0.256E_s \left( \frac{t}{L} \right)^3$$

From question (i)

$$E^*_{isotropic} = 16E_s \left( \frac{t}{L} \right)^3$$

The ratio of the Young's moduli of the two foams is

$$\frac{E^*_{anisotropic}}{E^*_{isotropic}} = \frac{0.256}{16} = 0.016$$

**Assessor's comments:**

*This was the least popular question on the paper. Most candidates were able to compute the relative densities of the foams and were able to identify that the length scales in the two foams are related via the relative density, which is the same for both foams. Many candidates adopted the correct approach to computing the relative Young's modulus, but did not identify the correct bending mechanism to compute the effective Young's modulus. Rather than considering a fully clamped beam, a cantilever was often considered. In computing the ratio of Young's moduli, some candidates assumed that it would scale by a factor  $L^3$ , which leads to an incorrect answer. A number of candidates did not attempt part b(ii).*

**Question 6**

(a) Viscoelasticity is a combination of elastic and viscous response exhibited in polymers. Hookean springs are used to simulate the elastic response in polymer deformation:  $\sigma = E\varepsilon$   
Dashpots are used to model the viscous (time-dependent) response in polymer deformation:  $\sigma = \eta\dot{\varepsilon}$ , where  $\eta$  is viscosity and  $\dot{\varepsilon}$  is the strain rate.

**(b)**

(i) The total strain in the system  $\varepsilon(t)$  is applied to both branch number one, whose strain is equal to  $\varepsilon_1(t)$ , and to the second branch, whose strain is  $\varepsilon_2(t)$

$$\varepsilon(t) = \varepsilon_1(t) = \varepsilon_2(t)$$

Moreover, within the series spring/dashpot branch number two, the strain is the sum of the elastic strain in the spring 2 ( $\sigma_2 / E_2$ ) and the viscous strain in the dashpot

$$\begin{aligned}\varepsilon_2(t) &= \varepsilon_{spring2}(t) + \varepsilon_{viscous}(t) \\ &= \frac{\sigma_2(t)}{E_2} + \varepsilon_{viscous}(t)\end{aligned}$$

where  $\sigma_2(t)$  is the stress carried by both members of the second branch.

Since the stress on the dashpot is also that within spring 2 ( $\sigma_2(t)$ ) we can sum their strain rates by differentiating the previous equation with respect to time

$$\frac{d\varepsilon_2}{dt} = \dot{\varepsilon}_2(t) = \frac{1}{E_2}\dot{\sigma}_2(t) + \frac{1}{\eta}\sigma_2(t)$$

Compatibility of the two branches of the assembly requires, in rate form

$$\dot{\varepsilon}(t) = \dot{\varepsilon}_1(t) = \dot{\varepsilon}_2(t)$$

$$\dot{\varepsilon}(t) = \frac{1}{E_1}\dot{\sigma}_1(t) = \frac{1}{E_2}\dot{\sigma}_2(t) + \frac{1}{\eta}\sigma_2(t)$$

where  $\sigma_1(t)$  is the stress carried in the first branch.

(ii) Consider the response to a step-change in total strain of magnitude  $\varepsilon_0$ , applied at time  $t = 0$ .

At times  $t > 0$ , the strain in both branches is constant

$$\dot{\varepsilon}(t) = \dot{\varepsilon}_1(t) = \dot{\varepsilon}_2(t) = 0$$

$$\text{Thus } 0 = \frac{1}{E_2}\dot{\sigma}_2(t) + \frac{1}{\eta}\sigma_2(t)$$

The first order differential equation leads to the decaying exponential solution form

$$\dot{\sigma}_2(t) = -\frac{E_2}{\eta}\sigma_2(t)$$

$$\sigma_2(t) = \sigma_{20} \exp\left(-\frac{E_2}{\eta}t\right)$$

where  $\sigma_{20}$  is the stress carried by both members of the second branch just after the instantaneous application of the total strain jump ( $t = 0^+$ ).

Assuming that at this short-time limit the total strain in the second branch is completely accommodated by spring 2

$$\sigma_2(t) = E_2 \exp\left(-\frac{E_2}{\eta}t\right) \varepsilon_0$$

At  $t > 0$ , the strain in the first branch is simply  $\varepsilon_1(t) = \varepsilon_0$ , so the stress is constant, of magnitude  $\sigma_1(t) = E_1 \varepsilon_0$ .

The stress  $\sigma_2(t)$  relaxes exponentially with time constant  $\tau$  given by

$$\tau = \frac{\eta}{E_2} = \frac{2 \times 10^{12}}{400} = 5 \times 10^9 \text{ s}$$

The total stress,  $\sigma(t)$ , is the sum of the stresses carried by the two parallel-loaded branches:

$$\sigma(t) = \sigma_1(t) + \sigma_2(t)$$

Adding the two stress contributions,  $\sigma_1(t)$  and  $\sigma_2(t)$ , gives the total stress

$$\sigma(t) = E_1 \varepsilon_0 + E_2 \exp\left(-\frac{E_2}{\eta} t\right) \varepsilon_0 = E_r(t) \varepsilon_0$$

$$\therefore E_r(t) = E_1 + E_2 \exp\left(-\frac{E_2}{\eta} t\right)$$

Assessor's comments:

*This was the second least popular question on the paper. A reason for the low average mark for this question is that many candidates did not attempt large parts of the question, notably part (b). Almost all candidates could write down the governing equations for a spring element and a dashpot element, but a number struggled to express the strain rate for a spring and dashpot system in series in terms of the model parameters and the stress. For part (b) of the question, some candidates did not realise that the total strain rate in the spring and in the spring-dashpot system must be equal. Students that could not derive the differential equation which could then be solved to give an expression for the stress were often able to guess the equation. Some candidates did not identify that when a constant strain is imposed that the strain rate is zero. A number of candidates followed an unnecessarily complex procedure to solve the differential equation using Laplace transforms.*