ENGINEERING TRIPOS PART IB 2008
Paper 5
ELECTRICAL ENGINEERING

CRIB

Q 1.

(a) Often used as output stages in audio amplifiers where amplification with low distortion is required; power output from 1W upward. Can source or sink current. Well suited to battery-operated equipment, since efficiency is high and quiescent current low.

The circuit offers the following advantages:-

- Greater power output than from equivalent devices in class A
- Greater efficiency than Class A
- Load typically shared by two matched complementary devices
- Very low consumption in quiescent state
- Power consumption roughly proportional to power output
- Low distortion can be achieved by careful adjustment of biasing (Class AB1)
- Negative feedback can be applied to further reduce distortion

(b) Consider the maximum amplitude sine wave that can be produced at the output. Assume ideal devices, in which the emitter can rise/fall to the collector potential.

In practice this cannot be achieved, and V_{CE} would not normally be allowed to fall below a few volts.

Let max peak output amplitude be V_P. $V_P = (V_{CC} - V_{EE})/2 = 15$ V

Hence the peak output current $I_{peak} = V_P/R_L = 15/8 = 1.875$ A

Power output at maximum amplitude $0.5\, V_P I_P = 0.5 \times 15 \times 1.875 = 14.06$ W RMS

For the V_{CC} rail, the average DC power input = $V_{CC} \times I_{CCav}$. For V_{EE} it is $V_{EE} \times I_{EEav}$

For average currents I_{CCav} & I_{Eeav}: $I_{CCav} = \frac{1}{2}\frac{1}{\pi}\int_0^{\pi} I \sin \omega t . d\omega t = \frac{1}{2\pi}[-\cos \omega t] = I_{peak}/\pi$

Hence I_{CCav} is $V_{CC}/(\pi \times R_L)$; similarly for I_{EEav}. The average DC power consumed is therefore $15 \times 15/(\pi \times 8)$ per rail, and hence twice this for the two rails.

Hence the average DC power from the two rails is: $15 \times 1.875 \times 1/\pi \times 2$

There was great confusion between peak and peak-to-peak voltages and current. Some candidates could not calculate the power in a sine wave or deduce an expression for the DC power consumed.

$$\text{The efficiency } \varepsilon = \frac{0.5 \times 15 \times 1.875}{15 \times 1.875 \times \frac{1}{\pi} \times 2} = \frac{\pi}{4} = 78.5\%$$

The maximum efficiency is much greater than that achieved in a capacitor-coupled class A stage is 25%. In Class A, the efficiency is often considerably less; also the power consumption is maintained even when no signal is presented at the input.

Several considered only the power consumed in R1 and R2 and ignored that drawn from the supply to drive the load, and got crazy values for η.

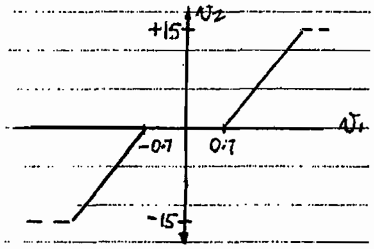
Here, complementary devices are connected as emitter followers with a common output terminal. They are biased so that with a rising input waveform, T1 just begins to conduct (V_{BE} just rising into forward-biased region) as T2 stops conducting (V_{BE} dropping below forward-biased region).

Without diodes, T1 & T2 conduct for <50% of the cycle. Severe crossover distortion is seen. The diodes keep the two bases about 0.7V + 0.7 = 1.4 V different in potential. As a result the crossover region shrinks, so reducing the crossover distortion. However, more current may flow when zero signal is applied.

(d) At any instant, only one device is actually conducting, T1 for positive half-cycles and T2 for negative half-cycles. T1 and T2 are complementary, and have matched small signal parameters. Since they are never operating simultaneously, the SSEC shows only one device. Consider the SSEC for T1 (exactly as for T2).

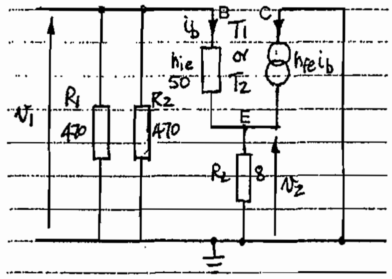
NB the diodes can be regarded as fixed voltage sources of 0.7V, and appear as shorts in the SSEC.

Many ended up with the wrong SSEC. Common errors: two transistors shown – only one is operating at any instant; wrong connections – had the device been marked E, B, C this would have been avoided; many split the load into two of 4Ω or 16Ω.



(without D1 & D2)

Transfer function with crossover distortion



Small-signal equivalent circuit

Voltage gain is v_2/v_1. Using KVL in the base circuit: $v_1 = i_b h_{ie} + v_2$ (1)

Using KCL at the emitter of the device:

$$i_b + h_{fe} i_b = \frac{v_2}{R_L} \qquad i_b(1+h_{fe}) = \frac{v_2}{R_L} \quad \text{and} \quad i_b = \frac{v_2}{R_L(1+h_{fe})} \qquad (2)$$

Eliminating i_b from (1)

$$v_1 = v_2\left(\frac{h_{ie}}{R_L(1+h_{fe})} + 1\right)$$

Hence

$$\frac{v_2}{v_1} = \frac{1}{\frac{h_{ie}}{R_L(1+h_{fe})} + 1} = \frac{1+h_{fe}}{\frac{h_{ie}}{R_L} + 1 + h_{fe}}$$

Substituting values, the voltage gain is about 0.89.

However, there is high current gain; it is quite easy to provide voltage gain in earlier (class A) stages.

(e) Amplifiers like these are used to develop high amplitude output waveforms spanning from rail to rail. Small-signal theory is a set of approximations based on calculus of variations, and assumes that the signal being considered is small relative to the supplies. This is not so here. Another reason is that without the diodes there is a severe non-linearity in the crossover region, in which small-signal approximations do not hold.

Q 2.

(a) Advantages of negative feedback

- System gain may be determined externally
- Reduced effect of characteristics of the op-amp
- Distortion and non-linearities are reduced
- Bandwidth is extended
- Modifies input/output resistance Rin/Rout – depends on feedback mode used:
    - Voltage feedback: Rin increased, Rout decreased
    - Current feedback: Rin decreased, Rout increased

Not many could remember more than a couple of these advantages.

(b) The –3dB point will be reached when $\omega_u T = 1$, hence $\omega_u = 1/T$. Hence the upper corner frequency without feedback is thus 10 Hz. This is extremely low, and the device could not be used unaided as (say) an audio amplifier.

(c) To determine the gain $G = \frac{v_2}{v_1}$ with negative feedback, start by writing:

$$v_2 = A(\omega)(v_1 - \beta v_2) \text{ where } \beta \text{ is the feedback factor}$$

$$\beta = \frac{R_1}{R_1 + R_2}$$

Rearranging,

$$G = \frac{A(\omega)}{1 + \beta A(\omega)}$$

assuming the op-amp has zero output resistance and infinite input resistance.

$$G = \frac{A}{1+\beta A} = \frac{A_0 / (1 + j\omega T)}{1 + \beta A_0 / (1 + j\omega T)} = \frac{A_0}{1 + j\omega T + \beta A_0}$$

$$= \frac{A_0 / (1 + \beta A_0)}{(1 + j\omega T + \beta A_0) / (1 + \beta A_0)} = \frac{A_0 / (1 + \beta A_0)}{1 + j\omega T / (1 + \beta A_0)}$$

This is in the form: $\frac{A_0 / k}{1 + j\omega T / k}$, where $k = 1 + \beta A_0$.

Write G_0 = midband gain = $\frac{A_0}{1 + \beta A_0}$

Hence $$G = \frac{G_0}{1 + j\omega T / (1 + \beta A_0)}$$

Considering this expression, we see that the upper corner frequency is pushed up by a factor $1 + \beta A_0$ relative to the case without feedback.

(d) Substituting the given values:-

(i) $\beta = 1/11$ $1 + \beta A_0 = 9092$ Hence $\omega_c = 90.92$ kHz

(ii) $\beta = 1/501$ $1 + \beta A_0 = 200.6$ Hence $\omega_c = 2006$ Hz

(e) This behaviour can be described in terms of the Gain-Bandwidth product, which characterises the op-amp and determines the maximum bandwidth obtainable under different feedback conditions. The Gain-Bandwidth product is defined as $A_0 \times \omega_u$.

For this amplifier it is $10^5 \times 10 = 10^6$ Hz, and it remains applicable even when negative feedback is applied to fix the gain.

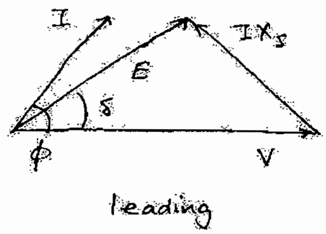
Using this approach, if the feedback is set for unity gain, a maximum bandwidth of 1MHz is expected. If the feedback is set for a gain of ×10, a bandwidth of about 100 kHz is seen; and if the feedback is set for a gain of ×500, a much lower bandwidth of 2kHz is seen.

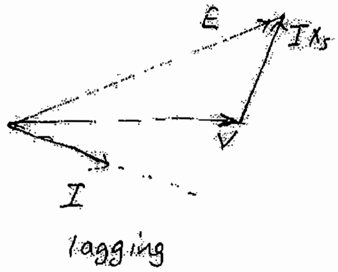
In all cases the gain (more accurately β) times the bandwidth obtained is equal to the GBW product.

Not many candidates were aware of the connection with the Gain-Bandwidth product

Q3

a)





b) operating chart = phasor diagram x $(3V/X_S)$.
x axis = Q, y axis = P.

c) S = 1000 MVA V_L = 60 kV $X_s = 3\ \Omega$

P = 640 MW $\cos\phi = 0.8$ lag

$V_p = 60/\sqrt{3} = 34.64$ kV

$I_p = P/(3V_p\cos\phi) = 640/3 \times 34.64 \times 0.8 =$ **7.70 kA**

IX = 23.09 kV

E_p = 51.90 kV, E_L = 89.89 kV

d) $E_{p,new} = 1.2 \times 51.90 = 62.28$ kV. IX' = 30.95, **I'_p = 10.32 kA**

e) operating chart

$\cos\phi_{max} = 0.8$,

$3V^2/X = 1{,}200$, $3VE/X = 2{,}000$ S = 1,000, cosine rule
so $\cos(90+\phi_{min}) = 0.65$, **$\cos\phi_{min} = 0.76$**

Q4 a)
Delta parallel $V_L = 415$ V
Do not do star delta transform.

As R and X are in *parallel*, do not calculate Z., you can get immediately

$P = 3V^2/R = 3 \times 415^2/12 =$ **43.1** kW
$Q = 3V^2/X = 3 \times 415^2/8 =$ **64.6** kVA   S = 77.65 =

$\cos\phi = 43.1/77.65 =$ **0.555**

Star $V_p = 240$ V

As R and C in series, must calculate I_p.
Z = 11.18 ohm $I_p = 240/11.18 = 21.4$ A

$P = 3I^2 R = 3 \times 21.4^2 \times 10 =$ **13.74 kW**
$Q = 3I^2 X = -3 \times 21.4^2 \times 5 =$ **-6.97 kVA**

S = 15.36 kVA $\cos\phi = 2/\sqrt{5} =$ **0.89**

Total P, Q, S
P = 56.8 kW, Q = 57.6 kVA, **S = 80.96 kW**

$I_L = S/3^{1/2} V_L = 91{,}320 / 1.732 \times 415 =$ **112.6 A**   **Cos ϕ = 0.702**

b)
for reduction to $\cos\phi = 1$, need $-Q = 57.6 = 3V^2/X_C$ for C's in star.

$X_C = 3.\ 240^2/57.6\ 10^3 = -3.0$ ohm
C = 1.1 mFarads

Or reduction to $\cos\phi=0.95$ $\tan\phi=0.33$

$Q' = P \tan\phi = 56.8 \times 0.33 = 18.67$ kVA

$dQ = Q - Q' = 57.6 - 18.67 = 38.93$ kVA

$X_c = 3V^2/dQ = -4.44$ ohm   or C = 0.72 mF

c) new S = P, $I_L = 56.8/\ 1.732 \times 415 = 79$ A
70% of original, 30% down.

New S = 59.79 kW $I_L = 59.8/\ 1.732 \times 415 = 83.2$ A
74% of original 26% down
Not worth the extra large capacitors to get the last reduction if PF.

Q5

c) 4 pole pairs, p =4

Synchronous speed = $w_s = 2\pi 50/4 = 78.5$ rad/s or 750 rpm. actual speed = 720 rpm

s = (750 – 720) / 750 = **0.04**

$R'_2/s = 2.5/0.04 = 75$ ohm

As impedances in the magnetising arm as large (but not negligible), move this arm to the left of R1; so

$$\frac{1}{Z} = \frac{1}{77.5+4j} + \frac{1}{560j} + \frac{1}{1200} = 0.0129 - 0.00066j - 0.0018j + 0.000833 = 0.0137 - 0.0025j$$

R= 70.53 Ω, X = 12.62 Ω, **Z = 71.68 Ω.**

$I_p = 415/Z =$ **5.79A**

$$T = \frac{3I'^2_2}{\omega_s}.\frac{R'_2}{s} \sim 3\,(5.79)^2\,75\,/(2\,\text{pi}\,12.5) = \mathbf{96.1\ N.m}$$

Q6 (a)

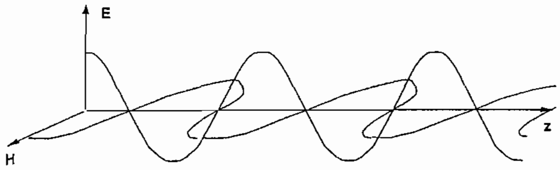
$E = E_0 u_x \exp j(wt-\beta z)$

Substitute into wave eqn, to give $-\beta^2 E_0 \exp j(wt-\beta z) = -w^2 E_0 \exp j(wt-\beta z) . \mu_0\varepsilon_0$

Giving $\frac{\beta}{\omega} = \sqrt{\mu_0 \varepsilon_0}$

The speed of propagation (phase velocity) is given by $\frac{\omega}{\beta} = 1/\sqrt{\mu_0 \varepsilon_0}$

b)



$H = H_0 u_y \exp j(wt-\beta z)$

Defining impedance as $\eta_0 = E/H$ (by equiv to V/I).

Putting E into Faraday-Maxwell eqn, $\frac{\partial E}{\partial x} = -\mu_0 \frac{\partial H}{\partial t}$ gives $\frac{E}{H} = \frac{\mu_0 \omega}{\beta} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

$H = E_0/\eta_0 \exp j(wt-\beta z)$

From $\frac{1}{2} E \times H = E_0^2 / 2\eta_0$

c) $\eta_0 = 377\ \Omega$,

so $P/\pi d^2/4 = 0.001 / (10^{-6} \times 3.1416/4) = 1.27\text{e}3\ W/m^2$. $= E^2 / 2\text{x}377$,

so E = **980 V/m**, H = 980 / 377 = **2.60 A/m.**

d) $\eta_L = \eta_0 /(4)^{1/2} = \eta_0/2$

$$\rho_R = \frac{E_R}{E_0} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-1/2}{3/2} = -\frac{1}{3}$$

$E_R = -1/3\ E_0$, $E_0 + E_R = E_T$, so $E_T = 2/3\ E_0$

Or, $\rho_R^{\ 2} = 1/9$. $\rho_R^2 + \rho_T^2 = 1$, so $\rho_T^2 = 8/9$. $\rho_T^2 = E_T^2/(\eta_T/\eta_0) = (8/9).(1/2)$, so $E_T^2 = 4/9$, $E_T = 2/3\ E_0$.

$E_0 = 980$ V/m so E_T = **654 V/m** .

Transmitted power = $E_T^2/2\eta_L = (2/3\ E_0)^2 /(2\ \eta_0/2)$ = **1.13 kW/m²**

Q7 a)

$v = 1/(LC)^{1/2} = 1/(360.10^{-9}. 80.10^{-12})^{1/2}$ = **1.86x10⁸ m/s**

$Z = (L/C)^{1/2}$ = **67 ohms**

If $\mu_r = 1$, $\varepsilon_r = (3.10^8/1.86.10^8)^2 = \mathbf{2.6}$

b) wavelength $\lambda = v/f = 1.86.10^8 / 300.10^6 = \mathbf{0.62}$ m.

for open circuit, length = $\lambda/4$ = **0.155 m**

for short circuit, length = $\lambda/2$ = **0.31 m**

c) $$\rho_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100-67}{100+67} = 0.197 \sim \mathbf{0.2}$$

$$\text{VSWR} = \frac{V_F + V_B}{V_F - V_B} = \frac{1.2}{0.8} = 1.5$$

d) $V_{in} = 0.5\ V_{GEN} = 50$ V.

$$\text{Power} = \frac{V_F{}^2}{Z_0}(1 - \rho_L{}^2) = \frac{50^2}{67}(1 - 0.04) = 35.8W$$