2008 IB PAPER 8 SELECTED TOPICS

SECTION A	INTRODUCTORY BUSINESS ECONOMICS	MR M KITSON
SECTION B	CIVIL AND STRUCTURAL ENGINEERING	DR S P G MADABHUSHI
SECTION C	MECHANICAL ENGINEERING	DR HEM HUNT
SECTION D	AEROTHERMAL ENGINEERING	DR CA HALL
SECTION E	ELECTRICAL ENGINEERING	PROF W I MILNE
SECTION F	INFORMATION ENGINEERING	PROF R CIPOLLA
SECTION G	BIOLOGICAL AND MEDICAL ENGINEERING	DR M L OYEN
SECTION H	MANUFACTURING, MANAGEMENT AND DESIGN	DR J M ALLWOOD

SECTION A *Business Economics* **CRIBS**
Answer not more than ***one*** *question from this section*

1
(a) Explain the concept of profit maximisation. [4]

Profits are maximised when the next unit produced and sold, adds as much to total revenue as it does to total cost. Profit maximisation occurs when marginal revenue = marginal cost (MR=MC). If MR exceeds MC profit can be increased by increasing production, if MC exceeds MR profit can be increased by cutting back on production.

(b) Explain the concept of contestable markets. [6]

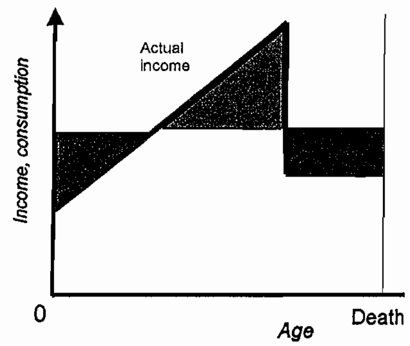
A contestable market is one in which there is one firm (or a small number of firms) but because of freedom of entry and exit, the firm faces competition and so operates like a perfectly competitive firm. The threat of "hit and run entry" from new firms may be sufficient to keep the industry operating at a competitive price and output. Contestable markets are different from perfect competitive markets as it is possible for one incumbent firm to dominate the industry. There are 3 conditions for contestability:

- Perfect information and the ability / right to use the best available technology
- Freedom to market / advertise and enter a market
- The absence of sunk costs

(c) Compare and contrast the Keynesian consumption function model with the Life Cycle model of consumption. [10]

According to the Keynesian consumption function, current consumption (C) is determined by current personal disposable income (Y). The implications of the Keynesian Consumption function are: the distribution of income will affect total consumption; economies may suffer from 'underconsumption' as they grow; Government's can expand demand through fiscal policy. For instance a change in income taxes will lead to a change in current personal disposable income leading to a change in consumption and in aggregate demand.

The Life-Cycle model considers consumption also being as a constant proportion of long-run or normal income. The LC model emphasises the age of the consumer, and proposes that he/she attempts to smooth consumption over a lifetime in which income fluctuates widely. As shown below the hypothesis suggests, under normal conditions, dissaving in youth and old age.



The implications of the LC model are that current consumption doesn't vary as much as current income and that temporary income changes have less impact on spending than permanent income changes. If a tax cut is considered to be temporary, it will have little impact on consumption – and the extra disposable income may be saved in anticipation of compensating income tax rises in the future. If a tax cut is considered to be permanent it will lead to rise in consumption. Therefore fiscal policy is not a powerful tool unlike in the Keynesian case.

2

(a) Describe the model of perfect competition. [6]

The model is based on the following:
Assumptions of perfect competition:
Many firms

Identical cost functions, U shaped
Homogeneous products
Free entry and exit to market
Perfect information

The outcome of perfect competition is:

P=min AC=MC

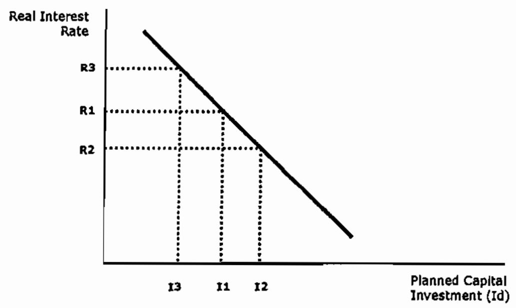
(b) Why is perfect competition normally considered to be more desirable than monopoly? [5]

Economic welfare is normally lower with a monopoly as total surplus is lower under monopoly than under perfect competition. Producer surplus is higher, and consumer surplus lower under monopoly. The deadweight losses of monopoly is the lost surplus. There may be distributional issues associated with the mix of surplus, usually we assume the loss of consumer surplus is more serious than an equivalent loss of producer surplus. Prices are lower under perfect competition, if min AC is the same in both cases.

(c) What impact would the following have on the level of investment in the macroeconomy:

(i) an increase in the rate of interest; [3]

An increase the interest rate will decrease the amount of autonomous investment in the macro economy. This is because the downward sloping marginal efficiency of capital curve indicates that there are less profitable investment opportunities at higher interest rates. See below:



An increase in interest rates from R2 to R1 will result in fewer capital projects being profitable (comparing interest rates with expected rates of return on capital)

(ii) an expected increase in future economic growth; [3]

If economic growth is expected to increase then the Accelerator model suggests that the desired size of capital stock will also increase. Net capital investment is the amount by which the required capital stock changes. It follows that the amount of investment depends on the size of the expected change in output. If the latter is expected to increase then the desired size of the capital stock needs to be increased, and – then investment will increase. Some candidates may mention the role of 'animal spirits' and the limitations of the accelerator model

(iii) an increase in the in the real exchange rate? [3]

This may work through various mechanisms. First, if the higher exchange rate reduces net exports trade it will reduce demand and may reduce income induced investment (see 2cii above). But this will mainly be in tradeable sectors – that is sectors that produce exports or import substitutes. Second, it may decrease the price of imported capital goods – this may impact on investment depending on the availability of capital goods in the domestic economy. Third, if the change in the exchange rate has any subsequent impact on interest rates this may also influence investment through the impact on the marginal efficiency of capital.

SECTION B

Q.3 Solution

(a) Stability ratio, $N = (\gamma z - \sigma_T)/s_u$

where γ = unit weight of clay
z = depth of tunnel axis below ground level
σ_T = support pressure applied to tunnel face
s_u = undrained strength of the clay [5]

(b) In the case of an open face ($\sigma_T = 0$) tunnel in Cambridge in Gault Clay

$N = (20 \times 20) / 200 = 2$

In the case of an open face ($\sigma_T = 0$) tunnel in Bangkok in marine clay

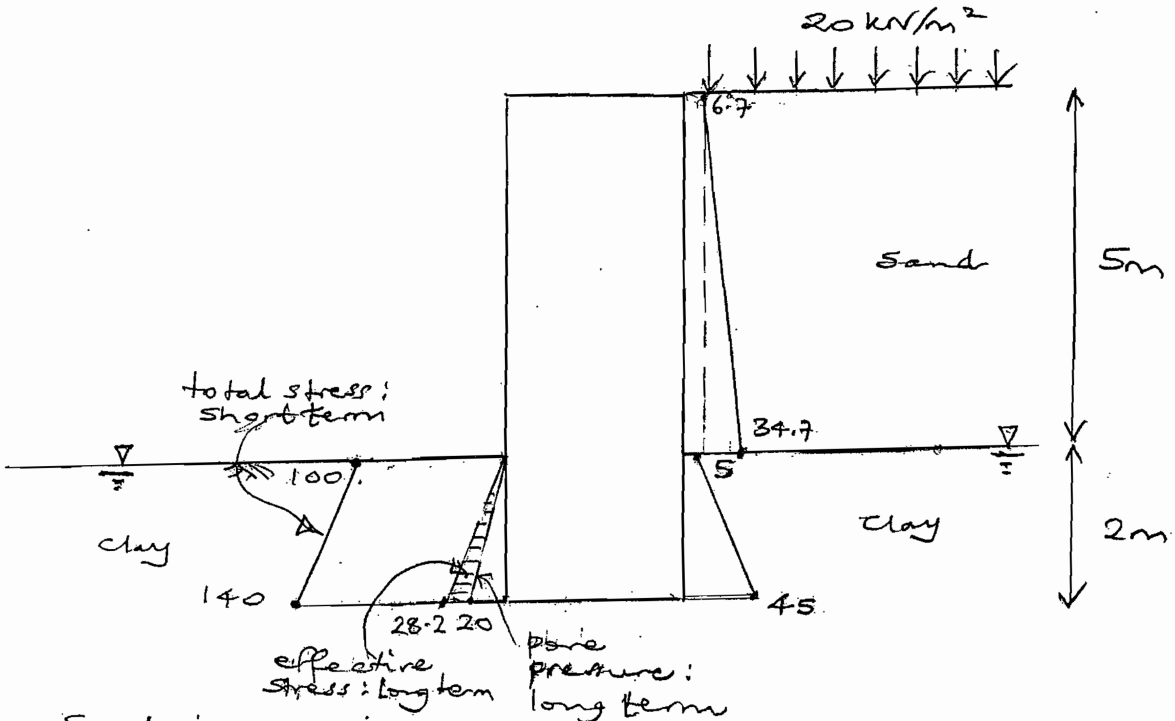
$N = (20 \times 17) / 40 = 8.5$

If N is less than about 5, the tunnel will be stable. Hence an open face tunnel in Cambridge will be stable whereas an open face tunnel in Bangkok will be unstable. [5]

(c) *Box jacking* is very suitable for constructing a tunnel beneath a live railway (eg in Boston recently), where the cut and cover technique is being used for the tunnel either side of the railway (but clearly cannot be used across the railway itself). It involves construction of a large excavation on one side of the railway for the construction of the reinforced concrete box, which is then jacked from this excavation. Reaction blocks have to be designed for the jacks to react against. Friction has to be reduced during jacking – often by a sophisticated system of greased cables. The front edge of the box has a steel cutting edge incorporated. The ground is excavated at the front of the box during the jacking process. In very weak ground, freezing techniques are used to stabilize the ground. A key advantage of the process is that a large tunnel can be constructed safely at extremely shallow depths beneath a live railway with disruption minimised, providing that ground settlements are well controlled. A disadvantage is the susceptibility of the jacking to unknown obstructions in the ground, eg uncontrolled fill and old foundations. [5]

(d) *Compensation grouting* involves injection of grout into the ground between the tunnel and the building foundation in a controlled manner, the aim being to compensate for the ground movements being caused by the tunnelling. The principal aim is to reduce the potential differential settlement of the building, thereby limiting damage. The grout is injected from tube-à-manchettes (TAM's) which are installed in the ground prior tunnelling, usually from an adjacent shaft but sometimes from an existing tunnel. The control of the grouting – when and where to grout, and how much – depends on measurements of the building and ground response using instrumentation. *Instrumentation* is installed on the building (levelling and/or electrolevels) and in the ground (levelling and extensometers) to monitor settlement and ground movements, and the grout is injected in response to the measurements. [5]

Q. ~~4~~ Solutions



Sand

$\phi' = 30°$, $K_a = \dfrac{1-\sin\phi'}{1+\sin\phi'} = \dfrac{0.5}{1.5} = 0.3\dot{3}$

Clay

$\phi' = 25°$ $\quad K_a = \dfrac{1-\sin\phi'}{1+\sin\phi'} = 0.41$

$K_p = 1/K_a = 2.46$

(a)

Active pressures

(i) Sand backfill (dry, ∴ no water pressure, ∴ $\sigma_v' = \sigma_v$)

horizontal stress due to surcharge $= K_a\sigma_v' = 0.3\dot{3} \times 20$
$= \underline{6.7 \text{ kPa}}$

at bottom of sand, $\sigma_v' = 20 + 5\times 17 = 105$ kPa

∴ $\sigma_h' = K_a \sigma_v' = 0.33 \times 105 = \underline{34.7 \text{ kPa}}$

(ii) clay

top of clay, $\sigma_v = 105$ kPa

$\sigma_h = \sigma_v - 2S_u = 105 - 2\times 50 = \underline{5 \text{ kPa}}$

bottom of clay $\sigma_v = 105 + 2\times 20 = 145$ kPa

$\sigma_h = \sigma_v - 2S_u = 145 - 2\times 50 = \underline{45 \text{ kPa}}$

Passive pressures

top of clay $\quad \sigma_v = 0$, $\sigma_h = \sigma_v + 2S_u = 0 + 2\times 50 = \underline{100 \text{ kPa}}$

bottom of clay $\quad \sigma_v = 2\times 20 = 40$ kPa, $\sigma_h = 40 + 2\times 50 = \underline{140 \text{ kPa}}$

(b) Total active force, P_A

$= (6.7 \times 5) + \frac{1}{2}(34.7 - 6.7) \times 5 + (5 \times 2) + \frac{1}{2} \times 40 \times 2$ kN/m

$= 33.5 + 70 + 10 + 40 = \underline{153.5 \text{ kN/m}}$

Total passive force, P_P

$= (100 \times 2) + \frac{1}{2} \times 40 \times 2 = \underline{240 \text{ kN/m}}$

Friction force on base $= P_F = 150$ kN/m

∴ Factor of safety against sliding $= \dfrac{P_P + P_F}{P_A} = \dfrac{240+150}{153.5} = \underline{\underline{2.5}}$

(c) On passive side, clay softens, and long term conditions are reached.

At surface $\sigma_v' = 0$, $\sigma_h' = 0$

At bottom of clay $\sigma_v = 2 \times 20 = 40$ kPa

water pressure $u = 2 \times 10 = 20$ kPa

∴ effective stress $\sigma_v' = \sigma_v - u = 40 - 20 = 20$ kPa

$\sigma_h' = K_a \sigma_v' = 0.41 \times 20 = 8.2$ kPa

$u = 20$ kPa

∴ $\sigma_h = \sigma_h' + u = 20 + 8.2 = \underline{28.2 \text{ kPa}}$

(d) Total active force $P_A = 153.5$ kN/m, as before

Total passive force $P_P = \frac{1}{2} \times 28.2 \times 2 = 28.2$ kN/m

Friction force $P_F = 150$ kN/m, as before

Factor of safety against sliding $= \dfrac{P_P + P_F}{P_A} = \dfrac{28.2 + 150}{153.5} = \underline{\underline{1.2}}$

The effect of the clay softening is to reduce the factor of safety against sliding from 2.5 to 1.2, with the stability relying on the friction acting on the base of the wall.

A reinforced concrete wall AC supports an excavation that is 7 m deep, and projects a further 2 m into the ground to D, as shown in Fig xxx. Tie rods, at 2 m intervals along the length of the wall, at position B, are connected to remote anchor blocks. Each rod is tensioned to a fixed tension force of 1000 kN.

The cube strength of the concrete is 30 N/mm^2, and the yield strength of the reinforcing bars is 460 N/mm^2. The ground pressures shown in the figure are those that must be resisted at the ultimate load condition.

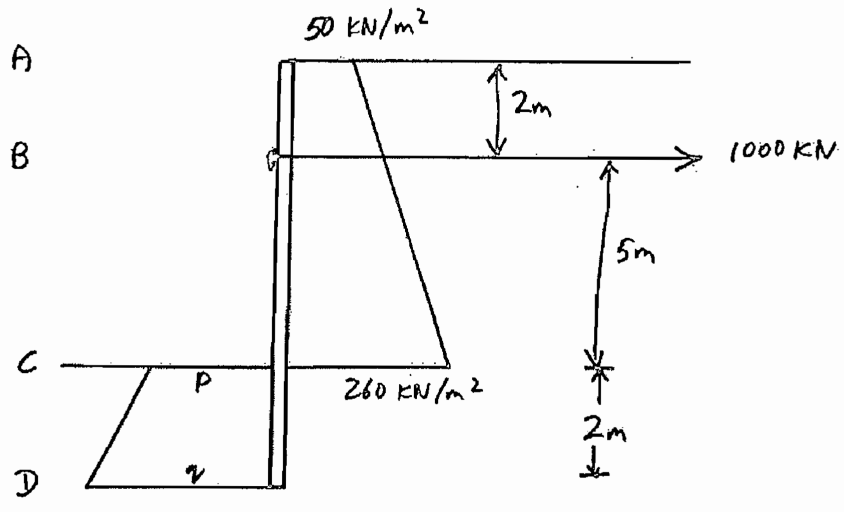
(a) If the pressure applied by the ground is p at C and q at D, and is assumed to vary linearly between those points, find p and q .

(b) Calculate the shear force and bending moments at A B C and D, and also at any other point where the shear force is zero. Sketch the resulting shear force and bending moment diagrams.

(c) If the wall is to be singly reinforced, calculate the minimum effective depth of the section for the most heavily loaded section.

(d) If the actual effective depth of the wall is 40% bigger than the value calculated in (c), and it is uniform throughout the wall, determine a suitable layout of reinforcement in the wall.

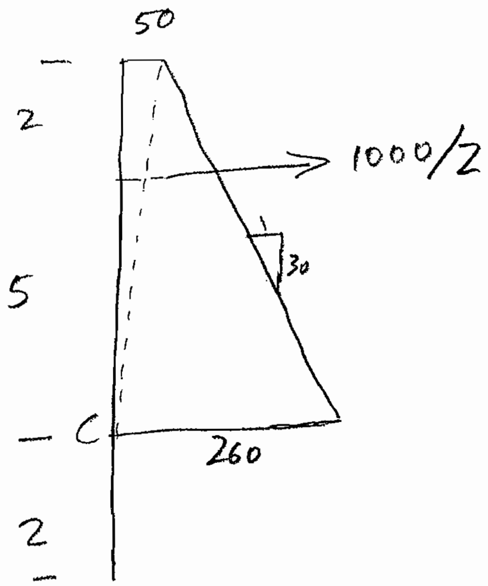
(e) What special factors might need to be taken into account when designing the reinforcement for this wall.



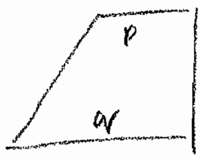
1B Paper 8 Qu 5

N.B. 2m centres for tie

5 (a)



(for one metre)



Find shear force and moment at C by looking up

$$F = \frac{(260+50)}{2}.7 - 500 = 585 \text{ kN/m}$$

$$M = \frac{50}{2}.7.\frac{2}{3}.7 + \frac{260}{2}.7.\frac{1}{3}.7 - 500 \times 5$$

$$= 817 + 2123 - 2500$$

$$= 440 \text{ kNm/m}$$

Looking down

$$F = 585 = \frac{(p+q)}{2}.2 = p+q$$

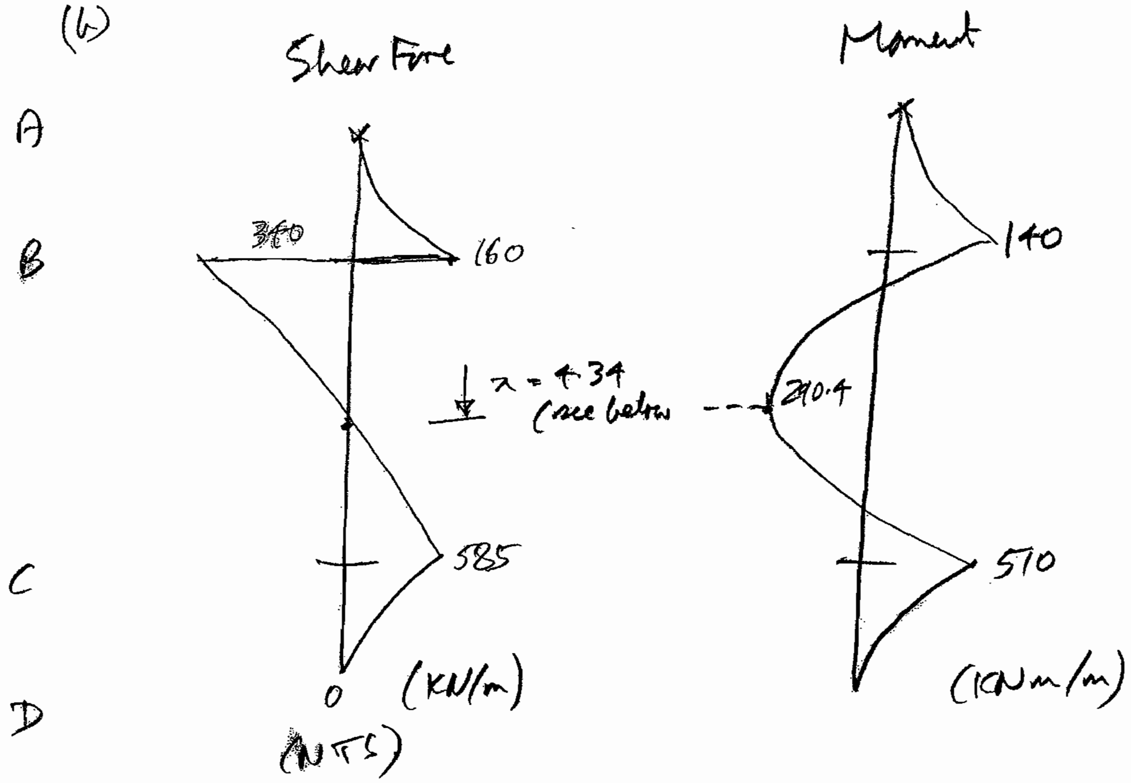
$$M = 440 = \frac{p}{2}.2.\frac{2}{3} + \frac{q}{2}.2.\frac{4}{3} = \frac{2p}{3} + \frac{4q}{3}$$

$$\Rightarrow p + 2q = 660$$

$$\therefore q = \underline{75 \text{ kN/mm}^2} \Rightarrow p = \underline{510 \text{ kN/mm}^2}$$

(2)

(b)



x when F=0

$$50x + \frac{30x^2}{2} = 500$$

$$x = \frac{-50 + \sqrt{2500 + 4 \cdot 15 \cdot 500}}{30} = 4.34\text{m}$$

$$M_B = 2\left(\frac{50}{2}\cdot\frac{4}{3} + \frac{110}{2}\cdot\frac{2}{3}\right) = 140 \text{ KNm/m}$$

$$M_x = \frac{50}{2} \cdot 4.34 \cdot 4.34 \cdot \frac{2}{3} + \frac{180 \cdot 2}{2} \cdot 4.34 \cdot 4.34 \cdot \frac{1}{3} - 500 \cdot 2.34$$

$$= -290.4$$

(c) Maximum moment = 510 KNm/m

$$0.15 f_{cu} b d^2 = 510 \cdot 10^6 = 0.15 \cdot 30 \cdot 1000 \cdot d^2$$

$$\Rightarrow d \geqslant \underline{337 \text{ mm}}$$

(d) Actual d = 472 mm ($= 1.4 \times 337$)

(accept any round number nearby 470, 480, 500 etc)

(3)

[Span/depth = $\frac{5000}{470}$ = 10.6 – reasonable]

· Calculate required steel area .

Guess neutral axis depth = 0.4.d = 189 mm

$M = 0.87 . f_y . A_s (d - \frac{x}{2})$

$510.10^6 . = 0.87. 460. A_s (472 - \frac{189}{2})$ (377.5)

$\Rightarrow A_s = 3375$ mm²/m

Check n.a depth

$(0.87 f_y . A_s = x b . 0.4 . d . f_{cu}$

$x = 2.18 . \frac{f_y}{f_{cu}} . \frac{A_s}{bd}$

$x = 0.239$

∴ n.a. depth = 112 mm

new lever arm = 416 mm

∴ new As = 3063 mm²/m

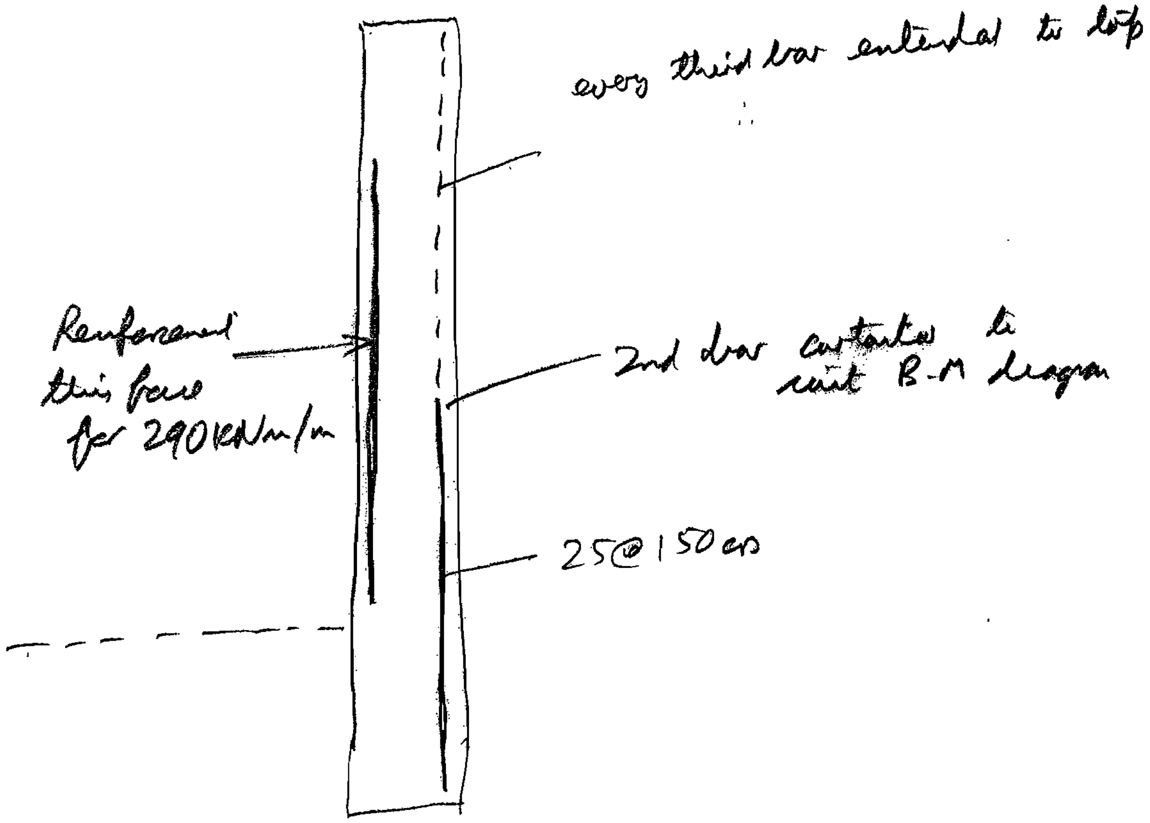
[new x = 0.217 depth = 102.4 lever arm = 420
As = 3034 mm²/m] not really needed – just show they know method

20 mm bars @ 100 crs = 3142 mm²/m

→ 25 @ 150 = 3272 mm²/m

32 @ 250 = 3217 mm²/m

Probably choose this



(e) Special rebar needed to deal with concentrated load from anchorage. Cannot assume that the 140 kNm/m is uniformly spread across width.

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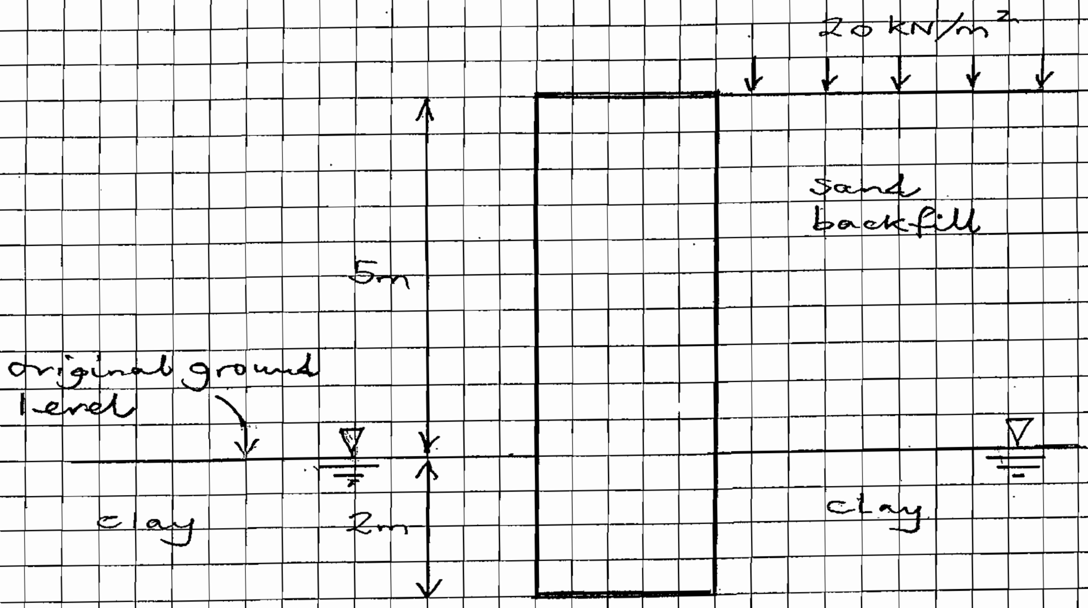
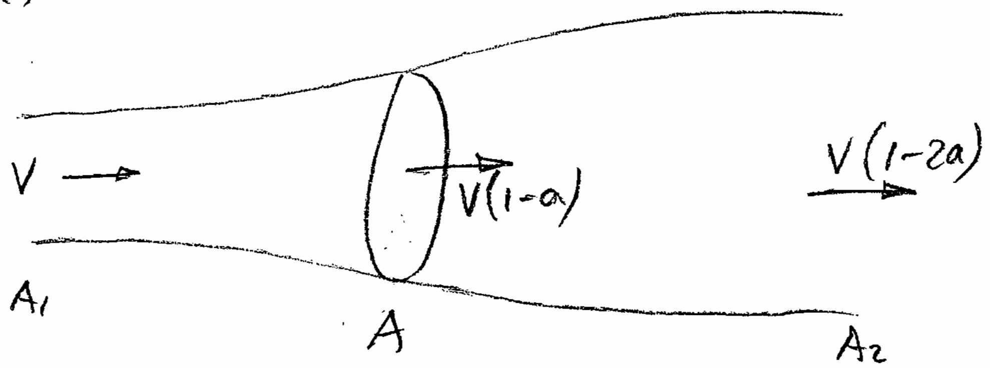


FIGURE 1

Section C

1

6 (a)



(i) continuity $\therefore$ $\rho A_1 V = \rho A V(1-a) = \rho A_2 V(1-2a)$

$$\therefore \quad A_1 = (1-a) A \quad = 0.75 A$$

$$A_2 = \left(\frac{1-a}{1-2a}\right) A \quad = 1.5 A$$

(ii) Rate of flow of K.E. into C.V.

$$= \underset{(\text{at } A_1)}{\tfrac{1}{2} \dot{m} V^2} - \underset{(\text{at } A_2)}{\tfrac{1}{2} \dot{m} \left(V(1-2a)\right)^2}$$

with $\dot{m} = \rho A_1 V$

so power gathered by C.V.

$$= \tfrac{1}{2} \rho A (1-a) V^3 \left(1 - (1-2a)^2\right)$$

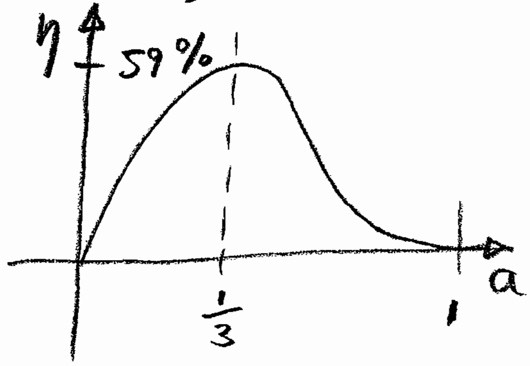
"Ideal" power $= \tfrac{1}{2} \rho A V^3$

$\therefore$ efficiency $\eta = (1-a)\left(1-(1-2a)^2\right)$

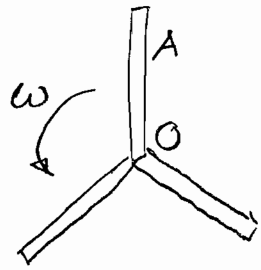
and for $a = 0.25$ $\quad \eta = \frac{9}{16} = \underline{\underline{56\%}}$

6(a) cont. The Betz Limit is the highest possible value of η, for $a = \frac{1}{3}$

Any value of $a \neq \frac{1}{3}$ must give $\eta < 59\%$



6(b) Assume rotor is a simple assembly of three uniform blades



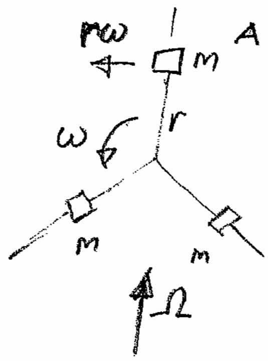
(i) If the rotor stops in τ seconds then $\dot{\omega} = \frac{\omega}{\tau}$ $\therefore$ Torque $= J\dot{\omega} = \frac{J\omega}{\tau}$

Shared between three blades

$\therefore$ root Bending Moment $= \frac{J\omega}{3\tau}$

(ii) Yaw gives rise to a gyroscopic couple $Q = J\omega\Omega$ but this is not shared equally between the blades. In the diagram the blade in position A takes $\frac{2}{N}$ of the couple where N is the number of blades, so for $N = 3$ root B.M $= \frac{2J\omega\Omega}{3}$ for the blade at A.

6(b) cont   For full marks, need to explain the "$\frac{2}{N+1}$". Consider three blade elements of mass m



The peak coriolis acceleration is at A : $a_{corr} = 2(r\omega)\Omega$

Root BM for A is $m a_{corr} r$

$= 2mr^2\omega\Omega$

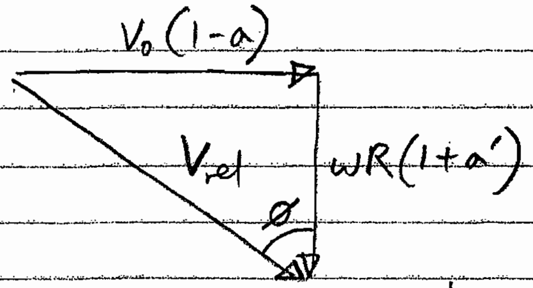
Total couple $Q = J\omega\Omega$ $J = 3mr^2$

Hence root BM for "A" is $\frac{2}{3}Q$

(For N blades root BM $= \frac{2}{N}Q$)

This is true for all elements ∴ true for whole rotor

7 (a)



$$\phi = \alpha + \theta \qquad \tan\phi = \frac{(1-a)V_0}{(1+a')\omega R}$$

$$\lambda = \frac{\omega R}{V_0} \qquad \therefore \quad \phi = \tan^{-1}\frac{(1-a)}{(1+a')\lambda}$$

(b) $\theta = 0.65°$, $\lambda = 8$, $a = 0.2$, $a' = 0.01$

(i) Angle of attack α

$$\alpha = \phi - \theta = \tan^{-1}\frac{(1-0.2)}{(1+0.01)\times 8} - 0.65°$$

$$= 5.65° - 0.65°$$
$$= 5.00°$$

(ii) $C_L \simeq 2\pi\alpha$ (α in radians) $C_D \simeq 0.04$
for $0 < \alpha < 15°$

$$\therefore C_L = 2\pi \times 5° \times \frac{\pi}{180°} = 0.55 \quad \text{and} \quad C_D = 0.04$$

Resolve forces acting on blade section:

$$F_N = F_L \cos\phi + F_D \sin\phi$$

and $F_T = F_L \sin\phi - F_D \cos\phi$

7 (cont.)

All forces may be normalised by $\frac{1}{2}\rho V_{rel}^2 c$ to find coefficients:

$$C_L = \frac{F_L}{\frac{1}{2}\rho V_{rel}^2 c} \qquad C_D = \frac{F_D}{\frac{1}{2}\rho V_{rel}^2 c}$$

$$C_N = \frac{F_N}{\frac{1}{2}\rho V_{rel}^2 c} \qquad C_T = \frac{F_T}{\frac{1}{2}\rho V_{rel}^2 c}$$

where ρ is the air density and c is the blade chord dimension

Hence:

$$\frac{F_N}{F_T} = \frac{C_N}{C_T} = \frac{C_L \cos\phi + C_D \sin\phi}{C_L \sin\phi - C_D \cos\phi}$$

$$= \frac{0.55 \times \cos 5.65° + 0.04 \times \sin 5.65°}{0.55 \times \sin 5.65° - 0.04 \times \cos 5.65°}$$

$$= 38.4$$

Elective crib Q8 2008 (HRS)

8. (a) (i) $I(x) = Ad^2 = A_o\left(\frac{x}{L}\right)d_o^2\left(\frac{x}{L}\right)^2 = A_o d_o^2\left(\frac{x}{L}\right)^3$

Maximum stress: $\frac{\sigma_{max}(x)}{d(x)} = \frac{M(x)}{I(x)} \Rightarrow \sigma_{max}(x) = \frac{WL/3}{A_o d_o^2} d_o\left(\frac{x}{L}\right) = \frac{WL}{3 A_o d_o}\left(\frac{x}{L}\right)$

Maximum at $x = L$: $\sigma_{max} = \frac{WL}{3 A_o d_o}$

(ii) Mass $m = \rho \int_0^L A dx = \frac{\rho A_o L}{2}$

Eliminate A_o: $m_\sigma = \frac{\rho W L^2}{6 d_o \sigma_f}$

(b) Mass $m = \frac{\rho A_o L}{2}$ as before, and $\delta = \frac{W L^3}{6 E A_o d_o^2}$

Eliminate A_o: $m_\delta = \frac{\rho W L^4}{12 E d_o^2 \delta}$

Criterion for stiffness-limited is $m_\delta > m_\sigma$, i.e. $\frac{\sigma_f L^2}{2 E d_o \delta} > 1$ or $\frac{\sigma_f}{2 E (d_o / L)(\delta / L)} > 1$

(c) (i) Evaluate σ_f/E: GFRP: 2 Bamboo: 2
As this ratio of properties is the same for bamboo as GFRP, so the inequality still holds (don't need to know the values of δ/L and d_o/L). Hence also stiffness-limited.

Mass for adequate stiffness: $m_\delta = \frac{\rho W L^4}{12 E d_o^2 \delta}$

Hence mass ratio, bamboo:GFRP = $\frac{(\rho / E)_{bamboo}}{(\rho / E)_{GFRP}} = \frac{0.7/18}{1.8/28} = 0.60$

(ii) Combination of lower mass and much lower embodied energy/kg: much more favourable for bamboo, reducing impact of blades on life cycle energy. Significance for design as a whole depends on relative importance of blades compared to tower, nacelle and foundation (typically 25%). The lower mass may also reduce the superstructure size required, giving some secondary weight savings and reducing embodied energy in remainder of machine.

Manufacturing energy: usually relatively unimportant compared to embodied energy, but in any case similar for GFRP and bamboo as moulding/layup technologies similar.

Energy in use: maybe some reduction in transport energies for bamboo due to lower weight blades, but depends on distances material or finished blades moved (e.g. bamboo blades made and installed in China – lower impact than shipping GFRP from UK to China, or bamboo from China to UK). Energy produced should be the same – aerodynamic shape and stiffness designed to be the same. Maintenance may not be identical for two materials – bamboo may have lower lifetime.

Disposal: neither can be recycled, but both can be burned as an energy source for electricity generation, with bamboo producing less waste.

Elective crib Q8 2008 (HRS)
Overall payback period: given likely dominance of embodied energy, payback period for bamboo expected to be shorter than for GFRP (but not in proportion to the embodied energy in the blades, due to the contribution from the remainder of the turbine). CO_2 evaluation even more favourable, as bamboo is itself a carbon sink.

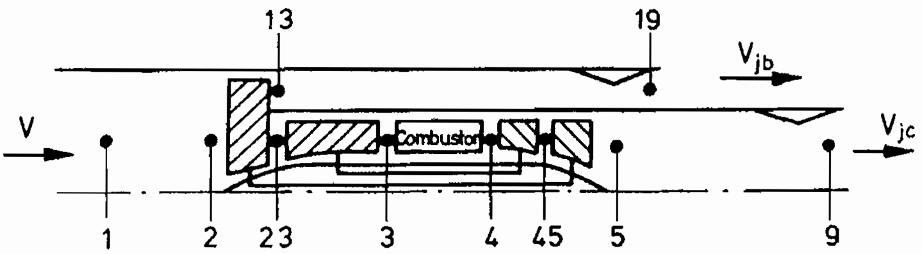
Examiner comments:

Average mark 12.7/20.
Question generally well done, or very poorly done. Poor answers revealed basic lack of beam theory from IA Structures, with some students still vaguely dividing any load by an area to get a tensile stress. In (a) many students failed to distinguish between the maximum stress at a given x (i.e. the outer fibre, but varying with x) and the maximum in the blade as a whole (at the root, x=L). In (c,ii) few noted that the payback period would be based on the embodied energy of the whole turbine rather than the blades alone, so over-stated the significance of the lower energy of the bamboo blades.

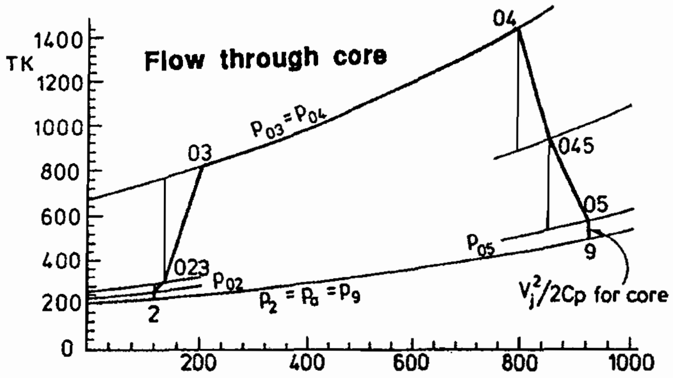
Section D

IB Paper 8 Aerothermal Elective 2008 Solutions

9. a)



Turbofan (two-shaft)



As the Bypass Ratio increases, the power output required from the low pressure turbine increases (stations 045→ 05). Hence, the distance from point 045 to point 05 on the T-s diagram increases and the vertical separation of points 05 and 9 reduces (corresponding to lower jet velocity).

b)

$$\eta_{is,c} = 0.87 = \frac{\left(\frac{p_{03}}{p_{023}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_{03}}{T_{023}} - 1} \Rightarrow \frac{T_{03}}{T_{02}} = \frac{(25)^{\frac{\gamma-1}{\gamma}} - 1}{0.87} + 1 = 2.734 \Rightarrow T_{03} = \underline{792.8\text{ K}}$$

$$\frac{\Delta h_0}{U^2} = \frac{c_p\left(T_{03} - T_{023}\right)}{n_{stage}U^2} = \frac{1005 \times (792.8 - 290)}{10 \times 355^2} = \underline{0.401}$$

c)

Work balance for HP shaft, $\dot{m}c_p\left(T_{03}-T_{023}\right)=\dot{m}c_p\left(T_{04}-T_{045}\right)$

$\Rightarrow T_{045}=T_{04}-\left(T_{03}-T_{023}\right)=1750-(792.8-290)=\underline{1247.2\text{ K}}$

$$\eta_{is,t}=0.85=\frac{1-\dfrac{T_{045}}{T_{04}}}{1-\left(\dfrac{p_{045}}{p_{04}}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$\Rightarrow \frac{p_{045}}{p_{04}}=\left(1-\frac{1-\dfrac{T_{045}}{T_{04}}}{0.85}\right)^{\frac{\gamma}{\gamma-1}}=\left(1-\frac{1-\dfrac{1247.2}{1750}}{0.85}\right)^{\frac{1.4}{.4}}=0.236 \qquad \Rightarrow \frac{p_{04}}{p_{045}}=\underline{4.24}$$

d)

Work balance for LP shaft,

$\dot{m}c_p\left(T_{023}-T_{02}\right)+BPR.\dot{m}c_p\left(T_{013}-T_{02}\right)=\dot{m}c_p\left(T_{045}-T_{05}\right)$

$T_{013}=T_{023}$, Therefore, $\left(BPR+1\right)\left(T_{013}-T_{02}\right)=\left(T_{045}-T_{05}\right)$

$$\Rightarrow BPR=\frac{\left(T_{045}-T_{05}\right)}{\left(T_{013}-T_{02}\right)}-1=\frac{480}{290-40}-1=\underline{11}$$

(i.e. a very high BPR - at the upper limit of today's turbofan designs)

10. a)

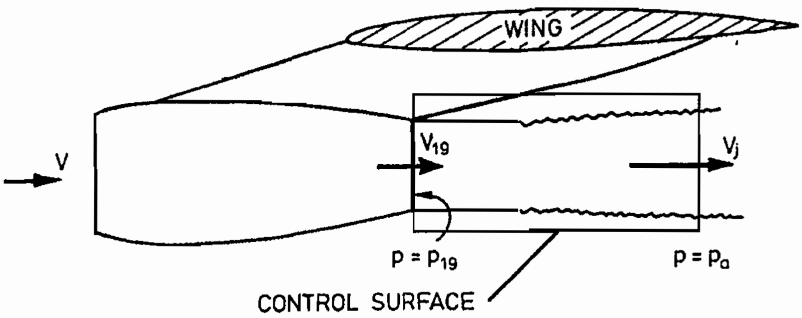
The *net thrust* is the thrust which is actually available to the aircraft, the *gross thrust* is the thrust that would be produced under the same conditions with the engine stationary. The difference between the two is the *ram drag* or the inlet air momentum.

$$F_N = \dot{m}(V_j - V) \qquad F_G = \dot{m}V_j \quad \Rightarrow \underline{F_N = F_G - \dot{m}V}$$

b)

The expression $(F_G + p_a A_N)$ only depends on conditions <u>inside</u> the engine, whereas F_N also depends on the aircraft flight conditions (Mach number, altitude).

To show this, consider the momentum equation for the control volume that extends downstream of the exit nozzle as shown:



$$A_N(p_{19} - p_a) = \dot{m}(V_j - V_{19})$$

$\Rightarrow F_G + A_N p_a = A_N p_{19} + \dot{m}V_{19}$ - The terms on the right-hand-side of this equation are all fixed by the conditions inside the engine provided the nozzle is choked (thus preventing any influence from downstream, within the atmosphere)

c)

(i) $T_{02} = T_a\left(1 + \frac{\gamma - 1}{2}M^2\right) = 217(1 + 0.2 \times 0.7^2) = \underline{238.3\text{ K}}$

$$p_{02} = p_a\left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} = 15(1 + 0.2 \times 0.7^2)^{3.5} = \underline{20.8\text{ kPa}}$$

(ii) $V = M\sqrt{\gamma R T_a} = 0.7 \times \sqrt{1.4 \times 287 \times 217} = \underline{206.7\text{ m/s}}$

$$F_G = F_N + \dot{m}V = 5000 + 20 \times 206.7 = \underline{9.134\text{ kN}}$$

d)

$$\left(\frac{\dot{m}\sqrt{c_p T_{02}}}{p_{02} A_N}\right)_{test} = \left(\frac{\dot{m}\sqrt{c_p T_{02}}}{p_{02} A_N}\right)_{cruise}$$

$$\dot{m}_{test} = \dot{m}_{cruise} \frac{\sqrt{T_{02,cruise}}}{\sqrt{T_{02,test}}} \frac{p_{02,test}}{p_{02,cruise}} = 20 \frac{\sqrt{238.3}}{\sqrt{288}} \frac{101}{20.8} = \underline{88.3 \text{ kg/s}}$$

$$\left(\frac{F_G + p_a A_N}{p_{02} A_N}\right)_{test} = \left(\frac{F_G + p_a A_N}{p_{02} A_N}\right)_{cruise}$$

$$F_G = p_{02,test}\left(\frac{F_G + p_a A_N}{p_{02}}\right)_{cruise} - p_{a,test} A_N = 101 \times \left(\frac{9134 + 15000 \times 0.06}{20800}\right) - 101 \times 0.06$$

$F_G = \underline{42.66 \text{ kN}}$ $\quad\Rightarrow\quad V_j = \frac{F_G}{\dot{m}} = \frac{42660}{88.3} = \underline{483 \text{ m/s}}$

e)

$$\left(\frac{ND}{\sqrt{\gamma R T_{02}}}\right)_{test} = \left(\frac{ND}{\sqrt{\gamma R T_{02}}}\right)_{cruise}$$

$$N_{cruise} = N_{test}\left(\frac{\sqrt{T_{02,cruise}}}{\sqrt{T_{02,test}}}\right) = 10000 \times \left(\sqrt{\frac{238.3}{288}}\right) = \underline{9096 \text{ rpm}}$$

11. a)

Thermal efficiency, $\eta_{th} = \dfrac{\Delta KE}{\dot{m}_{fuel} LCV} = \dfrac{\Delta \text{KE}}{\text{fuel heat release}}$

Propulsive efficiency, $\eta_p = \dfrac{F_N V}{\Delta KE} = \dfrac{\text{power to aircraft}}{\text{power to jet}}$

Specific fuel consumption relates the Net Thrust (which is the quantity that is needed by the aircraft) to the fuel consumption (which is what needs to be minimised). It therefore characterises the overall fuel performance of the engine rather than a particular aspect and is thus the most commonly used (especially in industry).

$$sfc = \frac{\dot{m}_{fuel}}{F_N} = \frac{\dot{m}_{fuel} LCV}{F_N V}\frac{V}{LCV} = \frac{V}{LCV}\frac{\dot{m}_{fuel} LCV}{\Delta KE}\frac{\Delta KE}{F_N V} = \underline{\frac{V}{LCV}\frac{1}{\eta_{th}\eta_p}}$$

b)

$$sfc = \frac{V}{LCV}\frac{1}{\eta_{th}}\left(\frac{V_j + V}{2V}\right) = \frac{V_j + V}{2\eta_{th} LCV} = \frac{M_j\sqrt{\gamma R T_j} + M_a\sqrt{\gamma R T_a}}{2\eta_{th} LCV}$$

$$\Rightarrow sfc = \underline{\frac{\sqrt{\gamma R T_a}}{2\eta_{th} LCV}\left(M + M_j\sqrt{\frac{T_j}{T_a}}\right)}$$

c)

(i) $C_L = \dfrac{L}{0.5\rho A V^2}$

$$A = \frac{mg}{0.5\rho M^2(\gamma R T_a)C_L} = \frac{200000\times 9.81}{0.5\times 0.311\times 0.85^2\times(1.4\times 287\times 216.7)\times 0.5}$$

$\Rightarrow$ $\underline{\text{A} = 401\ \text{m}^2}$

(ii)

$$M = \sqrt{\frac{mg}{0.5\rho A(\gamma R T_a)C_L}} = \sqrt{\frac{200000\times 9.81}{0.5\times 0.439\times 401\times(1.4\times 287\times 226.4)\times 0.5}}$$

$\Rightarrow$ $\underline{\text{M} = 0.70}$

$$\frac{sfc_{9.5km}}{sfc_{12km}} = \frac{\sqrt{T_{a,9.5km}}}{\sqrt{T_{a,12km}}}\left(\frac{M_{9.5km} + M_j\sqrt{T_j/T_a}}{M_{12km} + M_j\sqrt{T_j/T_a}}\right) = \frac{\sqrt{226.4}}{\sqrt{216.7}}\left(\frac{0.70 + 1.3\sqrt{1.15}}{0.85 + 1.3\sqrt{1.15}}\right) = 0.9538$$

$$sfc_{9.5km} = sfc_{12km} \times 0.9538 = \underline{0.0172 \text{ x } 10^{-3} \text{ kgs}^{-1}\text{N}^{-1}}$$

(iii) To minimise fuel burn, $\frac{V \times L/D}{sfc}$ needs to be maximised, which is close to $\frac{M \times L/D}{sfc}$. At the lower cruise altitude, although the *sfc* is slightly lower (by about 4%), the Mach number is very much lower (by about 18%). This means that the overall fuel burn will be significantly higher at the low altitude and therefore the higher altitude is preferred.

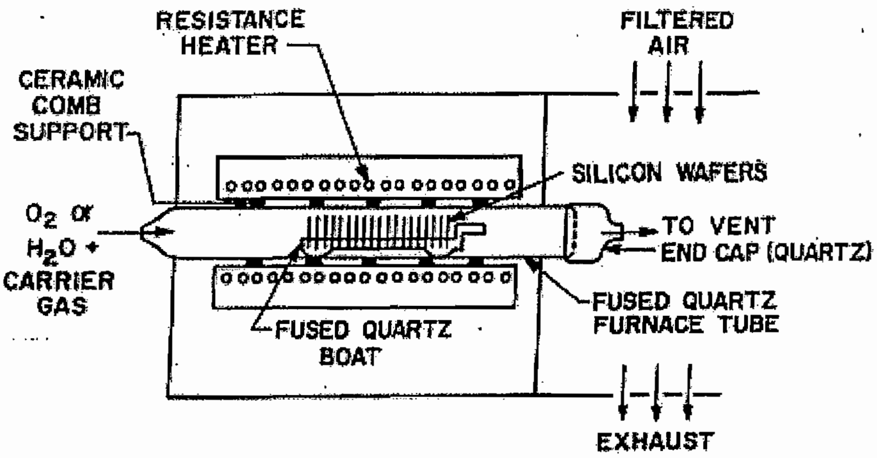
12 (a) SILICON DIOXIDE

Successful MOSFET operation depends upon a good quality gate insulator. Silicon dioxide is the standard insulator and it is also used in many of the different process steps as a barrier layer. It is therefore vital to have a reliable method of producing such material.

Oxidation of silicon is achieved by heating silicon wafers in a furnace in the presence of an oxidising atmosphere such as oxygen or water vapour. The two common approaches are:

Wet oxidation where the oxidising atmosphere contains water vapour. Temperature is approx 900-1100 C. This leads to a fast growth rate – depends on temperature but is of order 0.5 microns per our at 1100 C.

Dry oxidation where the oxidising atmosphere is pure oxygen. Growth temperatures have to be in the region of 1200 C to achieve acceptable growth rates of 0.1 microns/hour.

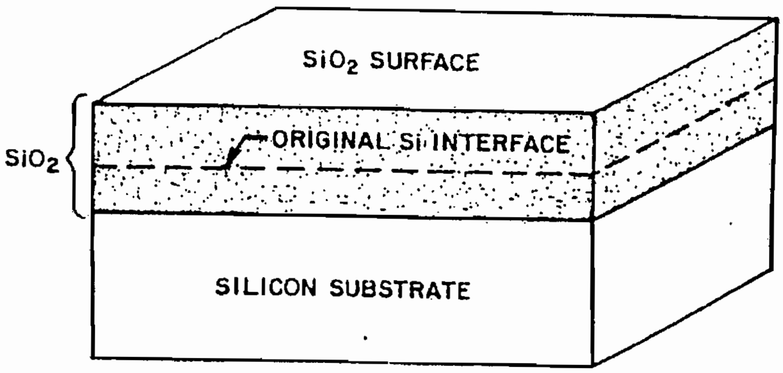


Schematic cross section of a resistance-heated oxidation furnace. The silicon wafer loading area is shown in a laminar hood.

Wet	$2H_2O + Si$	$\rightarrow$	SiO_2 (solid) + $2H_2$
Dry	$Si + O_2$	$\rightarrow$	SiO_2 (solid)

The silicon for the silicon dioxide film is donated by the slice and therefore as the oxide grows silicon is consumed. If the required oxide thickness is t then the silicon thickness is reduced by 0.44t and the overall thickness is increased by 0.56t.

As device dimensions continue to decrease process temperatures will also have to decrease and thermal growth of oxide may have to be replaced by lower temperature methods of insulator production. At present deposited low temperature oxide properties are much inferior to thermally grown. Also the use of alternative oxides for gate insulators is currently underway.



Growth of SiO_2.

(b) Hafnium Oxide, Zirconium Oxide, tantalum oxide, etc.

(c) Vol of 1 mol of Si

$$\frac{\text{Mol wt of Si}}{\text{Density of Si}} = \frac{28.9}{2.33} = 12.4\text{m}^3/\text{mol}$$

Vol of 1 mol of SiO_2

$$\frac{\text{Mol wt of SiO}_2}{\text{Density of SiO}_2} = \frac{60.08}{2.21} = 27.18\ \text{cm}^3/\text{mol}$$

Since 1 mol of Si is converted to 1 mol of SiO_2

$$\frac{\text{Thickness of Si} \times \text{area}}{\text{Thickness of SiO}_2 \times \text{area}} = \frac{\text{Vol of 1 mol Si}}{\text{Vol of 1 mol SiO}_2} = \frac{12.4}{27.18} = 0.456$$

So to grow a layer of 40 nm of oxide 18.25 nm of Si is consumed.

Total thickness of oxide plus silicon will be $\left(200 \times 10^{-6} - 18.25 \times 10^{-9}\right) + 40 \times 10^{-9}$ m.

(If they just quote from notes and say 0.45 then they correctly work out the thickness of Si and Si + SiO_2, then they should get 50% of mark – need to work out to get 100%.)

13 (a) scaling – book work

(b) $E = V/L = 0.5/ 5.10^{-8} = 10^7$ V/m.

$v = \mu E = 3.3x10^7$

$t = 3.3x10^7 / 5x10^{-8} = 1.51x10^{-15}$ s

(c) $Vg = eNd^2/ (2\varepsilon) = 4.68x10^{23}\ m^{-3}$

14 (a) semiconductors and insulators have band gaps. Insulators have bigger band gaps.

(b) intrinsic semi conductor by thermal excitation of their undoped electrons and holes. In doped semiconductors, the dopants provide free carriers, one per dopant, so carrier density = number of dopant atoms. Conductivity is controllable unlike in a metal, over many orders of magnitude.

(c) Electrons are accelerated by applied field. They are then randomly scattered by phonons or defects after an average time τ. This slows them down to zero velocity, after which they go through the cycle again.

$F = eE = m^* v/\tau$

$v = eE\tau/m^*$

m^* is the effective mass, a quantity that plays role of electron mass in this equation.

So mobility $\mu = v/E = e\tau/m^*$ (eqn in data sheet)

Conductivity $\sigma = n.e.\mu = n.e^2\tau/m^*$

Carrier velocity has a maximum value of the scattering limited velocity, v_s.

(d) $v = \mu E$, $E = v/\mu = 1.25x10^5/0.25 = 5x10^5$ V/m

$L = V/E = 1/ 5x10^5 = 2x10^{-6}$ m.

$t = L/v = 2x10^{-6} / 1.25 \text{ x } 10^5 = 1.6 \text{ x } 10^{-11}$ s

$\mu = e\tau/m^*$ so $\tau = \mu m^*/e = 0.25 \text{ x } 0.2 \text{ x } 9.1x10^{-31} / 1.6x10^{-19} = 2.8x10^{-13}$

an order of magnitude less than transit time, so approximations are OK.

Part IB – Paper 8 – Information Engr. Section F

15 We will discuss the correction of the three problems in turn.

a) <u>3° tilt of the horizon</u>:

In order to correct for this, it will be necessary to rotate the image 3° in an anti-clockwise direction. We need to define a mapping from a point (u, v) in the input image A to each point (p, q) in the output image B. By defining (u, v) as a function of (p, q), we can iterate through all the integer pixel locations (p, q) in B and fill them with appropriate interpolated samples from A. This prevents any pixels in B from being unfilled, or left blank. Interpolation is necessary because (u, v) will in general be real numbers, not integers. Bi-linear interpolation is simple & likely to be satisfactory. (Bi-cubic or spline interpolation are more complex options.)

15 (a) (cont.)

If (u,v) & (p,q) are measured with respect to the centres of A & B, then we can write

$$\begin{bmatrix} u \\ v \end{bmatrix} = kR \begin{bmatrix} p \\ q \end{bmatrix}$$

where R is a rotation matrix

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \quad \theta = 3^\circ$$

& k is a constant, slightly less than unity, which ensures that the corner pixels of B, still fall just within the limits of A after the required rotation by ~~R(θ)~~ R. This will avoid any undefined pixels in B, and will 'zoom' the image by a factor $\frac{1}{k}$. k must be chosen as a function of θ.

b) <u>Faces under-exposed</u>

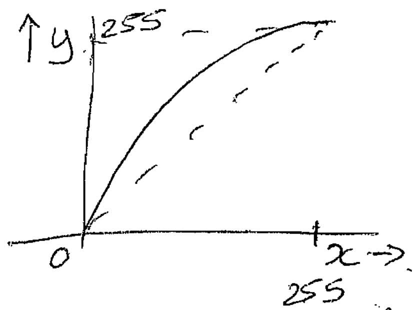
A simple way to adjust for under-exposure is to use gamma correction. In this

15(b) (cont)

method, if x is the luminance level of the image at a given pixel, where $0 \leq x \leq 255$, then we normalise it to unit range & raise it to some power γ, to get the new luminance level using:

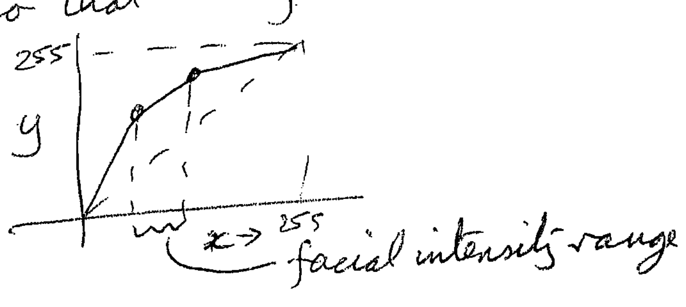
$$y = 255 . \left(\frac{x}{255} \right)^{\gamma}$$

If $\gamma < 1$ then the conversion rule looks like



and the intensity of the darker regions of the image is increased. To preserve colour balance, the R, G & B components of the pixel must be scaled by the same amount, so we multiply each of them by $\frac{y}{x}$.

A more precise method would be to use a histogram of the facial area to see the intensity of the face, & then choose a piecewise linear law for y vs. x so that the facial intensities are ~~amplified~~ increased the most; eg



15(b) (cont)

The piecewise linear law avoids giving excessive gain to the very dark image regions, which is a potential difficulty with gamma correction when γ is significantly less than 1 (where gradient is proportional to $x^{(\gamma-1)} \rightarrow \infty$ as x tends to zero.)

15(c) <u>Camera noise.</u>

Camera noise in the lower-lit areas of the image will become more noticeable after the lighting correction to brighten up the faces. Lowpass filtering can be applied to parts of the image to reduce the visibility of the noise, but unfortunately this will also blur the image. To minimise the blurring, while still reducing the noise, it is necessary to use adaptive lowpass filtering, based on detection of edges in the image. Horizontal and vertical highpass filters can be used to detect vertical & horizontal edges

15(c) (cont)

regions in the image (by rectifying the filter output, ie taking its absolute value) and this can control the tap weights of an ~~[illegible]~~ adaptive lowpass filter. A suitable filter would be:

$$y_n = \alpha_1(x_{n+1} - x_n) + x_n + \alpha_2(x_{n-1} - x_n)$$

When $\alpha_1 = \alpha_2 = 0$, this performs no filtering, & when $\alpha_1 = \alpha_2 = \frac{1}{4}$, this performs maximum smoothing with tap weights of $[\frac{1}{4}\ \frac{1}{2}\ \frac{1}{4}]$ for x_{n-1}, x_n, x_{n+1}.

To control each α, we can use a law such as:

$$\alpha_i = \frac{1/4}{1 + \left(\frac{e_i}{t}\right)^2}$$

where e_i is the edge intensity at location i & t is a threshold, set by the user.

We do this adaptive filtering in both the horizontal and vertical directions, and if necessary repeat the filtering to get stronger noise removal in smooth areas of the image. With appropriate choice of threshold, edges of reasonable contrast will not be blurred.

Paper 8 (Information Engineering)

15 (a) Consider smoothing an image, first with a Gaussian of standard deviation σ_1, then with a Gaussian of standard deviation σ_2:

$$s(x) = g_{\sigma 2}(x) * (g_{\sigma 1}(x) * I(x))$$

Since convolution is associative, we can write this as the convolution of the image with the kernel $g_{\sigma 2}(x) * g_{\sigma 1}(x)$:

$$s(x) = (g_{\sigma 2}(x) * g_{\sigma 1}(x)) * I(x)$$

The easiest way to evaluate the convolution of two Gaussians is to find their Fourier transforms and then multiply them in the frequency domain. If $g_\sigma(x) \leftrightarrow G_\sigma(\omega)$, then:

$$\begin{aligned}
G_\sigma(\omega) &= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{2\sigma^2}\right) e^{-j\omega x} dx \\
&= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[-\left(\frac{x^2}{2\sigma^2} + j\omega x\right)\right] dx \\
&= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2\sigma^2}\left(x^2 + 2j\omega\sigma^2 x\right)\right] dx \\
&= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2\sigma^2}\left((x + j\omega\sigma^2)^2 - j^2\omega^2\sigma^4\right)\right] dx \\
&= \exp\left(-\frac{\omega^2\sigma^2}{2}\right) \times \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{(x + j\omega\sigma^2)^2}{2\sigma^2}\right) dx \\
&= \exp\left(-\frac{\omega^2\sigma^2}{2}\right) \text{ (since the integral is a standard Gaussian)}
\end{aligned}$$

the computational cost.

Hence

$$g_{\sigma 2}(x) * g_{\sigma 1}(x) \leftrightarrow G_{\sigma 2}(\omega) \times G_{\sigma 1}(\omega) = \exp\left(-\frac{\omega^2\sigma_2^2}{2}\right) \times \exp\left(-\frac{\omega^2\sigma_1^2}{2}\right)$$

or

$$g_{\sigma 2}(x) * g_{\sigma 1}(x) \leftrightarrow G_{\sigma 2}(\omega) \times G_{\sigma 1}(\omega) = \exp\left(-\frac{\omega^2(\sigma_2^2 + \sigma_1^2)}{2}\right)$$

The expression on the right is the Fourier transform of a Gaussian with standard deviation $\sqrt{\sigma_2^2 + \sigma_1^2}$. Hence, the convolution of two Gaussians with variances σ_1^2 and σ_2^2 is a Gaussian with variance $\sigma_1^2 + \sigma_2^2$. It follows that consecutive smoothing with a series of 1D Gaussians, each with a particular standard deviation σ_i^2, is equivalent to a single convolution with a Gaussian of variance $\sum_i \sigma_i^2$.
(b) In practice, only a discrete set of scales can be considered, giving rise to an *image pyramid.* For a given image size, we an *octave* of scales is examined, corresponding to Gaussians with standard deviations from σ to 2σ. The image is then subsampled by a factor of 2 and the process is repeated for the next octave.

To improve efficiency, the above can be performed in an incremental manner. Within the octave σ to 2σ, for i-th interval (our of s) of the pyramid, we want $\sigma_i = 2^{i/s}\sigma$. To achieve this incrementally:

$$G(\sigma_{i+1}) = G(\sigma_i) * G(\hat{\sigma})$$

Then from (a):

$$\hat{\sigma} = \sqrt{\sigma_{i+1}^2 - \sigma_i^2}$$

and since

$$\sigma_{i+1} = 2^{(i+1)/s}$$

we have:

$$\hat{\sigma} = \sigma_i\sqrt{2^{2/s} - 1}$$

The separability property of the Gaussian function should also be used to reduce the computational cost.

Image Processing - Paper 8

QUESTION

Consider a model of 10x10 pixel black and white images where each pixel is black with probability $\theta = 0.6$ and all pixels are independent.

(a) What is the most probable image under this model and what is its probability?

(b) Assume that θ is unknown, and that we wish to learn it from data using Bayesian methods. Let us start by giving θ a uniform prior on the interval [0,1]. What is the posterior distribution of θ after observing only two black pixels and no white pixels? What is the posterior probability that $\theta > \frac{1}{2}$?

(c) Explain why a model of black and white images where each pixel is independent is a very poor model of real world images, and suggest a better model.

17

ANSWERS

The answers are as follows:

(a) the most probable image is all black and has probability 0.6^{100}

(b) $p(\theta) = 3\ \theta^2$ and the definite integral from 0.5 to 1 of $3\ \theta^2$ is 7/8.

(c) This model is unrealistic because it assumes that all pixels are independent. It is clear from real world images that pixels are highly correlated. In particular, nearby pixels are strongly correlated, so it makes sense to have a model where it is more probable for nearby pixels to either both be black or both be white. A simple model of this kind is $(1/Z) \exp \{ \sum_{ij \in N} \theta (x_i - 0.5) (x_j - 0.5) \}$ where the sum is taken over neighboring pixels.

CRIB- 2008 Paper 8, Section G, Engineering for the Life Sciences

18

(a) In the context of reinforcement learning write brief notes on the following *[4]*

(ii) *Primary reinforcers;*

A primary reinforcer is a stimulus that is hard-wired to increase the probability of actions that lead to that stimulus. Examples include food, drink and sex. 1

(ii) *Secondary reinforcers;*

A secondary reinforcer is a stimulus that has to be paired with a primary reinforcer (or an established secondary reinforcer) to increase the probability of actions that lead to the stimulus. An example is money. 1

(iii) *Temporal difference error.*

The temporal difference error is the difference between the estimated value of the current state and the sum of the current reward and estimated (discounted) value of the next state: $r+\gamma V(s(t+1))-V(s(t))$. The error can be used to update value functions in reinforcement learning. 2

(b) How is the position of the eye within the orbit estimated? Provide experimental evidence for your answer. *[4]*

Rather than sensed, the position of the eye is predicted based on a copy of the motor command acting on the eye muscles, termed efference copy. When the eye is moved without using the eye muscles by pressing with your finger on the open eye through the eyelid, the retinal locations of visual objects change, but the predicted eye position is not updated, leading to the false perception that the world is moving. (Or paralysing the eye muscles and trying to move the eye leads to a similar percept).

(c) Answer the following in the context of inverse model learning. *[4]*

(i) Describe the direct inverse model learning approach.

In direct inverse model learning, motor commands are randomly generated (motor babbling) and produce outcomes. An inverse model monitors the outcome and tries to predict the motor commands that caused them. The inverse model is updated using supervised learning. 2

(ii) What are the problems of using direct inverse model learning?

There are three problems. First the learning is not goal-directed in that motor babbling is not aimed at reaching a particular goal. Second, in training the inverse model input is actual outcomes and, therefore, needs to be rewired for use to take in desired outcomes. Finally it fails for many nonlinear systems because it learns the average motor command to achieve an outcome which may not itself be a solution. 2

(d) Answer the following in the context of human arm stiffness. *[4]*

i) What is the stiffness ellipse of the human arm and how can it be experimentally estimated?

The stiffness ellipse of the arm is the stiffness of the arm (restoring forces per unit displacement) assessed at the hand for different directions. It is measured by using a robotic interface to rapidly displace the hand a short distance in many directions and measure the restoring force/unit displacement. 2

ii) How and why might the stiffness change when using a screwdriver?

The screwdriver is an inherently unstable system in that any deviations from perpendicular to the surface are unstable when applying a downward force. By increasing the stiffness of the arm (possibly in all directions) stability can be maintained. 2

(e) Answer the following in the context of Bayes rule. *[4]*

(i) State Bayes rule as it applies to estimating state based on sensory input

Prob(state|sensory feedback)= Prob(sensory feedback|state)P(state)/P(sensory feedback) 1

or

Posterior is proportional to likelihood x prior

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

(ii) Explain why the image below is perceived as a cube.

As the world is three-dimensional but images are projected onto a two dimensional retina, information is lost. Therefore, any image is consistent with infinitely many different three-dimensional worlds. To determine a percept, the prior probability of different states of the world is used. In our environment, right angles and cubes are common so the percept is biased towards a solution that has a high prior and therefore a high posterior, a cube. In this case there are two equally good solutions and this percept is bistable. 3

(a) When the flagellar motors rotate in the "forward" sense, the normal-form, left-handed corkscrew-like flagellar filaments associate in a bundle, which propels the bacterial cell forwards smoothly. When some of the motors switch to reverse rotation, the proximal part of those filaments change form to right-handed corkscrews; which then dis-engage from the bundle and thrash around independently, causing the cell to "tumble" in an aimless way. When the motors resume rotation in the forward sense, the flagellar bundle re-forms; and the cell moves off in a direction determined by its random orientation at the end of the tumbling phase.

In this way the cell executes a "random walk". The direction of rotation of the motors is controlled by machinery within the cell, which in turn responds to the concentration of nutrient molecules, as detected by "receptors" on the cell's surface. When the rate of detection is found to increase with time, the smooth-swimming phase is prolonged; so the random walk is biassed towards the source of the nutrients which the cell is seeking.

(b) All proteins consist of a strong but flexible chain of polypeptide units, to each of which is attached an amino acid. The identity of each amino acid is determined by a 3-letter sequence in a DNA molecule which is being read by the machinery which assembles amino acids onto the chain.

There are 20 different kinds of amino acid. Some are large, others small. But the most important feature is whether they are hydrophobic (water-hating) or hydrophilic (water-seeking); and the latter may have positive or negative electric charge.

The way in which the chain folds up depends entirely on the sequence of amino acids along the chain. The main driving-force is the desire of the hydrophobic amino acids to be buried in the interior, well away from the surrounding water in the cell.

Proteins are thus very versatile, and they constitute a whole range of materials having different mechanical and electrical properties, as well as components of machinery (eg. the flagellar motor), enzymes (which act as catalysts for chemical reactions by holding 2 molecules in the proper spatial relationships) and subtle, bi-stable building-blocks (as in bacterial flagellar filaments) — etc, etc..

(c) A crucial feature of living cells is that all of their components must <u>self-assemble</u>. But complex organisms – such as flies, plants, mammals... – have many different types of cell, which perform widely different functions.

The complete prescription for all of these different cell types must be transmitted from generation to generation, if life is to continue.

Nature arranges that the complete prescription for building every type of cell, which is contained in the long DNA molecule, is contained in <u>every</u> cell. Thus each specialist type of cell uses ("transcribes") only a small fraction of the DNA which it contains.

The growth of an organism occurs by successive division of cells into two "daughter" cells. Before any cell divides, its complete DNA is duplicated, so that one copy can go into each daughter cell.

The advantage of this scheme is that life can continue; and also that evolution – which depends on random small changes in the DNA message as a result of copying errors, ionising radiation, etc. can produce a whole range of species from a single "primordial ancestor".

(d) The evolutionary relationships elucidated by Darwin, on the basis of his patient and intelligent contemplation of the world of living things, were established long before the machinery of inheritance was understood.

Technological developments over the past 50 years make it possible now to determine the precise sequence of DNA code-letters for different organisms. The DNA sequence of the human genome, some 10^9 letters, was determined a few years ago; and by now the complete DNA sequences of many different organisms have been established.

If Darwin was correct in thinking that all living things are descended from a single primordial ancestor, through a single "tree of life", then it should be possible to map the branching-points of the tree by comparing in detail (by means of computer-searches) the DNA of different species.

A good example is the "cytochrome c" protein which exchanges oxygen, and which is found (in modified forms) in every living species that uses oxygen – i.e. almost everything. By examining the small differences in DNA sequence of the cytochrome c gene in many different species, one can map out the "tree of life", and this method is found to produce essentially the same tree which may be produced by other, including Darwinian, methods.

(e) The corkscrew-like filament of bacterial flagella is a very subtle piece of structural engineering.

It is known, by biochemical assays, to be built from a single type of protein-molecule building-brick, which self-assembles into a tubular structure by successive addition of building-blocks at the terminal construction-site.

If all of these bricks were in the same environment with respect to their neighbours, the tube would be straight; but in fact the tube is corkscrew-shaped, i.e. curved in particular.

The clever trick which Nature uses is to make the building block bi-stable; i.e. to arrange that it is equally happy to exist in two slightly-different geometrical configurations. Then it can be arranged that the 3D geometrical constraints of building the tube can be met only if some of the building blocks are in conformation A, while others are in conformation B. That in turn enables the tube to be curved.

Now a single-letter change in the gene of the protein building-block will in principle result in the change of a single amino acid from one kind to another. That, in principle, will change the overall shape of the building block by a small amount; which in turn can alter the proportions of the building-blocks taking up conformations A and B. In turn, this means that the tubular assembly will have different curvature and twist, and thus a different helical form.

In theory (and also as found in experiment) a flagellar filament with 11 near-longitudinal strands of building-bricks has 11 + 1 = 12 possible helical forms, constituting a well-defined "family". A single-letter change in the flagellin-protein's DNA can thus make the filament switch from one of these discrete wave-forms to another — a remarkable amplification!

20

(a)

The defect is osteochondral, which means it contains both a bone ("osteo") and a cartilage ("chondral") component.

Bone

Bone is approximately 50% mineral (hydroxyapatite) by volume and the remaining 50% is hydrated organic materials (Broken down further as 20% water and 30% organic material in terms of total volume; 90% of this organic material is the protein collagen). Bone is still, tough and strong while being lightweight, and it has an elastic modulus of order O(10 GPa).

Cartilage

Cartilage is approximately 75% water. Of the remaining material, it is all organic and there is no mineral. About two thirds of the organic material is collagen protein and the remaining third is "proteoglycans" which are sugar-protein macromolecule complexes. Cartilage is very compliant compared to bone, with an elastic modulus of around 0.1 MPa in compression and 1 MPa in tension. It also exhibits dominant time-dependent deformation due to fluid flow, and this time-dependence can be described by "poroelastic" theory based on Darcy's hydraulic permeability.

In between the bone and cartilage there is a thin interface of calcified cartilage. This is a hybrid tissue, composition similar to cartilage but with mineral deposited in some of the water space.

(b)

Implant 1 is a bioinert prosthesis based on joint replacement technology. It consists of a metal (stainless steel, Ti-6-4 or cobalt-chrome alloy) component and may also have a polymer surface. The metal attaches into the bone in the defect and anchors to the bone; either metal or polymer would provide a bearing surface that would articulate against the remaining cartilage on the opposing (femoral) side.

Implant 2 is a porous two-section scaffold that would be seeded with cartilage cells (chondrocytes) in the top and bone cells (osteoblasts) in the bottom portion. The scaffold could be a uniform bioresorbable polymer in both sections or could be tailored to have different mechanical properties in each part since the baseline stiffness of bone and cartilage differ by several orders of magnitude. The implant could be customized for the shape of the defect. The implant would be attached to the bone at the side with osteoblasts such that the portion with chondrocytes would articulate against the femoral cartilage. The idea is that the scaffold would resorb while the cells generated new tissue to fill in the gaps, eventually restoring the defect to its original state with viable cartilage attached to bone bonded into the native bone.

(c)

Mechanical—immediate

The tissue-engineered construct (implant 2) likely has much less stiffness than the native tissue and would provide less mechanical support immediately upon implantation compared with implant 1.

Mechanical—long-term

If the tissue-engineered construct (implant 2) filled in with native tissue, it might still be mechanically weak and compliant compared with the native tissue and might not be able to serve the demanding mechanical needs of the knee joint, which sees forces many times that of body-weight. The metal implant (1) would have long-term stability but the metal might stress-shield the surrounding bone.

Cell activity

The metal implant (1) is biologically inert (inactive). The only likely cellular response is negative, such as a fibrous capsule formation around the metal-bone interface, leading to implant loosening. Implant 2 is both resorbable and potentially bioactive depending on the scaffold material chosen. The cellular response of implant 2 would be positive in that the implant could remodel with time based on mechanical regulation of the cell responses.

Biocompatibility

Implant 2: Assuming the cells used to seed the scaffolds are autologous, the implant would likely be favourably biocompatible. Only the scaffold would be foreign material. If the cells are not autologous, an immune reaction is possible if not probable. Implant 1 is based on inert materials that have been proven in the body for many decades, and metals are successful as long-term implants from a biocompatibility perspective.

Reliability and failure mechanisms

Failure of the tissue-engineered implant (2) would amount to disappearance of the implant—unsuccessful filling of the defect with native tissue following degradation of the resorbable scaffold. This could be due in part to a failure of the implant integration by cell death. It is desirable for the bone side to be vascularized. However, if the cartilage became vascularized it would be possible for the tissue to de-differentiate from cartilage to fibrocartilage and not serve its mechanical function. The engineering metal implant (1) is unlikely to fail in such a dramatic manner, with the greatest risk due to aseptic loosening after fibrous capsule formation, requiring a more invasive revision surgery.

Articulating surface

The femoral natural cartilage would articulate against the implant. In the case of the tissue-engineered construct, if the surface was smooth and cartilaginous this would be advantageous, but if the surface was not smooth this could lead not only to implant failure but erosion of the tissue on the femoral side. Similarly, articulating against the metal implant could cause femoral cartilage erosion. The long term danger is that the femoral cartilage would erode completely and a full knee joint replacement would be required.

Patient factors

The patient is young but we don't know how active and that might factor in. We also need to ascertain the patient's willingness to undergo an experimental procedure (the tissue engineered construct) and participate in a clinical trial.

There are pros and cons to both approaches. There is no clear choice for the "right" answer here, although the patient's young age would suggest the tissue engineering approach may be preferable. However, this is yet developing technology and thus involves an element of risk, although in failure of implant 2 implant 1 would be an option that still remained viable.

SECTION H: Manufacturing, Management and Design

Solution to 21

(a) The profit and loss accounts for the present and two options are:

	Present	Option A	Option B
Revenue (new)	500,000	350,000	500,000
Revenue (recycled)		150,000	200,000
Total revenue	**500,000**	**500,000**	**700,000**
Direct labour	100,000	120,000	150,000
Direct material	200,000	150,000	350,000
Gross profit	**200,000**	**230,000**	**200,000**
Management and marketing	100,000	100,000	100,000
Building and equipment charge	60,000	60,000	60,000
Net profit	**40,000**	**70,000**	**40,000**

(b) If the price for recycled bags drops by 10%, the revised calculation of profit and corresponding % change in profit are:

	Present	Option A	Option B
Revenue (new)	500,000	350,000	500,000
Revenue (recycled)		135,000	180,000
Total revenue	**500,000**	**485,000**	**680,000**
Total costs	460,000	430,000	660,000
Net profit	**40,000**	**55,000**	**20,000**
% change in profit	0%	21%	50%

(c) The costs of recycled bag production for either option are not linked to the price of the recycled bags, so the company has very few options to reduce its sensitivity by managing its costs. In fact, the calculation of combined profit for the two options is rather misleading – as the recycled bags are in both cases less profitable than the conventional bag, and in Option B, the recycled bags are actually loss making. Accordingly, the company would be wise to prepare separate profit and loss accounts for the new bags, and to withdraw them rapidly if their price was unable to cover their costs.

Solution to Q22

a) Design mix: explains that a product consists of a number of 'levels' - core benefits, actual product (tangible and intangible), augmented product, meta product (business model)

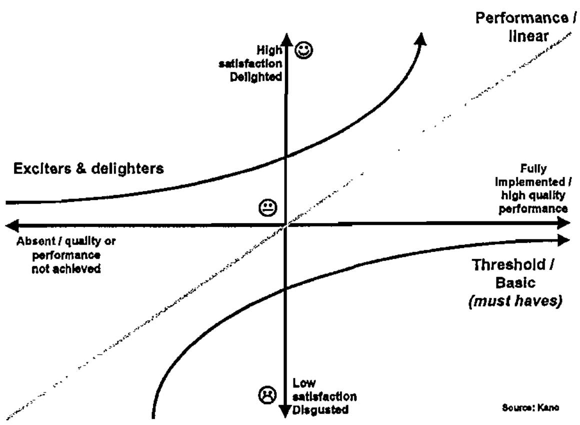
Differentiation can be gained along any of these dimensions. A cheap writing instrument might provide new core benefits, such as writing upside down or combining writing in ink and in pencil or offering mult-coloured writing! Differentiation along tangible performance or quality based dimensions, such as writing time, life, reliability. It might be differentiated through appearance or feel. It might be differentiated through its augmented attributes, such as service deals or finance packages. Finally, it might be provided with a new business model – pay per page!

b) Can establish 'importance' of requirements through interview or competitive analysis. However, this provides a one dimensional picture. Kano – developed by Noriaki Kano in the 90s. It is a good way to classify requirements based on perceptions of customer quality. Kano classifies attributes into 4 main categories:

Indifferent: attributes to which the customer pays no attention "If they are present, it is nice. If they are not present, it does not matter"

Expected attributes (Threshold / Basic): have to be present in the product in order to make it successful. Failure to offer these attributes represents secured failure for the product and customer remains neutral regardless of how well the feature is executed. Provide no differentiation (brakes).

One dimensional attributes (Performance / Linear): Characteristics that are directly correlated to customer satisfaction. Increased functionality results in increased customer satisfaction. Decreased functionality results in greater dissatisfaction. Price is related to these attributes



Attractive attributes (Exciters / Delighters): Customers get great satisfaction from a feature - and are willing to pay a premium. Satisfaction will not decrease (below neutral) if the product lacks the feature. Unspoken and unexpected by customers. Often satisfy latent or unknown needs.

For a novel writing instrument, would expect some examples for each characteristic. Examples: Basic – puts ink on paper. Linear – pen life. Delighter – variable nib thickness.

c) Can use a range of types of prototypes to test the concepts:

Would expect to see examples of these for the writing instrument.

TYPE	USES
Simple Sketch	*The simplest, cheapest and quickest way of evaluating lots of ideas for form, technical arrangement and usability. Often highly under-utilised.*
Block model	*Primarily for early testing of usability, ergonomics and form. Also useful to quickly evaluate a productÕs physical arrangement. Mainly use easy to work and cheap materials.*
Visual (physical) model	*Enables evaluation of visual and form aspects. Produced to look as realistic as possible. Good for testing product feel and form.*
3D CAD model	*Evaluation of overall form, assembly sequence, component fit and production issues. Can also plug into a range of complex analytical models.*
Functional (technical) model	*To test specific performance aspects. Not necessarily representative of production processes. Good for evaluating reliability, durability, performance, failure etc.*
Production prototype	*To evaluate all elements of performance, function, form, use and producibility. Made with processes representative of the final production method. Fully functional.*
Analytical / virtual models	*Mathematical models to support component and assembly optimisation. Often used for safety critical elements. Can be costly and answers are always approximations.*

d) Students should provide some reflection on the benefits of structured tools. They help focus attention on critical issues, they are good for encouraging teamwork. But, if used naively, they tend to suggest a 'right answer'.

Solution to Q23

(a) List the four main types of funding available for a firm. Give an example source for each type.

1. Grants: Source = government agencies (regional, national), the EU, charities.

2. Debt: Source = banks

3. Equity: Source = business angels, venture capitalists, public markets

4. Revenue: Customers (private and public organisations), may be paid in advance. [20%]

(b) Give examples of business models based upon:

(i) selling a product

(ii) selling a service

(iii) selling a service enabled by a product

(iv) selling a product with consumables

1. Sony (TV, laptop); Ford (cars, trucks); Unilever (ice cream, shampoo), etc2. Easyjet (flights); McKinsey (consulting services); Dojos (food), HSBC (financial services), etc

3. O2 (mobile phone calls + phones); Apple (ipod + itunes), etc

4.HP(printers + toner/ink); Gillette (razors and blades), etc

What are the strengths and weaknesses of each of these business models? [40%]

Sell a product: Pros = Increase revenue with higher volumes; Con = Need to develop, make, distribute, support, etc.

Sell a service: Pro = No manufacturing costs; Con = Can be hard to scale up the business (in many cases can only grow through recruiting and training lots of people).

Sell a product plus services: Pro = Long-term revenues on the back of the sale of each product, Con = Need infrastructure to provide services.

Sell a product plus consumables: Pro = Long-term revenues on the back of the sale of each product; Con = Need infrastructure to deliver consumables.

(c) Discuss the advantages and disadvantages of business models for a product-based start-up firm where the business operations are managed either:

(i) entirely in-house

(ii) in partnership with other firms [40%]

Entirely in-house

Advantages

You get all the value generated; you are in full control of what happens; coordination is simpler as no external parties involved, problems can be addressed quickly, communication gaps are minimised.

Disadvantages

You need to raise all the investment/capital; you need to purchase / lease equipment which may remain idle much of the time; you take all responsibility for all that goes wrong; you have to work within the limits of the resources you can access – there may be gaps in knowledge that are costly to fill; you are responsible for all equipment, materials, people, etc.

Form partnership with someone else

Advantages

You can focus efforts on where you can add most value; use experience and resources of other businesses; potentially increased speed to market as less setup costs / time required; shared responsibility for building the business; reduced level of investment/capital; reduced levels of risk; increased flexibility- better able to respond to changing market conditions.

Disadvantages

Share the value with other people / organisations, need to work with (and 'get-on with') other people / organisations. Managing partnerships can be very resource intensive; partners' activities can change; need to manage partnerships throughout whole lifecycle (setup, management, termination); need to be careful with type of agreement formed with partner (liability issues, clarity of responsibility for different activities).