ENGINEERING TRIPOS PART IB

Monday 2 June 2008 9 to 11

Paper 1

MECHANICS

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

The answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper
Single-sided graph paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 Figure 1 shows (in plan view) a rigid, uniform beam of length L and mass m at rest on a smooth, flat, level table. The beam is subjected to a horizontal force of constant magnitude F at right angles to, and at one end of, the beam.

(a) Show that the instantaneous centre of the beam's rotation is located at the point $x = 2L/3$. [4]

(b) Calculate the location on the beam at which the maximum bending moment occurs. [8]

(c) Express the magnitude of this maximum bending moment in terms of F and L and sketch the distribution of the bending moment along the beam. [8]

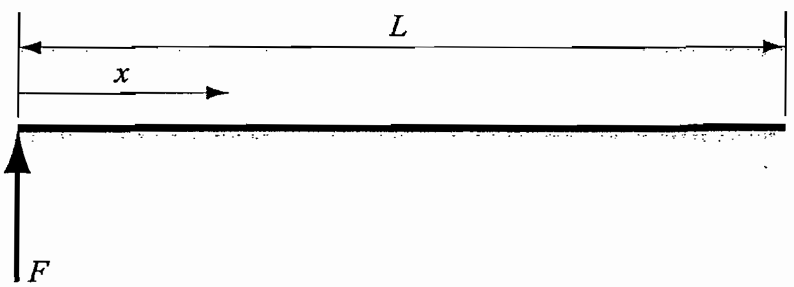

Fig. 1

2 A uniform, heavy bar AB of mass m and length $\sqrt{2}L$ is connected by frictionless pins to light links OA and BC that have lengths L and $2L$ respectively, as shown in Fig. 2. The motion of the mechanism is in the vertical plane, driven by the link OA which rotates with constant angular speed $\dot{\theta}$. For the instant shown find:

(a) find the angular speed $\dot{\psi}$ of the link BC and the angular speed $\dot{\varphi}$ of bar AB; [6]

(b) find the angular accelerations $\ddot{\psi}$ and $\ddot{\varphi}$; [8]

(c) hence calculate the torque required at O to drive the mechanism. [6]

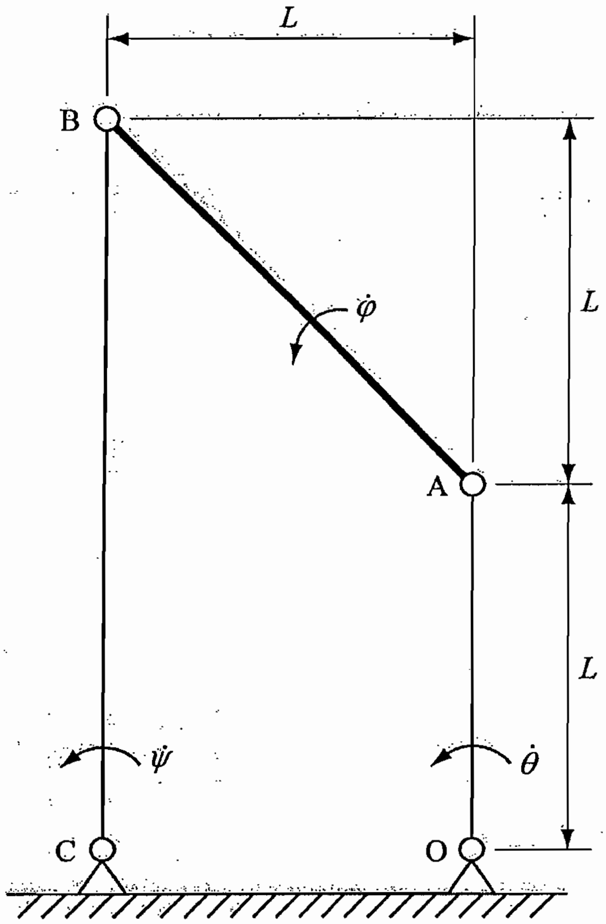

Fig. 2

3 A rigid light link OA of length L rotates around a smooth pivot at O with constant angular speed Ω as shown in plan view in Fig. 3. A rigid heavy bar AB of mass m and length L is pinned frictionlessly to OA at A. AB is held initially such that angle θ is 90° and is then released. Note that throughout the subsequent motion the angle θ is defined as the angle of AB relative to the direction of OA.

Assume that all motion is in the horizontal plane and that friction may be neglected.

(a) Determine the angular acceleration $\ddot{\theta}$ of AB relative to OA after AB is released. [8]

(b) Hence show that, after AB is released, θ satisfies:

$$\dot{\theta} = \Omega\sqrt{3\cos\theta}$$ [6]

(c) Write down the value of θ at which the tension in link OA is a maximum and calculate the magnitude of this maximum tension. [6]

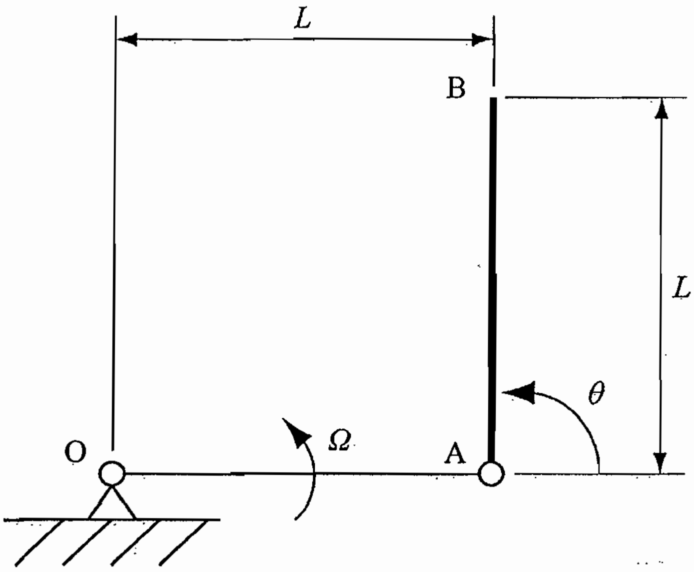

Fig. 3

SECTION B

4 A solid sphere of mass m and radius R rolls without slipping along a flat horizontal surface with speed u. The sphere rolls into a step of height h and sticks to the corner as shown in Fig. 4.

(a) Show that the sphere will not come out of contact with the ground if $h > 7R/5$. [10]

(b) Find the minimum value of initial speed u that will ensure that the sphere climbs all the way up onto the step, providing $h < 7R/5$. [10]

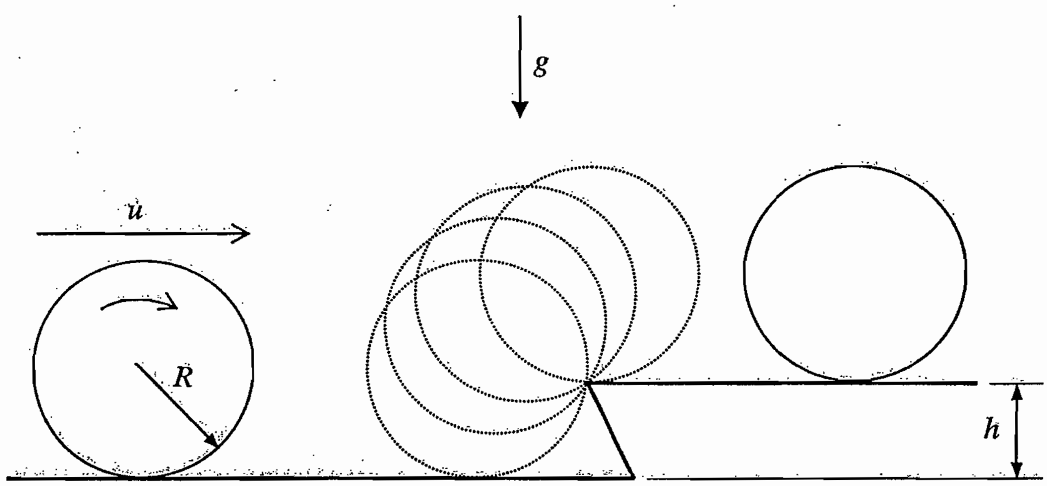

Fig. 4

5 Figure 5 shows a side elevation of a flywheel-driven toy car. The flywheel is a solid steel disc with a mass m and moment of inertia J. The flywheel spins with an angular velocity Ω on frictionless bearings with its axis of rotation parallel to the car wheels and is positioned within the car as shown in Fig. 5. The rear wheels of the car have a radius r and are driven via a non-slipping belt at a reduced angular velocity $\omega = \Omega/G$. The car wheels, body and transmission are light plastic components and their mass may be neglected.

The flywheel of the toy car is spun-up to an initial angular velocity of Ω_0 with the car stationary and the driven wheels just clear of the ground, as shown in Fig. 5. The car is then released and accelerates forwards in a straight line with the rear wheels slipping. The coefficient of friction between the rear wheels and the ground is μ.

(a) What is the horizontal acceleration of the car if it is just on the point of 'pulling a wheelie' (i.e. if the front wheels are just lifting off the ground)? [4]

(b) The normal contact forces between the ground and the front and rear wheels are N_1 and N_2 respectively. Show that the angular acceleration of the flywheel is:

$$\dot{\Omega} = \frac{-\mu N_2 r}{GJ}$$ [4]

(c) Find the inequality relating h, L, μ, r and G that must be true if the front wheels are to remain in contact with the ground as the car accelerates. [12]

The acceleration due to gravity is g.

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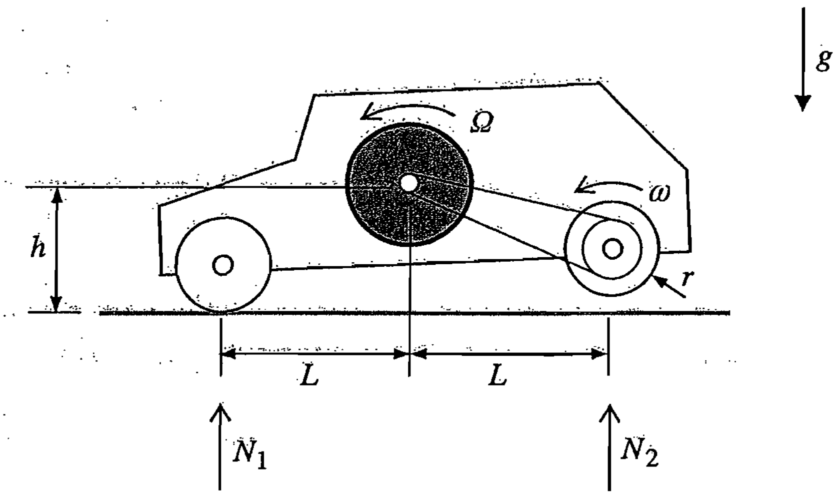

Fig. 5

(TURN OVER

6 Figure 6 shows a mechanism composed of three rigid links (BC, CDE and EF) and a hydraulic actuator (BD) connected by pin-joints. The fluid in the actuator acts on a piston of area A. Hydraulic fluid is to be pumped to the actuator at a varying volumetric flow rate Q in order to maintain a constant angular velocity ω of the rigid link EF.

(a) Find the relationship between Q, the hydraulic fluid flow rate, and dr/dt, the rate of change of the length of the actuator with respect to time. [2]

(b) Determine the flow rate Q necessary to achieve an angular velocity of ω for the rigid link EF when the mechanism is in the configuration shown in Fig. 6 with $\theta = 45°$. [8]

(c) Determine the rate of change of fluid flow rate with respect to time dQ/dt required to keep the angular velocity ω of EF constant at this instant. [10]

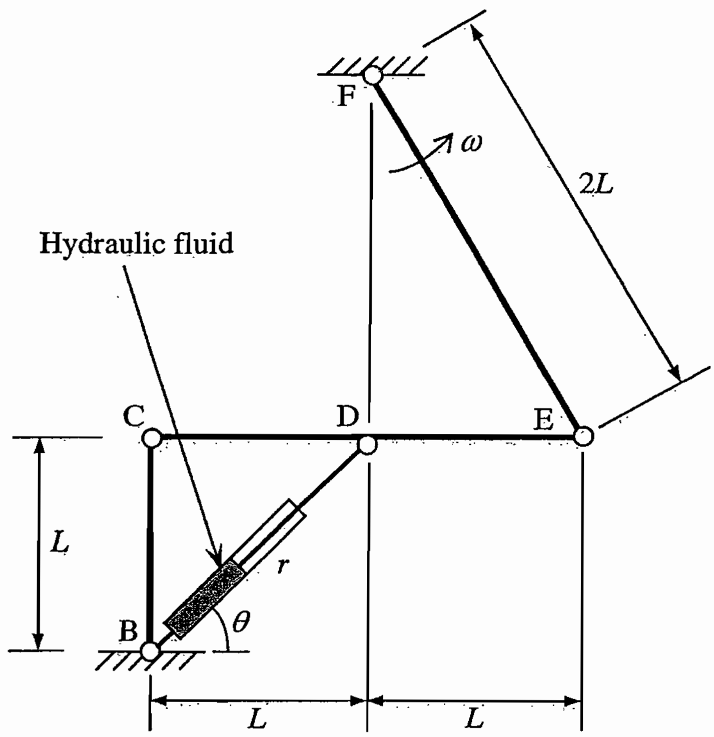

Fig. 6

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