ENGINEERING TRIPOS PART IB

Monday 2 June 2008 2 to 4

Paper 2

STRUCTURES

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 The pin-jointed structure shown in Fig. 1 consists of linear elastic bars of equal cross-sectional area A, made of the same material of Young's modulus E. The supports are positioned assuming that the bars are all of length L. In practice, bar III was manufactured too short by an amount e_0, where $e_0 << L$. Self-weight is to be ignored.

(a) Determine the number of redundancies. [2]

(b) Show that the state of self-stress is defined by equal bar tensions. [4]

(c) Using the Force Method, determine the actual bar forces in the structure in equilibrium with the load F applied to the central joint at an angle θ to the horizontal, as shown. [8]

(d) Find the components of displacement of the central joint when $\theta = 90°$. [6]

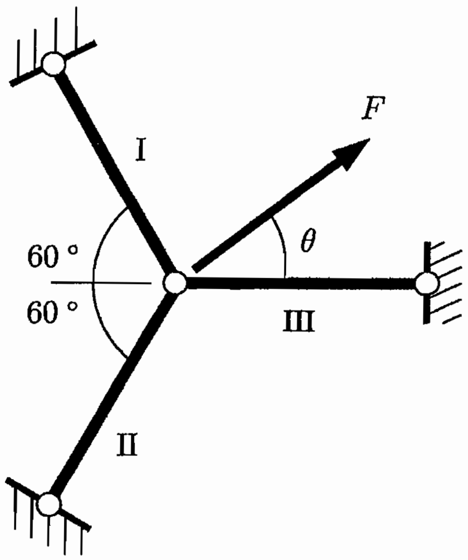

Fig. 1

2 (a) A thin-walled cylinder has a thickness t and a radius to mid-thickness R. Show that the second moment of area of the cross-section about any diameter is $\pi R^3 t$. Using this result, determine the polar second moment of area. [6]

(b) A $60\,°$ strain gauge rosette is attached to the outside surface of a thin-walled cylinder, made of steel with a yield stress of 275 MPa. The cylinder is unstressed initially and all gauges are set to zero. Upon loading, the gauges register the following strains: $\epsilon_0 = 100 \times 10^{-6}$, $\epsilon_{60} = -50 \times 10^{-6}$ and $\epsilon_{120} = 75 \times 10^{-6}$.

(i) If the strain gauge is aligned so that the component of strain, ϵ_{xx}, is measured directly by ϵ_0, show that the other strain components are given by:

$$\epsilon_{yy} = \frac{-\epsilon_0 + 2\epsilon_{60} + 2\epsilon_{120}}{3}, \quad \gamma_{xy} = \frac{2\epsilon_{60} - 2\epsilon_{120}}{\sqrt{3}}$$

[4]

(ii) Compute the principal stresses at the point of strain measurement. [6]

(iii) Determine the factor of safety according to the von-Mises criterion. [4]

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3 The planar, semi-circular arches in Fig. 2 have uniform cross-sections and are unstressed initially at ambient temperature. Both arches are then loaded at their apex by a vertical force V, as indicated, and deform within their own plane.

(a) Determine the reaction at each foot for the arch shown in Fig. 2(a). How does each reaction change when the temperature rises? [6]

(b) The arch in Fig. 2(b) has linear elastic bending stiffness EI and a linear coefficient of thermal expansion α.

(i) By using the principle of Virtual Work with the condition that the rotation at each foot is zero, compute the reaction at each foot at ambient temperature. [8]

(ii) Determine the change in ambient temperature needed to ensure that the reaction at each foot is purely vertical. [6]

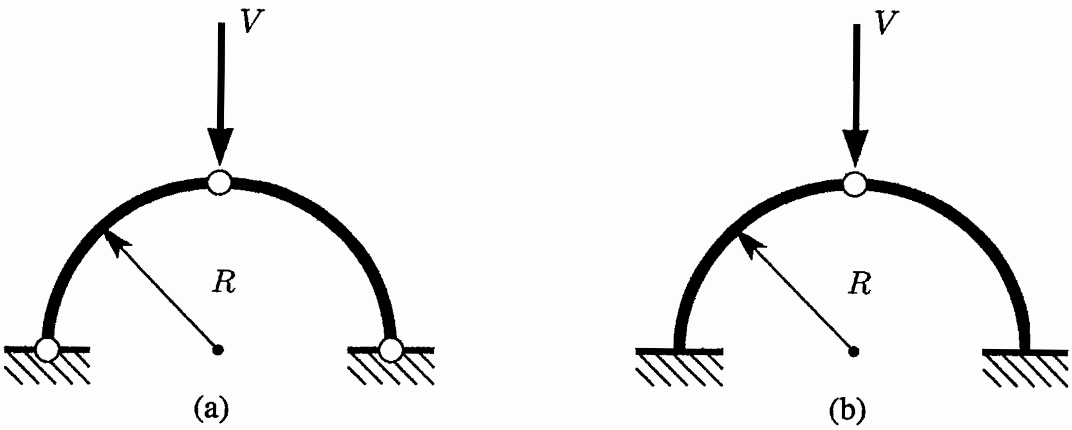

Fig. 2

SECTION B

4 The structure shown in Fig. 3(a) is a propped cantilever with a *rigid* vertical column connected *rigidly* at its mid-span. It carries a *total* distributed load W over the horizontal span and a force W at the tip of the column. The horizontal span has the uniform cross-section shown in Fig. 3(b), which is made from welded steel plate sections of width b and thickness t. Bending takes place about the cross-sectional major axis and the structure deforms within its own plane. There are no initial stresses before loading.

(a) Assuming $b >> t$, show that the plastic section modulus Z_p of the cross-section for major axis bending is approximated by:

$$Z_p = \frac{b^2 t}{2}$$

[6]

(b) Perform a *lower* bound analysis on the horizontal span to determine the *maximum* safe value of W. Sketch the overall bending moment diagram. [8]

(c) Perform an upper bound analysis to determine a value of collapse load. Comment on the relationship between your lower and upper bound estimates. [6]

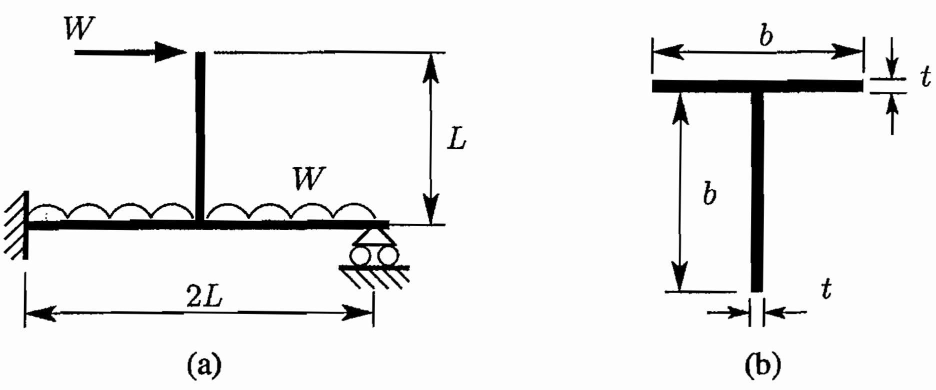

Fig. 3

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5 The inclined portal frame in Fig. 4 carries two forces H and V, where the vertical force is applied at mid-span. The frame has a uniform, fully plastic moment M_p, is unstressed initially and deforms within its own plane. Perform an upper bound analysis to determine the collapse conditions for:

(a) one beam mechanism; [4]

(b) one sway mechanism; [6]

(c) one combined mechanism. [6]

Combine all results into a single interaction diagram. [4]

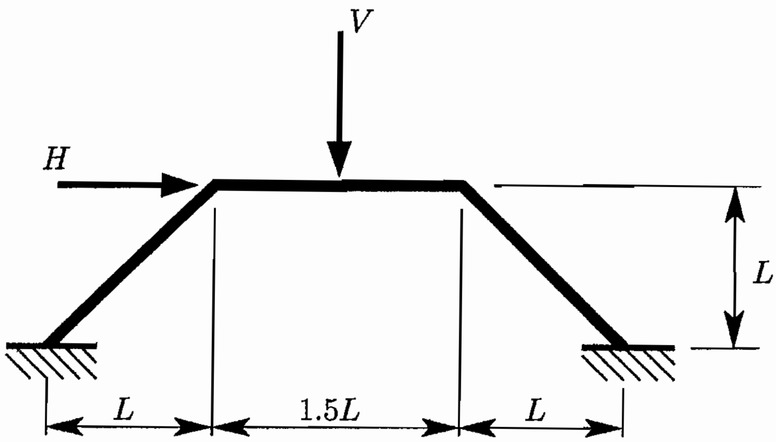

Fig. 4

6 The materials shown in Fig. 5 are uniform and isotropic, and are assumed to behave as ideal, rigid-plastic continua with a shear stress k . Where indicated, slip planes are shown as dashed lines.

(a) Two long, rigid anvils of breadth b indent a long strip of thickness b . A rigid block collapse mechanism is proposed as shown in Fig. 5(a). Show that the force per unit length F_1 required for successful indentation is:

$$F_1 = 2bk$$ [6]

(b) When a long strip of material is forced through the device shown in Fig. 5(b), its original thickness H is reduced according to the indicated geometry. Assuming that the contacting walls are long and smooth, estimate the required force per unit length F_2 for successful operation of the device. [6]

(c) A long rigid, right-angled triangular indenter impresses upon a long corner of material as shown in Fig. 5(c). Suggest a suitable collapse mechanism and estimate the force per unit length F_3 required for successful indentation when the contacting surfaces of the indenter and material are first *smooth* , and then *rough* . [8]

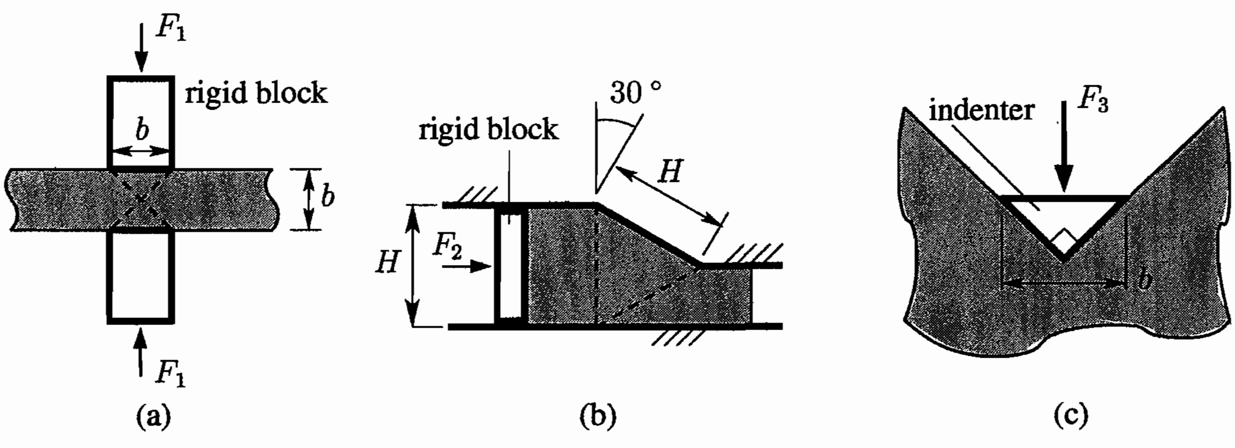

Fig. 5

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