ENGINEERING TRIPOS   PART IB

Tuesday 3 June 2008   9 to 11

Paper 3

MATERIALS

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1   Figure 1 shows the Ag − Al phase diagram.

   (a)   The Ag − Al system has one eutectic reaction. For *this* reaction, write down:

      (i)   the composition;

      (ii)   the phases involved;

      (iii)   the temperature of the reaction.   [4]

   (b)   An Ag − 20 wt% Al alloy is cooled slowly to room temperature from the melt. Describe the microstructural changes that occur, noting key temperatures and phase transformations. Illustrate your answer with sketches.   [6]

   (c)   The following alloys are solution heat-treated, quenched to room temperature and then reheated to 200 °C for 5 hours. Using the equilibrium phase diagrams in Figs 1 and 2, and Fig. 6.4 on page 30 of the Materials Data Book, suggest solution heat treatment temperatures for each alloy.

      (i)   Al − 50 wt% Ag (designated A);

      (ii)   Al − 10 wt% Mg (designated B);

      (iii) Al − 4 wt% Cu (designated C).   [4]

Table I gives the yield strength for alloys A, B and C after slow cooling to room temperature, and quenching and reheating to 200 °C. Explain briefly the differences between alloys B and C. Account for the behaviour of alloy A.   [6]

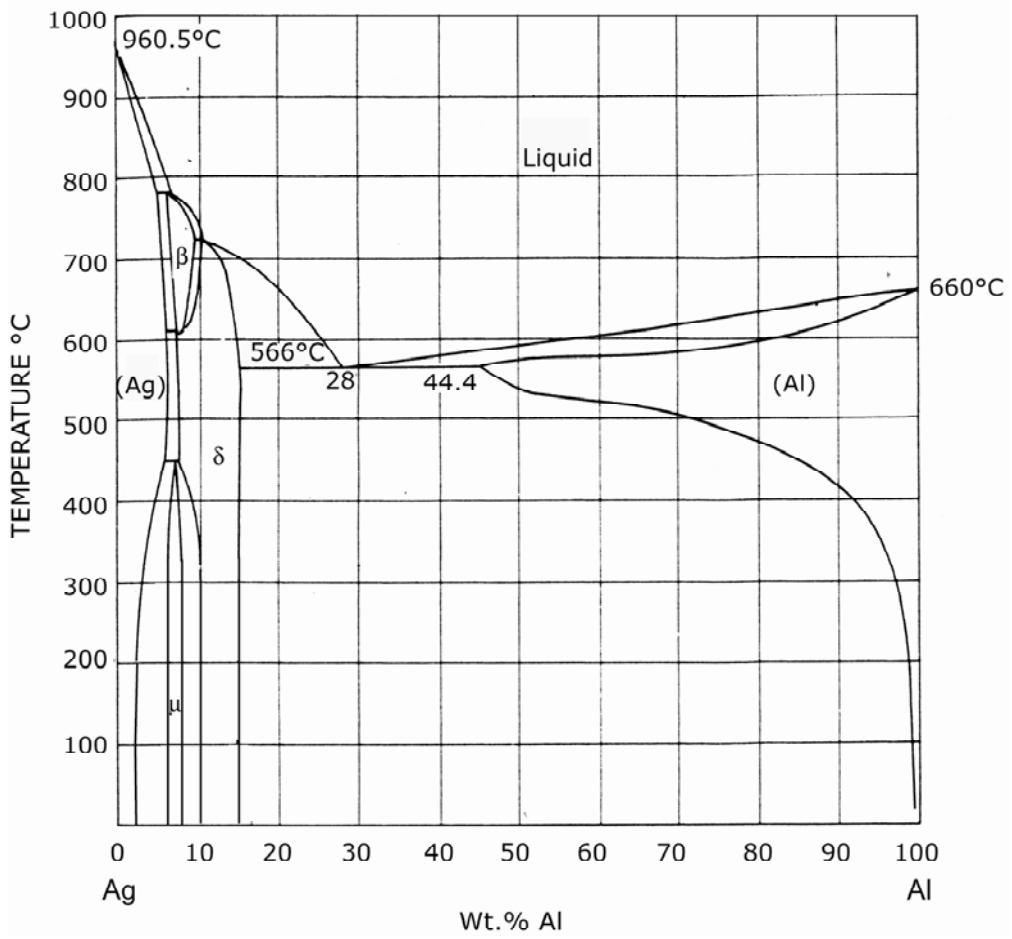| Alumimium alloy		Yield strength (MPa)	
		Slowly cooled to room temperature	Quenched and reheated to 200 °C
A	Al − 50 wt% Ag	120	450
B	Al − 10 wt% Mg	100	100
C	Al − 4 wt% Cu	100	350

Table I

Fig. 1
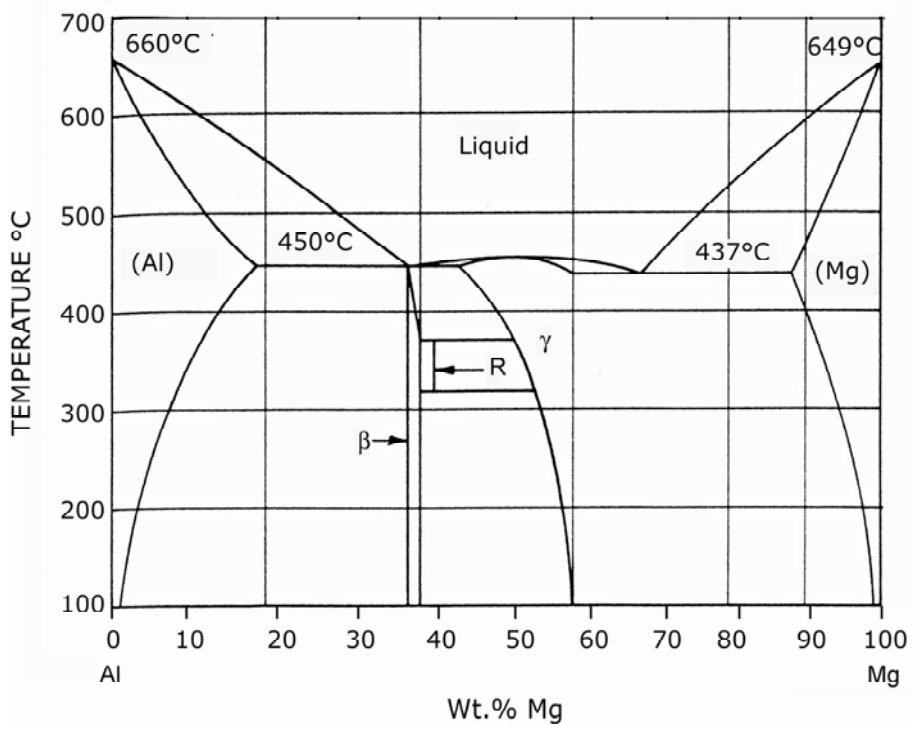

Fig. 2

2   (a)   Explain briefly the difference between *diffusive* and *displacive* phase transformations and give an example of each.   [4]

    (b)   Figure 3 shows the microstructures of three plain carbon steels and their corresponding values of hardness.   Identify the microstructures and phases present in these steels.   Estimate the carbon content of the steels in Figs 3(a) and (b), and explain your reasoning.   [5]
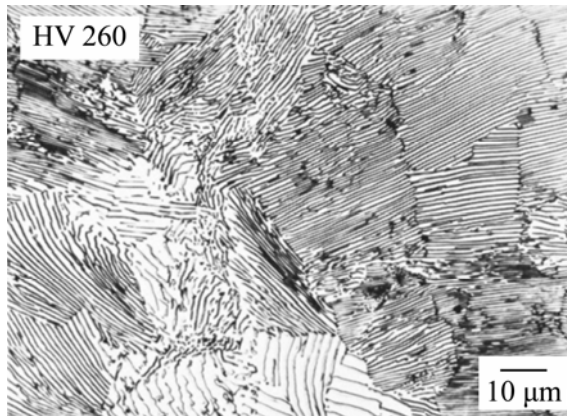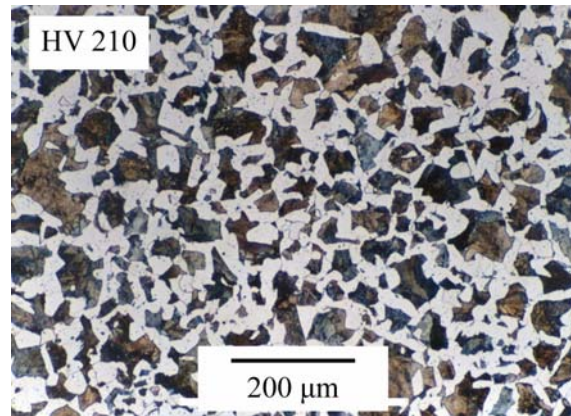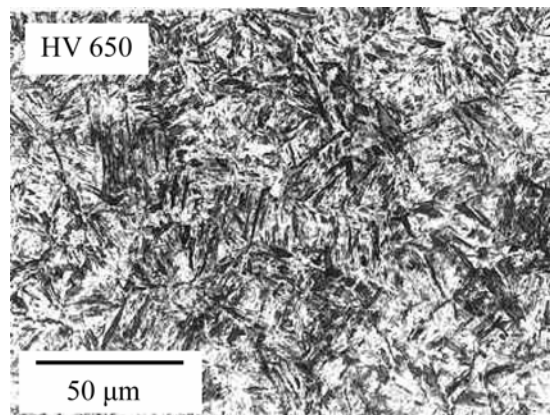

Fig. 3(a)   Fig. 3(b)

Fig. 3(c)

    (c)   Table II shows data for the diffusion coefficient, D, for carbon in bcc Fe. Construct an Arrhenius plot of the data in Table II. From your plot, estimate the value of D at 300 °C and the distance over which carbon can diffuse in one minute at this temperature. Estimate the ratio between the diffusion times needed to diffuse carbon the same distance in steel at 200 and 350 °C.   [6]

(cont.

Temperature (°C)	$D\,(\mathrm{m^2\ s^{-1}})$
200	7.0×10^{-16}
250	5.0×10^{-15}
350	1.0×10^{-13}
400	3.2×10^{-13}

Table II

(d)   The teeth on a large gearwheel, made from steel containing 0.15 wt% carbon, are case-hardened by carburising at a temperature of 950 °C. The diffusion of carbon into the steel is described by:

$$\frac{C(x,t) - C_o}{C_s - C_o} = 1 - \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

where C is the concentration of carbon at a distance x below the surface after time t , C_s is the concentration of carbon at the surface, C_o is the initial concentration in the steel, and D is the diffusion coefficient.

Calculate the time taken to obtain a carbon content of 0.8 wt% at a depth of 1.0 mm if the surface of the steel is maintained at a composition of   1.3 wt% carbon during carburising.

For the diffusion of carbon in γ-Fe, the activation energy Q is 84 kJ mol^{-1} and the corresponding pre-exponential factor D_o is 2×10^{-6} m^2 s^{-1}.   [5]

3   (a)   Define *Gibbs free energy* G and explain why it is a key parameter in the study of phase transformations. [4]

(b)   Show that the critical radius, r^* , for homogeneous nucleation of a spherical nucleus of a solid phase in a liquid phase is given by

$$r^* = -\frac{2\gamma T_E}{\Delta H_V (T_E - T)}$$

where γ is the surface energy per unit area of the solid phase-liquid interface, ΔH_V is the enthalpy change per unit volume associated with the transformation, T_E is the equilibrium temperature and T is the temperature of the system.

Explain briefly the difference between *heterogeneous* and *homogeneous* nucleation, and give an example of heterogeneous nucleation. Which type of nucleation do you expect to occur for a small degree of undercooling? [6]

(c)   An amorphous glass has a population density of 3×10^{13} m^{-3} spherical nuclei available for crystallisation. It is heat treated to form a completely crystalline grain structure. Assume that crystallisation follows the same nucleation as the solidification process in (b).

(i)   If the final average grain diameter in the crystalline material is 45 μm, estimate the number of nuclei per unit volume which are activated to form grains in the final product. [3]

(ii)   The spherical nuclei have radii evenly distributed between 2 nm and 5 nm as illustrated in Fig. 4. Use your answer in (i) to calculate the critical radius r^* of the nucleus. [4]

(iii)   What temperature should be chosen for the heat treatment in order to obtain an average grain size of 45 μm in the crystalline material? [3]

The surface energy between amorphous and crystalline glass is $\gamma = 0.18$ J m^{-2}, the enthalpy change per unit volume on crystallisation is $\Delta H_V = -1.5 \times 10^9$ J m^{-3} and the crystallisation temperature is $T_E = 850$ °C.
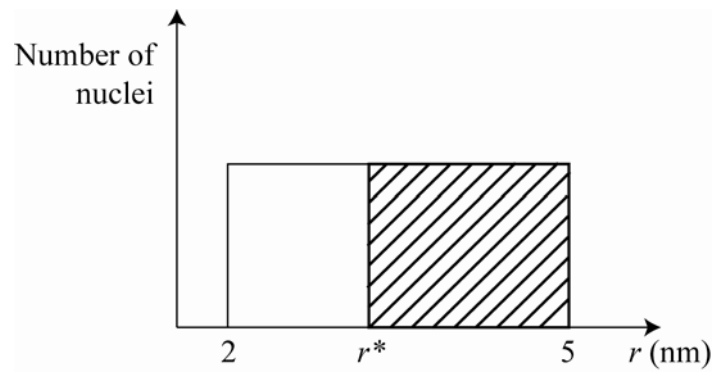
Fig. 4

SECTION B

4   (a)   Sketch a typical *creep* curve, and label carefully the different stages of creep. Describe briefly the contribution of each stage to creep life.   [3]

(b)   Explain briefly the mechanisms by which *dislocation creep* and *diffusion creep* occur.   [4]

(c)   A nickel-based alloy (ρ = 8900 kg m^{-3}) is to be used as a turbine blade. The root radius r_r and tip radius r_t of the blade, illustrated schematically in Fig. 5, are 0.2 m and 0.3 m , respectively.

(i)   For a constant angular speed ω , write down an expression for the acceleration as a function of the distance r from the centre of the turbine.   [2]

(ii)   Assuming that the cross-sectional area A of the blade is constant along its length, show that the stress σ at the root of the blade is given by:

$$\sigma = \frac{\rho\omega^2}{2}(r_t^2 - r_r^2)$$

[4]

(iii) The steady-state creep behaviour of the alloy can be described using the diffusional creep equation;

$$\dot{\varepsilon} = B\sigma \exp\left(-\frac{Q}{RT}\right)$$

where $\dot{\varepsilon}$ is the steady state strain rate, B is a constant, Q is the activation energy for creep and R is the gas constant.

(cont.

Show that the rate of change of the blade length with time is given by:

$$\frac{dl}{dt} = B\frac{\rho\omega^2}{2}\exp\left(-\frac{Q}{RT}\right)\int_{r_r}^{r_t} (r_t^2 - r^2)\, dr$$

If the turbine blade operates at a temperature of 900 °C and a rotational speed of 12000 rpm, determine the length change in the blade after continuous operation for 1000 hours.

(For this alloy, $Q = 135$ kJ mol^{-1} and $B = 4.0\times10^{-11}$ N^{-1} m^2 s^{-1}.)   [5]

(iv) What microstructural features can be modified to reduce diffusional creep deformation?   [2]
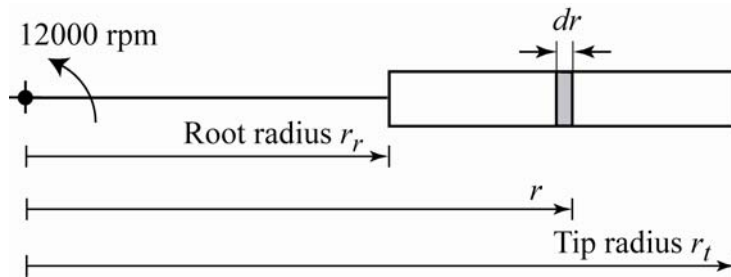

Fig. 5

5   (a)   Consider a two-dimensional foam having a square unit cell as shown in Fig. 6(a). Each cell has a side length L and wall thickness t. The density and Young's modulus of the foam are ρ^* and E^*, respectively, and the density and Young's modulus of the material from which the walls are made are ρ_s and E_s. Estimate:

(i)   the relative density of the foam ρ^* / ρ_s ;   [4]

(ii) the relative Young's modulus of the foam, E^* / E_s , when loaded in the vertical direction, in terms of the relative density, ρ^* / ρ_s . Assume that the deformation is dominated by bending ($t \ll L$) and that the end-clamped condition is applicable.   [8]

(b)   Consider two open-cell foams of the same relative density. Foam 1 has a square unit cell as shown in Fig. 6(a). Foam 2 has a rectangular unit cell. The cell has a width $4L_1$ and height L_1 as shown in Fig. 6(b). The foams are made of the same material and the walls in both foams are of the same thickness.

(i)   Express the cell dimension L_1 in terms of L .   [4]

(ii) Calculate the ratio of the Young's moduli of the two foams when loaded in the vertical direction, assuming the same deformation mode.   [4]
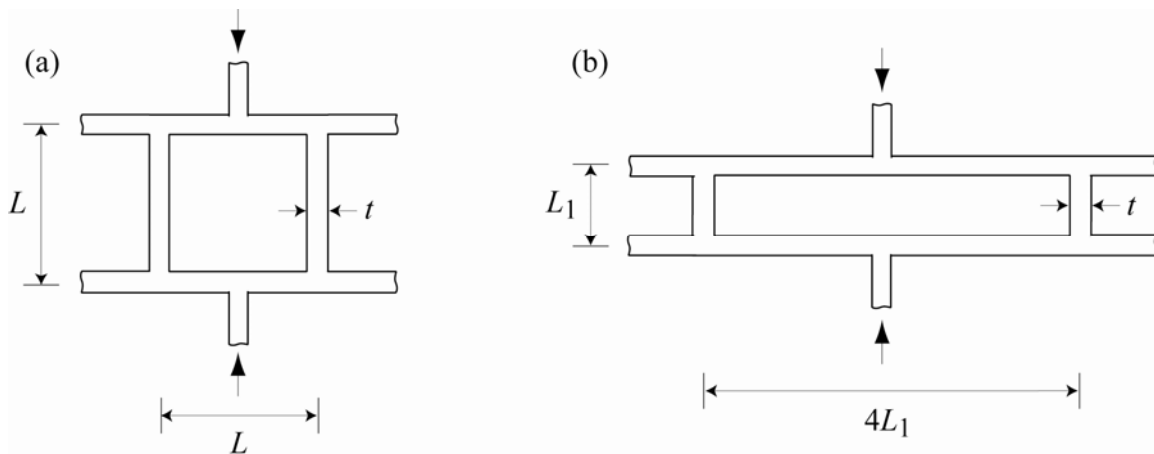

Fig. 6

6   (a)   Explain why dashpot and spring elements are useful in describing the viscoelastic behaviour of polymers.   Write down the governing equations for each element.   [4]

(b)   The deformation response of a certain polymer can be represented by the dashpot / spring arrangement shown in Fig. 7.   One branch of the model consists of a linear elastic spring of modulus E_1 .   A second branch, loaded in parallel with the first branch, consists of a linear elastic spring of modulus E_2 in series with a linear dashpot of viscosity η . The macroscopic stress and strain are $\sigma(t)$ and $\varepsilon(t)$ respectively, as shown in Fig. 7.

(i)   Derive the differential equations relating the total strain rate $\dot{\varepsilon}(t)$ to the stresses $\sigma_1(t)$ and $\sigma_2(t)$ in each of the two parallel-loaded branches.   [4]

(ii)   At $t = 0$, a strain ε_o is applied and held constant.   Find an expression for the stress contributions $\sigma_1(t)$ and $\sigma_2(t)$ in both branches.   If $E_2 = 400$ MPa and $\eta = 2 \times 10^{12}$ MPa s , calculate the relaxation time τ . Show that the relaxation modulus $E_r(t)$ is given by:
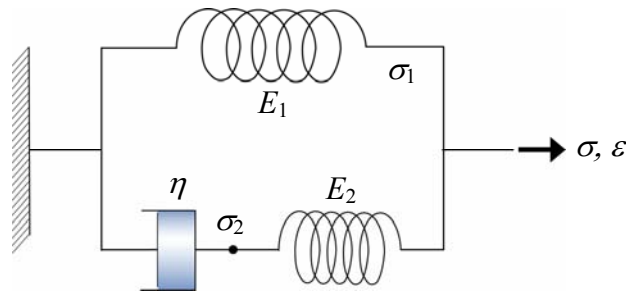
$$E_r(t) = E_1 + E_2 \, e^{-t/\tau}$$ [12]

Fig. 7

END OF PAPER