ENGINEERING TRIPOS PART IB

Tuesday 3 June 2008 2 to 4

Paper 4

THERMOFLUID MECHANICS

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

*Answer not more than **two** questions from this section.*

1 *It should be noted that parts (a) and (b) of this question are unrelated.*

(a) A hot water pipe has an outside diameter of 15 mm and a surface temperature of 60 °C. Material with thermal conductivity 0.03 $\mathrm{W\,m^{-1}\,K^{-1}}$ is to be used to insulate the pipe. The outside diameter of the insulation is 50 mm and the surface heat transfer coefficient is 6 $\mathrm{W m^{-2} K^{-1}}$. The surrounding room temperature is 10 °C.

(i) Show that for radial heat conduction through an annular layer of thermal conductivity λ with inner and outer radii of r_1 and r_2 respectively the thermal resistance (per unit length) is:

$$\frac{\ln(r_2/r_1)}{2\pi\lambda}$$ [2]

(ii) Assuming that the surface temperature of the hot water pipe does not change when the insulation is fitted, calculate the convective heat transfer loss per unit length, ignoring radiation. [3]

(iii) The insulation around the pipe can be considered as a grey body with emissivity of 0.8 and the room is a black body. Assuming that the surface temperature of the insulation is the same as in part (ii) above, calculate the heat transfer loss per unit length due to radiation. [3]

(b) A large metal heat sink operates at a steady uniform temperature, T_0. There is a circular hole of diameter d in the heat sink. Water is pumped through this hole to cool the heat sink. The water is supplied at temperature T_1 and the mass flow rate through the hole is $\dot{m}$. The arrangement is shown schematically in Fig. 1. The surface heat transfer coefficient between the water and the heat sink is h and the specific heat capacity of the water is c_p. Both h and c_p are constant along the hole and the water remains in the liquid phase.

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(i) Show that at a distance x along the hole the temperature $T(x)$ of the water is given by:

$$T(x) = T_0 - (T_0 - T_1)\exp\left\{\frac{-\pi d h}{\dot{m} c_p} x\right\}$$ [6]

The hole in the heat sink has diameter 5 mm and is 0.5 m in length. The water is at a pressure of 5 bar throughout the hole and the mass flow rate is 0.06 $\mathrm{kg\,s^{-1}}$. The water enters the hole at 25 °C and the water temperature at exit must be at least 20 °C below the saturation temperature.

(ii) Determine the flow regime within the hole. [2]

(iii) Estimate the Nusselt number for the water flow. [2]

(iv) Calculate the maximum operating temperature of the heat sink. [2]

Suitable correlations for the convective heat transfer are given in the Thermofluids Data Book. You may assume that liquid water at 5 bar has the following average properties: density = 970 $\mathrm{kg\,m^{-3}}$, dynamic viscosity = 0.56×10^{-3} $\mathrm{kg\,s^{-1}\,m^{-1}}$, thermal conductivity = 0.65 $\mathrm{W\,m^{-1}\,K^{-1}}$ and specific heat capacity = 4.22 $\mathrm{kJ\,kg^{-1}\,K^{-1}}$.

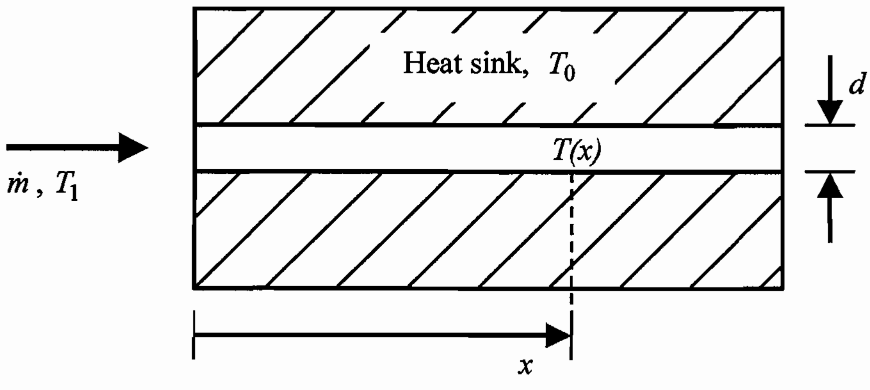

Fig. 1

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2 (a) Explain why the enthalpy h of superheated water vapour at low partial-pressure p and temperature T may be approximated by either of the two following expressions:

$$h(p,T) \approx h_g\big|_p + c_{ps}\left(T - T_g\big|_p\right),$$

or

$$h(p,T) \approx h_g\big|_T$$

where the subscript g refers to conditions on the saturated vapour line and c_{ps} is the specific heat capacity at constant pressure. [2]

(b) If the water vapour within an air-conditioning plant is modelled as a perfect gas, show that the specific gas constant is $R_s = 0.462\ \mathrm{kJ\,kg^{-1}K^{-1}}$. [1]

An air-conditioning plant, shown schematically in Fig. 2, operates at atmospheric pressure and consists of a cooler followed by a heater. Moist air enters the air-conditioning plant at $30\ °\mathrm{C}$ and relative humidity $\phi = 79\%$ (state 1). The condensate is removed beneath the cooler (state 2). The temperature of the condensate is the same as the moist air after the cooler (state 3). At state 3 the moist air is saturated. At exit from the air-conditioning plant the moist air is at $20\ °\mathrm{C}$ and relative humidity $\phi = 40\%$ (state 4). The mass flow rate of moist air entering the plant is $15\ \mathrm{kg\,s^{-1}}$.

(c) Show that the mass flow rate of water vapour entering the air-conditioning plant $\dot{m}_{s1}$ is $0.3125\ \mathrm{kg\,s^{-1}}$. [4]

(d) By considering the flow through the heater (state 3 to state 4) show that the temperature T_3 before the heater is approximately $6\ °\mathrm{C}$. Determine $\dot{m}_{s4}$ the mass flow rate of water vapour at exit. [4]

(e) Determine $\dot{Q}_{heater}$ the rate of energy input to the heater. [3]

(f) Determine $\dot{m}_{w2}$ the mass flow rate of condensate. [1]

(g) Determine $\dot{Q}_{cooler}$ the rate of energy extracted by the cooler. [5]

Throughout this question you may assume that the specific heat capacity at constant pressure for superheated water vapour at low partial-pressures is $1.86\ \mathrm{kJ\,kg^{-1}K^{-1}}$.

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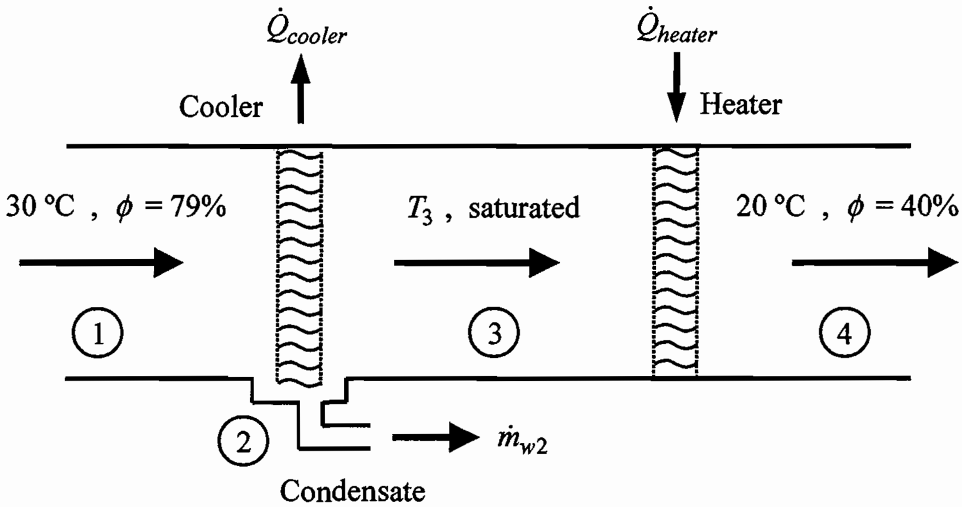

$\dot{Q}_{cooler}$
$\dot{Q}_{heater}$
Cooler
Heater
30 °C , ϕ = 79%
T_3 , saturated
20 °C , ϕ = 40%
1
3
4
2
$\dot{m}_{w2}$
Condensate

Fig. 2

3 An ideal refrigeration plant for a 'cold-store' is shown in Fig. 3. Both the compressor and turbine are reversible and the working fluid is R-134a. Saturated vapour enters the adiabatic compressor at $-15\ ^\circ\mathrm{C}$ and saturated liquid enters the adiabatic turbine at $50\ ^\circ\mathrm{C}$.

(a) Calculate the maximum possible coefficient of performance for a refrigeration plant operating between the above two temperatures. [2]

(b) Explain why in most practical refrigeration cycles the turbine is replaced by a throttle. [2]

(c) Sketch the thermodynamic cycle of the refrigeration plant on both '$p-h$' and 'T-s' diagrams. Indicate clearly how the cycle would be changed if either the turbine or the compressor were not reversible. [6]

(d) Show that for the reversible compressor the conditions at condenser entry correspond to 8.44 K of superheat. [3]

(e) Calculate the specific enthalpy of the refrigerant at entry to the evaporator for the reversible turbine. [2]

(f) Calculate the coefficient of performance for the refrigeration plant and comment on the value compared to that calculated in part (a). [5]

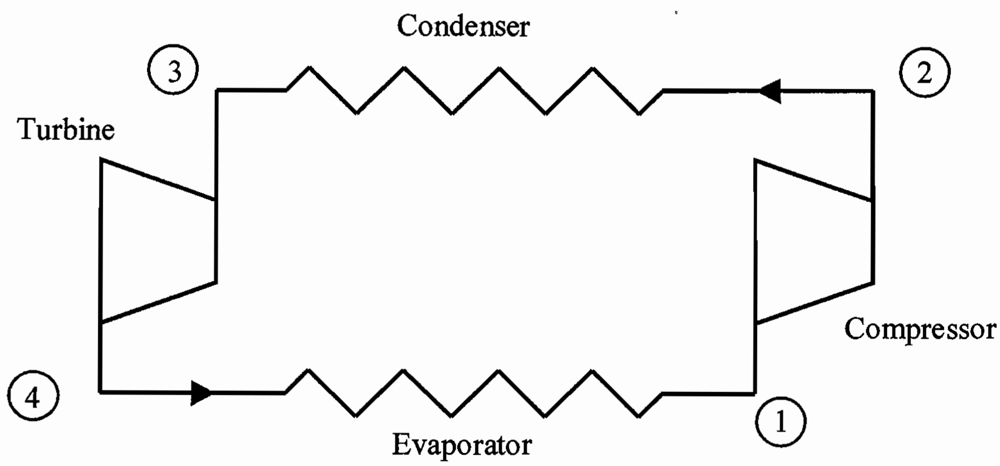

Fig. 3

SECTION B

Answer not more than two questions from this section.

4 For a cylinder of diameter d moving perpendicular to its axis with constant velocity V through an infinite fluid of density ρ and dynamic viscosity μ the drag force per unit length is F. Figure 4 shows how the drag coefficient C_D depends on the Reynolds number Re.

(a) Give appropriate definitions for the drag coefficient and Reynolds number. [2]

(b) Explain the physical interpretation of the Reynolds number. [2]

(c) Explain why at low Reynolds number $C_D \propto (Re)^{-1}$. [4]

(d) Explain carefully why the drag coefficient reduces when the Reynolds number is increased from A to B on Fig. 4. Include in your explanation sketches of the likely flow pattern around the cylinder at both these Reynolds numbers and comment on the changes in the relative sizes of the 'skin-friction drag' and 'form drag'. [8]

(e) Determine the drag force per unit length for the following:

(i) a 25 mm diameter cylinder moving at 60 ms^{-1} in air;

(ii) a 0.6 mm diameter 'fishing line' in water moving at 2 ms^{-1}.

Both can be assumed to be operating at sea level conditions. [4]

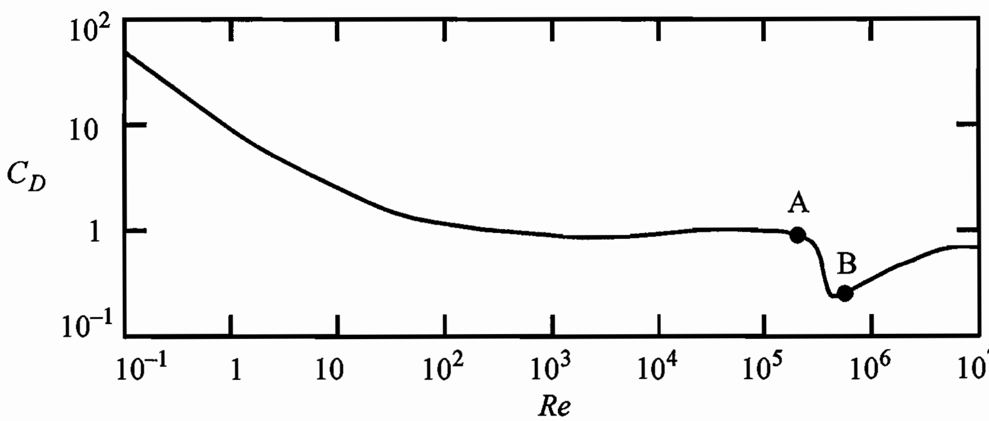

Fig. 4

5 An incompressible fluid with velocity V_1 and density ρ approaches a row of long flat plates, as shown in Fig. 5. The flat plates can be considered to have zero thickness and are spaced w apart at an angle β to the oncoming flow direction. Between each adjacent pair of plates there is a steady region of separated flow within which there is negligible velocity. This region causes a reduction of the flow area forcing the oncoming flow to form a 'free-stream jet' between the plates. Towards the end of the plates, the free-stream jet is parallel and occupies a fraction $1/b$ of the geometrical area between adjacent plates. Between the plates, there is no mixing of the separated region with the free-stream jet.

(a) Explain why, towards the end of the plates, the pressure p_j in the free-stream jet and in the separated region are the same. [2]

(b) Show that the velocity within the free-stream jet V_j is given by:

$$V_j = \frac{b V_1}{\cos\beta}$$ [2]

(c) By applying the steady flow momentum equation in the direction parallel to the plates (i.e. at an angle β to the oncoming flow) to the flow through the control volume indicated in Fig. 5 and ignoring any shear stress on the surface of the plates show that:

$$p_1 w \cos\beta + \dot{m} V_1 \cos\beta = p_j\, w \cos\beta + \dot{m} V_j$$

where p_1 is the pressure of the oncoming flow and $\dot{m}$ is the mass flow rate per unit height passing between each pair of plates. [4]

(d) Explain why Bernoulli's equation can be applied along a streamline between the oncoming flow into the free-stream jet. Show that:

$$b = 1 + |\sin\beta|$$ [8]

(e) Far downstream of the row of flat plates, the free-stream jets and the wakes formed by the separated regions mix to form a uniform flow. Calculate the flow angle α_2 far downstream of the row of flat plates. [4]

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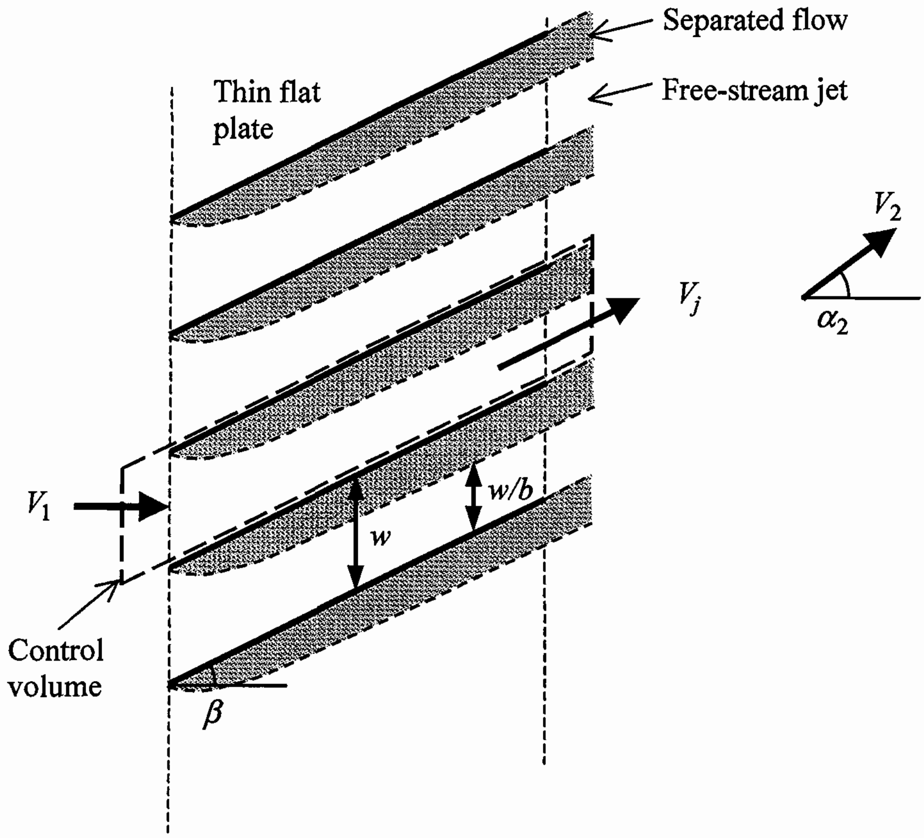

Fig. 5

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6 A large circular reservoir of diameter D and depth H feeds a viscous fluid into a vertical pipe of small diameter d and length h, as shown in Fig. 6. The fluid has density ρ and dynamic viscosity μ.

(a) Assuming that $D >> d$, explain why viscous forces are only expected to be important in the vertical pipe. [2]

(b) By considering a suitable control volume, or otherwise, show that the equation that governs the distribution of shear stress τ across the vertical pipe is:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau) = \frac{\partial p}{\partial x} - \rho g$$

where r is the distance from the centre of the vertical pipe, p is the pressure, g is the acceleration due to gravity and x is the distance down the vertical pipe. [4]

(c) Explain why the pressure gradient is constant along the vertical pipe. [2]

(d) Show that the velocity profile in the vertical pipe is:

$$v(r) = \frac{1}{16\mu}\left(\rho g - \frac{\partial p}{\partial x}\right)\left(d^2 - 4r^2\right)$$ [6]

(e) Find an expression involving ρ, g, H and h for the pressure gradient. [2]

(f) Calculate the volume flow rate along the vertical pipe. [4]

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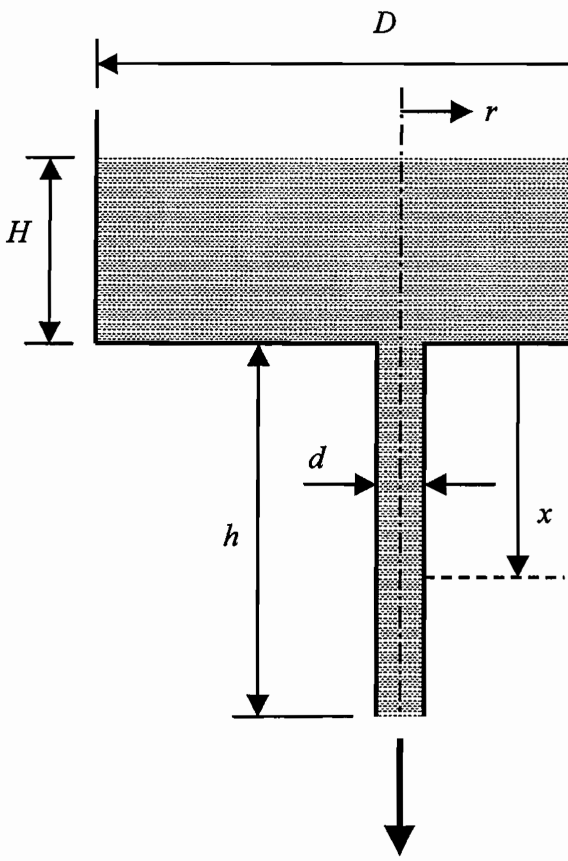

Fig. 6

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