ENGINEERING TRIPOS PART IB

Thursday 5 June 2008 2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than ***four*** *questions.*

Answer not more than ***two*** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Additional copy of Fig. 2.

STATIONERY REQUIREMENTS
Single-sided script paper,
semi-logarithmic graph paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

*Answer not more than **two** questions from this section.*

1 (a) Define the term frequency response for an asymptotically stable system having transfer function $G(s)$, and explain how it can be used to calculate the steady state response of such a system to inputs of the form $x(t) = \sin(\omega t)$, where ω rad s $^{-1}$ is a fixed frequency. [6]

(b) A linear system has the following transfer function:

$$G(s) = \frac{100s}{(s+1)(s+10)}$$

Determine:

(i) the response of the system to an input signal $x(t) = 0.1H(t)$, after any initial transients have died away;

(ii) the response of the system to an input signal $x(t) = 0.3\cos(0.3t)$, again after any initial transients have died away. [6]

(c) Sketch the Bode diagram for $G(s)$ on the semi-log paper provided. Determine from this the approximate range of frequencies where the input signal is passed unattenuated. [8]

2 (a) A linear system has impulse response $g(t)$. Under what condition is the system asymptotically stable? [6]

(b) A linear system has the following impulse response:

$$g(t) = \exp(-t) \sum_{k=0}^{\infty} \delta(t - k\tau)$$

where τ is a positive constant. Determine whether the system is asymptotically stable. [6]

(c) Show that the transfer function of the system described in part (b) is

$$G(s) = \frac{1}{1 - \exp(-(s+1)\tau)}$$

Consider a sine-wave input signal $x(t) = \sin(\omega t)$, where ω rad s^{-1} is a fixed frequency. Determine the smallest non-zero frequency ω for which the steady state output signal is in phase with the input signal, when $x(t)$ is input to the system $G(s)$. What is the gain of the system at that frequency? [8]

3 A linear system has transfer function

$$G(s) = \frac{10}{s(s^2 + 2\sqrt{2}s + 4)}$$

The system is to be controlled in the negative feedback configuration of Fig. 1 with a constant gain of k_p in the forward path.

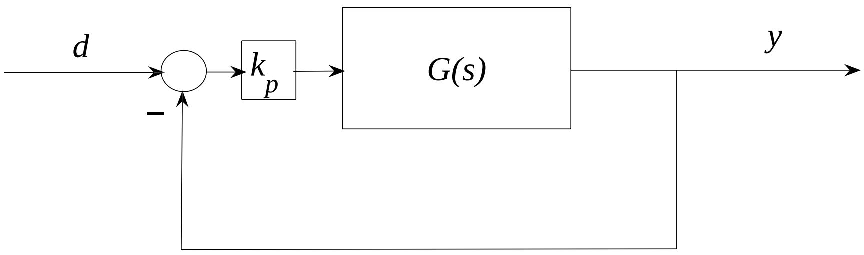


Fig. 1

(a) Determine the closed-loop transfer function from d to y. [4]

(b) A part of the Nyquist diagram for the system is shown in Fig. 2. Determine the gain and phase margins for the system, with $k_p = 1$. How would you expect the system to respond to changes in the desired signal d ? [10]

(c) When $k_p = 0.5$, determine the range of frequencies for which

$$\left| \frac{1}{1 + k_p G(j\omega)} \right| \geq 1$$

Explain the implications of this for system behaviour. [6]

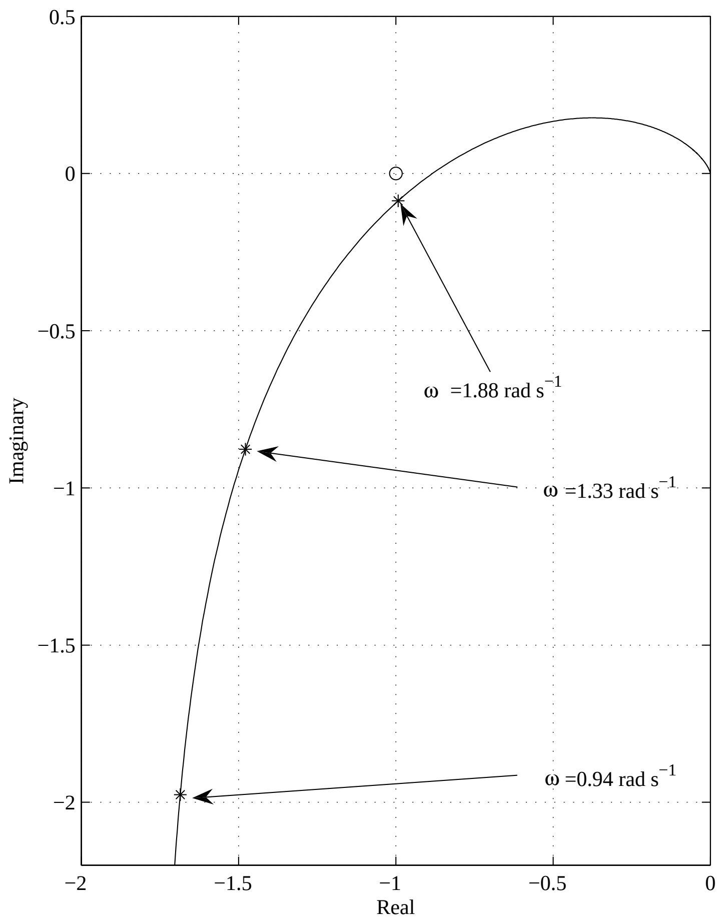


Fig. 2

Note: an additional copy of Fig. 2 is supplied at the end of this paper. This should be annotated with your constructions and handed in with your answer to this question.

SECTION B

Answer not more than ***two*** *questions from this section.*

4 A signal $f(t)$ is as shown in Fig. 3, where T and τ are constants. Note that $f(\tau) = f(\tau + T) = 1$.

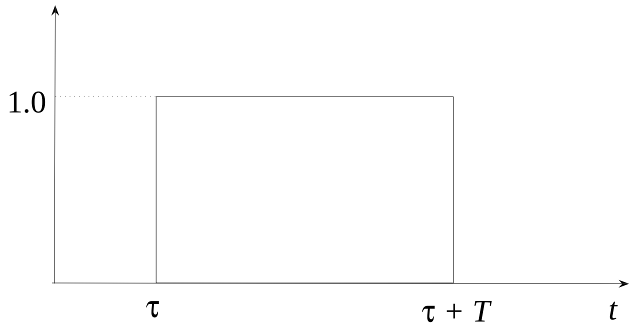


Fig. 3

(a) Show by direct integration that its Fourier Transform is

$$F(\omega) = T \exp(-j\omega(\tau + T/2))\text{sinc}(\omega T/2)$$ [5]

(b) It is proposed to sample $f(t)$ at regular times $t = \ldots 0, T/M, 2T/M, \ldots$, etc., where M is an integer constant. Now, take $\tau = 0$. Show that the Discrete Time Fourier Transform (DTFT) $F_s(\omega)$ of the sampled waveform is:

$$F_s(\omega) = \exp(-j\omega T/2)\frac{\sin((M+1)\omega T/(2M))}{\sin(\omega T/(2M))}$$ [8]

(c) Hence, using sampling theory or otherwise, explain why the following identity holds:

$$\sum_{k=-\infty}^{+\infty} (-1)^{kM}\text{sinc}(\omega T/2 + \pi k M) = \frac{\sin((M+1)\omega T/(2M))}{M\sin(\omega T/(2M))}$$ [7]

5 (a) Explain the meaning of channel capacity. [2]

(b) The binary erasure channel is a channel for which the transmitted bits are either received correctly with probability $1-\varepsilon$ or erased with probability ε, i.e. there are no bit-flips . The capacity of this channel is

$$C = 1-\varepsilon \quad \text{bits/channel use}$$

where ε is the erasure probability. Sketch the capacity of the channel as a function of ε and shade the region of rates that are not achievable. [2]

(c) Assume now that the erasure probability ε depends on the signal-to-noise ratio (SNR) of a given modulation. What sort of relationship would you expect between ε and SNR? Justify your answer. [3]

(d) If $\varepsilon = 1/\text{SNR}$, sketch the capacity of the channel as a function of SNR, shading again the region of rates that are not achievable. [3]

(e) A speech signal of bandwidth $B = 3$ kHz is to be transmitted over the above channel, but needs to be digitised first.

(i) How many samples per second do we need from this signal so that we can reconstruct it perfectly? [2]

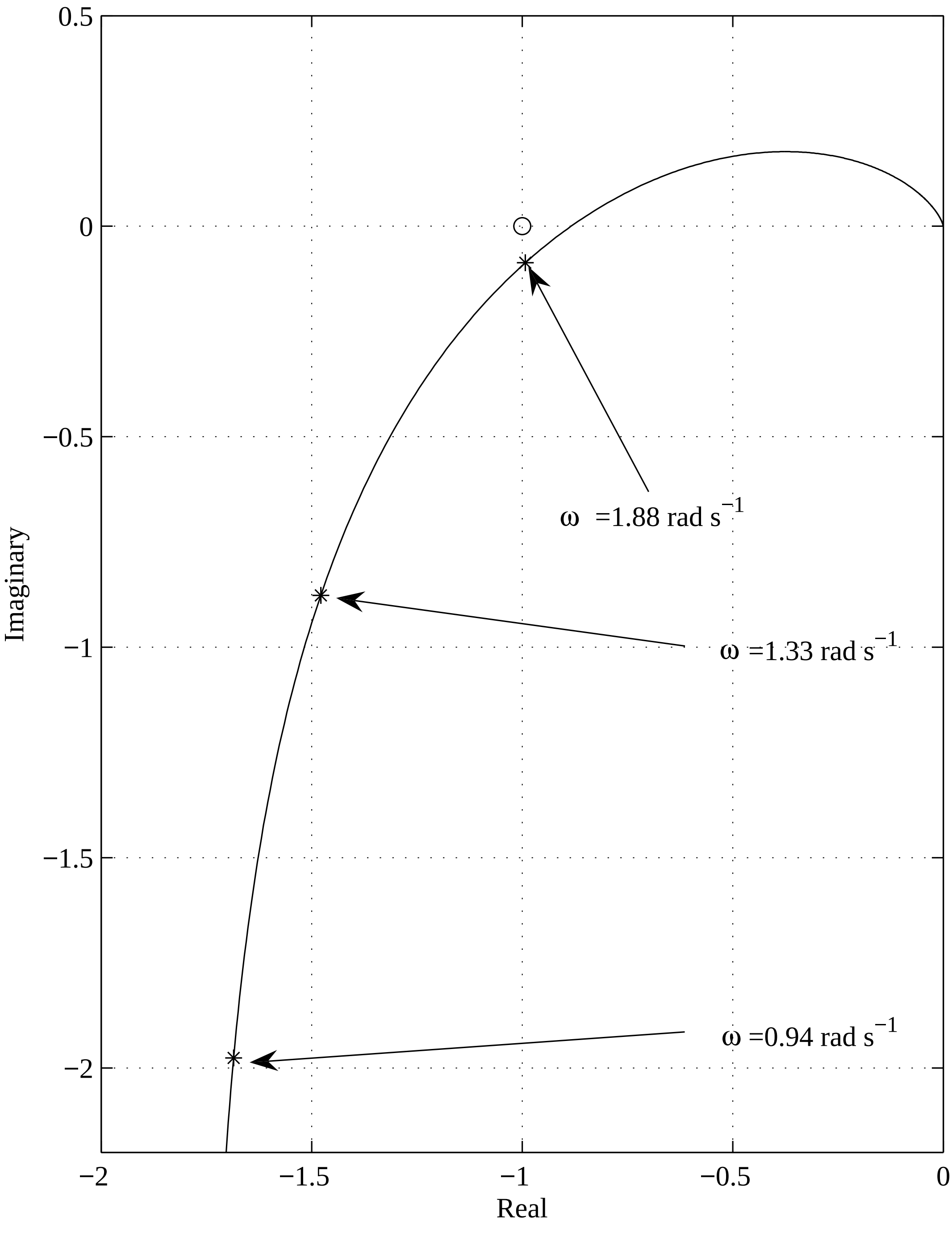
(ii) Show that the quantisation noise variance of a uniform quantiser is $\Delta^2/12$, where Δ is the quantisation step. [4]

(iii) Assume that we use a 10-bit uniform quantiser, that one channel use of the above binary erasure channel corresponds to $10\ \mu$s, and that a rate $r = 1/2$ code is used to correct the errors that occur in the channel. Is the corresponding rate achievable at $\text{SNR} = 10$ dB? [4]

6 (a) Describe the principal analogue and digital amplitude modulation techniques and outline the main differences between them. [6]

(b) Consider a signal with $B = 5$ kHz. Derive, sketch and compare the spectra obtained with double sideband (DSB) analogue modulation and with binary amplitude shift-keying (ASK), using the association $0 \rightarrow -A$ and $1 \rightarrow +A$. Assume sampling at the Nyquist rate, a 5-bit quantiser and a rectangular pulse. For the purposes of sketching, assume that the carrier frequency is much larger than B . [8]

(c) For the same set-up as in part (b), how many users can be accommodated in an FDMA cellular system with total bandwidth 20 MHz, with DSB and ASK modulations? For ASK modulated users, assume that frequencies beyond the second zero do not cause interference to other users. How would the result change if a triangular pulse were used instead of rectangular? [6]



Copy of Fig. 2. This should be annotated with your constructions and handed in with your answer to question 3.

END OF PAPER

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