ENGINEERING TRIPOS PART IB

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6 June 2008 2 to 4.30
2 to 3.30 Foreign Language Option

Paper 8

SELECTED TOPICS

*Answer **one** question from Section A. In addition:*

> *If you are not taking the Foreign Language option, answer **four** questions, taken from only **two** of Sections B – H. Not more than **two** questions from each section may be answered.*
> *If you are taking the Foreign Language option, answer **two** questions from **one** of Sections B – H.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Data Sheet for Section B (6 pages)
Data Sheet for Section E (2 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version 6

SECTION A *Business Economics*

*Answer not more than **one** question from this section.*

1 (a) Explain the concept of profit maximisation. [4]

(b) Explain the concept of contestable markets. [6]

(c) Compare and contrast the Keynesian consumption function model with the life cycle model of consumption. [10]

2 (a) Describe the model of perfect competition. [6]

(b) Why is perfect competition normally considered to be more desirable than monopoly? [5]

(c) What impact would the following have on the level of investment in the macroeconomy:

(i) an increase in the rate of interest; [3]

(ii) an expected increase in future economic growth; [3]

(iii) an increase in the real exchange rate? [3]

SECTION B *Civil and Structural Engineering*

Answer not more than **two** *questions from this section.*

You may refer to the data sheet as necessary at the end of the examination paper.

3 (a) Define stability ratio as a means of determining the safety of a tunnel being constructed in clay, in terms of the depth of the tunnel axis below ground level, the support pressure (if any) applied to the tunnel face and the unit weight and undrained shear strength of the clay. [5]

(b) Explain why it is safe to construct an open face tunnel in Cambridge in Gault clay at a depth of 20 m where the undrained shear strength is 200 $kN\ m^{-2}$ but not in the marine clay of Bangkok at the same depth where the undrained shear strength is 40 $kN\ m^{-2}$; assume that the unit weights of Gault clay and marine clay are 20 $kN\ m^{-3}$ and 17 $kN\ m^{-3}$ respectively. [5]

(c) Outline the essential features of tunnel construction by box jacking, illustrating your answer by reference to a recent project in Boston, USA and listing some advantages and disadvantages of the process. [5]

(d) A tunnel is to be constructed below a fragile masonry building and compensation grouting has been selected to protect the building from damage. Describe the principal features of the compensation grouting process and the instrumentation needed to ensure its success. [5]

4 A brick retaining wall is shown in Fig. 1. It is constructed by cutting a 2 m deep trench with vertical sides in the original ground, followed by building the wall to a height of 5 m above the original ground level. Dry sand backfill is placed against the completed wall, and a surcharge of 20 kN m^{-2} is applied to the surface of the backfill. The sand backfill has a unit weight γ = 17 kN m^{-3} and the critical state angle of friction φ' = 30 degrees. The original ground is a clay with a unit weight γ = 20 kN m^{-3}, undrained shear strength S_u = 50 kN m^{-2} and the critical state angle of friction φ' = 25 degrees. The water table is at the original ground level.

It should be assumed that the wall moves sufficiently to mobilise active pressures on the backfilled side and passive pressures on the other side, and that its vertical sides are smooth.

(a) Calculate the total horizontal stresses acting in the short term on each side of the retaining wall and sketch the stress distribution with depth. [8]

(b) Assuming the total horizontal friction force acting on the base of the wall is 150 kN (per metre length of wall), calculate the factor of safety against sliding in the short term. [4]

(c) Sometime after construction of the wall, pore pressures change in the clay on the passive side and cause it to soften such that long-term conditions are reached on that side. Calculate the total and effective horizontal stresses on the passive side of the wall. [5]

(d) Assuming that undrained conditions remain in the clay on the backfilled side, and that the friction force on the base of the wall remains unchanged at 150 kN (per metre length of wall), calculate the factor of safety against sliding due to the softening of the clay on the passive side. Comment on the effect of the clay softening on the stability of the wall. [3]

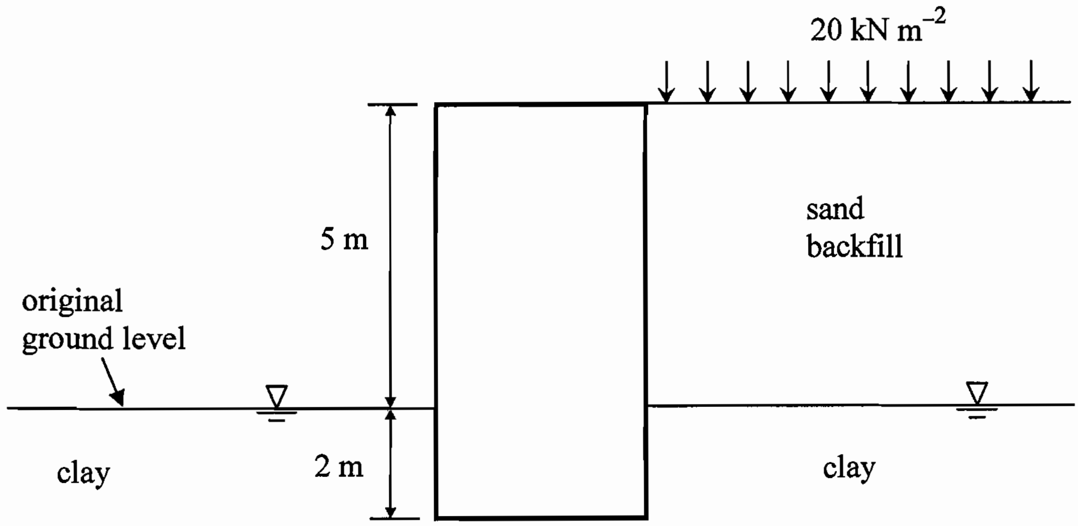


Fig. 1

5 A reinforced concrete wall AC supports an excavation that is 7 m deep, and projects a further 2 m into the ground to D, as shown in Fig. 2. Tie rods, at 2 m intervals along the length of the wall, at position B, are connected to remote anchor blocks. Each rod is tensioned to a fixed tension of 1000 kN.

The cube strength of the concrete is 30 MN m^{-2}, and the yield strength of the reinforcing bars is 460 MN m^{-2}. The ground pressures shown in Fig. 2 are those that must be resisted at the ultimate load condition.

(a) If the pressure applied by the ground is p at point C and q at point D, and if the pressure is assumed to vary linearly between those points, find p and q. [3]

(b) Calculate the shear force and bending moments at points A, B, C and D and the bending moment at any other point where the shear force is zero. Sketch the resulting shear force and bending moment diagrams. [4]

(c) If the wall is to be singly reinforced, calculate the minimum effective depth of the section for the most heavily loaded section. [6]

(d) If the actual effective depth of the wall is 40% bigger than the value calculated in part (c) above, and it is uniform throughout the wall, determine a suitable layout of reinforcement in the wall. [4]

(e) What special factors might need to be taken into account when designing the reinforcement for this wall. [3]

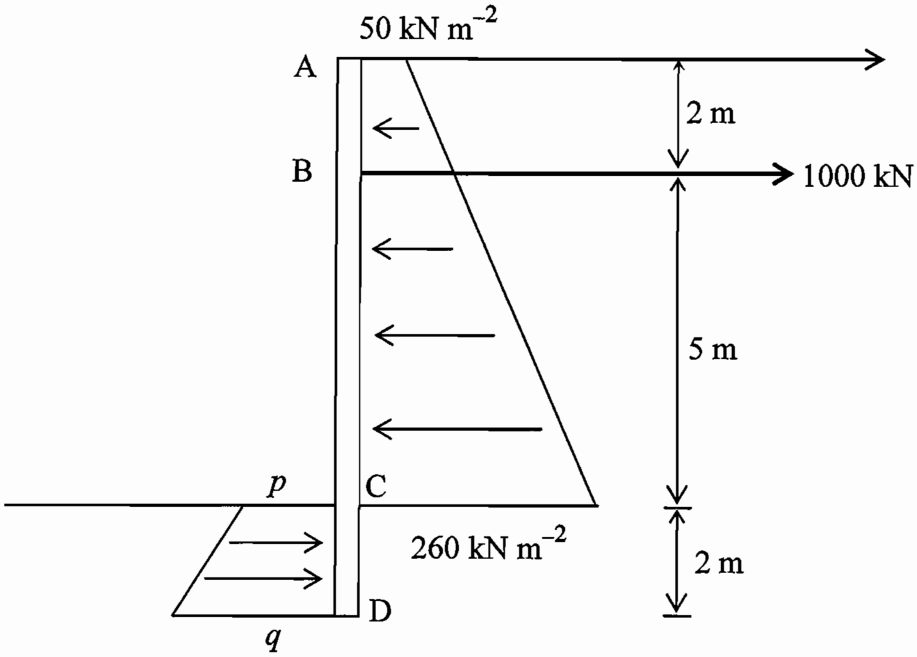


Fig. 2

SECTION C *Mechanics, Materials and Design*

*Answer not more than **two** questions from this section.*

6 (a) A horizontal-axis wind turbine of swept area A is operating in undisturbed air of density ρ moving at speed V with $a = 0.25$ where a is the axial induction factor as defined in Fig. 3.

(i) Use mass continuity for the control volume shown in Fig. 3 to find in terms of A the cross-sectional areas A_1 and A_2 of the control volume as shown in the figure. [2]

(ii) Use your results in (i) to find the power output of the turbine and hence deduce the efficiency of the turbine. Explain why the efficiency is less than the Betz Limit of 59%. [8]

(b) The wind turbine has three blades and the total moment of inertia of the rotor is J. The turbine is rotating steadily at angular velocity ω. Make suitable assumptions to estimate the maximum bending moment at the root of each blade:

(i) when bringing the rotor to rest over a period of τ seconds; [2]

(ii) when the turbine is yawing at a steady angular rate Ω. [8]

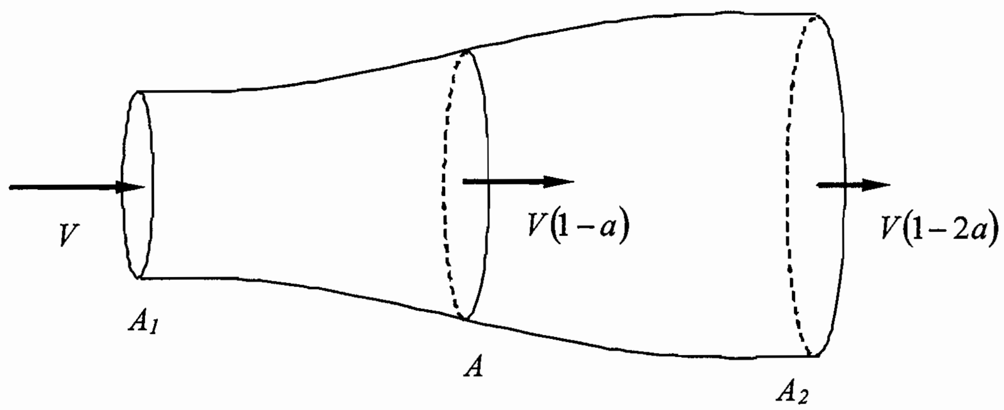


Fig. 3

7 Figure 4(a) shows the incident wind velocity and motion of the tip of a blade of a wind turbine of radius R ; a and a' are the axial and angular induction factors respectively. The turbine is rotating at an angular velocity ω in a free stream wind velocity V_0. Fig. 4(b) shows the direction of the apparent, or relative, wind velocity V_{rel} acting upon the same section of blade. The blade twist angle is θ and the angle of attack (in radians) is α. F_L and F_D are the aerodynamic lift and drag forces per unit length acting on the blade section. F_N and F_T are the resultant components of these forces acting in the normal and tangential directions respectively.

(a) Find an expression for the relative flow angle $\phi = \alpha + \theta$ in terms of the induction factors and the tip speed ratio $\lambda = \omega R / V_0$. [5]

(b) A typical wind turbine blade tip with twist $\theta = 0.65^\circ$ will operate under the following conditions: $\lambda = 8$, $a = 0.2$ and $a' = 0.01$. The lift and drag coefficients of the blade are approximately given by:

$$C_L \approx 2\pi\alpha \quad , C_D \approx 0.04 \quad \text{for } 0 < \alpha < 0.25 \text{ rad} \quad (0.25 \text{ rad} \approx 15^\circ)$$

(i) Find the angle of attack α of the blade tip. [5]

(ii) Determine the ratio F_N / F_T of the forces acting on the blade tip. [10]

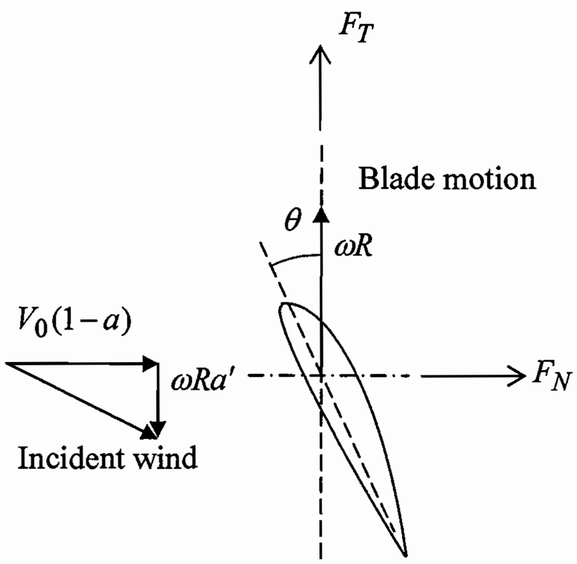


Fig. 4(a)

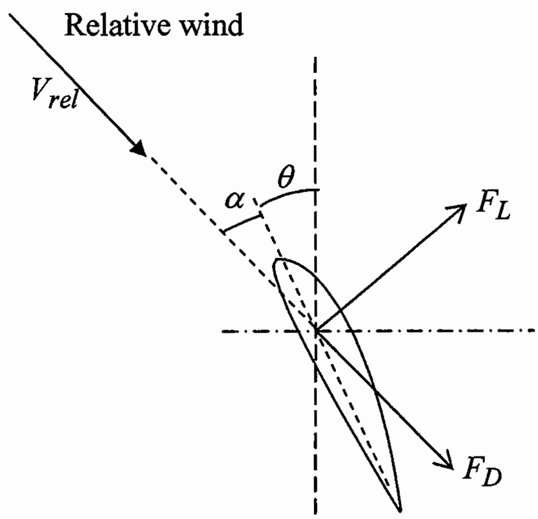


Fig. 4(b)

8 A wind turbine blade of length L is illustrated in Fig. 5. The spar depth d and cross-sectional area of the spar skins A vary linearly such that

$$\frac{d}{d_0} = \frac{A}{A_0} = \frac{x}{L}$$

where x is the distance from the blade tip and d_0 and A_0 are the values of d and A respectively at the root of the blade. The thickness of the spar skins may be assumed to be much less than the depth $2d$. The intensity of loading is assumed to vary linearly with x, leading to a bending moment M that is given by

$$M(x) = \frac{WL}{3}\left(\frac{x}{L}\right)^3$$

where W is the total wind load on the spar.

(a) (i) Show that the second moment of area of the blade section is given by

$$I(x) = A_0 d_0^2 \left(\frac{x}{L}\right)^3 .$$

Hence find an expression for the maximum stress in the outer surface of the skin. [3]

(ii) Show that the mass of the spar is given by $\rho A_0 L/2$ where ρ is the material density. Hence derive an expression for the mass of spar required to avoid failure, in terms of W, L, d_0 and relevant material properties. [3]

(b) The stiffness is limited by a maximum tip deflection δ, given by

$$\delta = \frac{W L^3}{6 E A_0 d_0^2} .$$

Derive an expression for the mass of a spar of specified stiffness, in terms of W, L, d_0, δ and relevant material properties. Hence find a criterion that identifies whether the design is stiffness or strength limited. [4]

(c) A stiffness-limited GFRP spar with material properties given in Table 1 is designed in this way with specified δ/L and d_0/L . An alternative design is to be considered, using laminated bamboo, with properties also given in Table 1 where symbols have their usual meaning.

(i) Assuming that W, L, $d(x)$ and δ are unchanged, show that this design will also be stiffness-limited, and evaluate the ratio of the masses of the bamboo and GFRP spars. [5]

(ii) Discuss briefly how the change from GFRP to bamboo might influence the estimation of the energy payback period for the turbine, with reference to each of the four stages in the life cycle. [5]

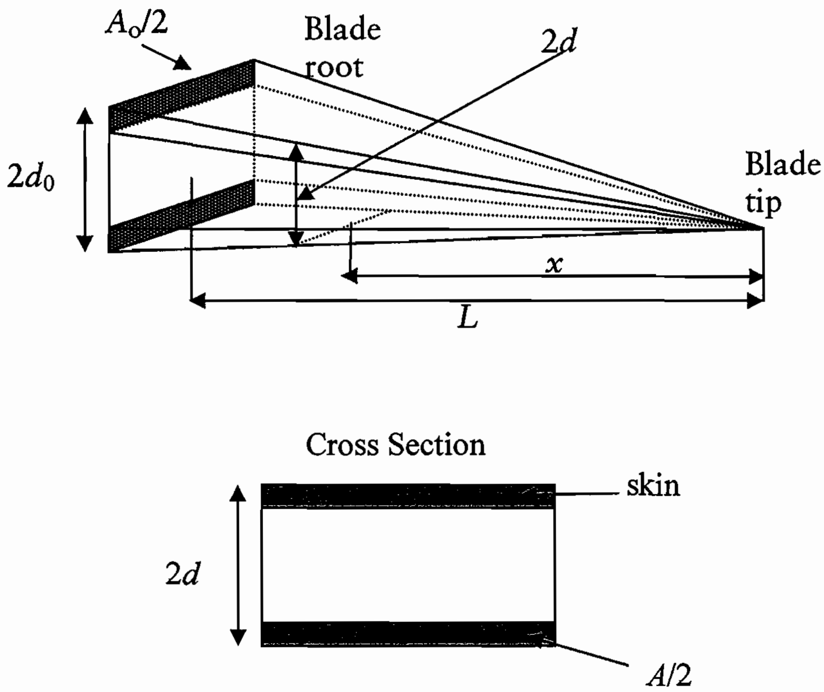


Fig. 5

	E (GPa)	σ_f(MPa)	ρ (Mg m^{-3})	Embodied energy (MJ/kg)
GFRP	28	56	1.8	110
Bamboo	18	36	0.7	1.5

Table 1

(TURN OVER

SECTION D *Aerothermal Engineering*

*Answer not more than **two** questions from this section.*

9 (a) Sketch the layout of a high bypass ratio 2-shaft turbofan engine and label the key components. Sketch a temperature-entropy diagram for the core stream and on this diagram indicate the changes in thermodynamic states that correspond to the components labelled in your sketch of the engine layout. Describe how the temperature-entropy diagram of the core changes as the bypass ratio of the engine is increased. [8]

(b) A two-shaft turbofan is fitted to an aircraft. At cruise, after passing through the fan, the flow entering the core has a stagnation temperature of 290 K and a stagnation pressure of 61 kPa. The core compressor has a stagnation pressure ratio of 25 and an isentropic efficiency of 87%. The mean blade speed of the compressor is $U_m = 355\,\mathrm{ms}^{-1}$ and there are 10 stages. Determine the stagnation temperature at exit from the core compressor and calculate the compressor stage work coefficient, $\Delta h_0 / U_m^2$. [5]

(c) The high-pressure turbine powers the core compressor and has an isentropic efficiency of 85%. The stagnation temperature at exit from the combustor is 1750 K. Calculate the stagnation temperature at entry to the low-pressure (LP) turbine and determine the stagnation pressure ratio across the high-pressure turbine. [4]

(d) The stagnation temperature drop across the LP turbine is 480 K. If at inlet to the engine the stagnation temperature is 250 K find the bypass ratio of the engine. Assume that at exit from the fan, the stagnation temperature in the bypass duct is 290 K (the same as for the core air-stream). [3]

You may neglect the mass flow rate of the fuel and assume that the combustion products behave as a perfect gas with the same properties as air. Stagnation pressure losses in the combustor can also be neglected. Assume that for air, $\gamma = 1.4$, $R = 287\ \mathrm{J\,kg^{-1}\,K^{-1}}$ and $c_p = 1005\ \mathrm{J\,kg^{-1}\,K^{-1}}$.

10 (a) Describe the difference between *gross thrust* F_G and *net thrust* F_N . Write an equation to show how these terms are related to one another. [4]

(b) A simple turbojet engine operates with a choked convergent exit nozzle with throat area A_N . The non-dimensional relationship relating the engine thrust to the engine mass flow can be written

$$\frac{F_G + p_a A_N}{p_{02} A_N} = f\left\{\frac{\dot{m}\sqrt{c_p T_{02}}}{p_{02} A_N}\right\}$$

where all the terms have their usual meanings. Explain carefully why the gross thrust, rather than the net thrust, is used in the above relationship. [4]

(c) A turbojet of the type described in part (b) is used to power an aircraft that cruises at a Mach number of 0.7 at an altitude where the ambient temperature is 217 K and the ambient pressure is 15 kPa. The mass flow rate of air through the engine is 20 kg s^{-1} and the net thrust generated is 5 kN.

(i) Find the stagnation temperature and pressure at inlet to the engine. [3]

(ii) Calculate the flight speed and the gross thrust. [2]

(d) A stationary test is run at sea-level with the engine operating at the same non-dimensional condition as at cruise. The ambient pressure during the test is 101 kPa and the ambient temperature is 288 K. If the nozzle area is 0.06 m^2 (for both the test and at cruise), find the mass flow rate through the engine, the gross thrust and the jet velocity during the sea-level test. [5]

(e) During the sea-level test, the engine rotational shaft speed is 10,000 rpm. Calculate the rotational shaft speed during cruise. [2]

Assume that for air, $\gamma = 1.4$, $R = 287$ J kg^{-1} K^{-1} and $c_p = 1005$ J kg^{-1} K^{-1}.

11 (a) Define thermal efficiency η_{th} and propulsive efficiency η_p for a jet engine. Explain why the specific fuel consumption *sfc* is a more commonly quoted parameter for a jet engine and show that it is given by

$$sfc = \frac{V}{LCV}\frac{1}{\eta_{th}\eta_p}$$

where V is the flight speed and LCV is the lower calorific value of the fuel. [5]

(b) Using the Froude equation for propulsive efficiency, $\eta_p = 2V/(V_j + V)$, show that the result of part (a) can be re-written as

$$sfc = \frac{\sqrt{\gamma R T_a}}{2\eta_{th} LCV}\left(M + M_j\sqrt{\frac{T_j}{T_a}}\right)$$

where M is the flight Mach number, V_j is the jet velocity, M_j is the jet Mach number, T_a is the ambient temperature and T_j the static temperature in the jet. [4]

(c) A civil aircraft has a total mass of 200 tonnes. It cruises at an altitude of 12 km at a flight Mach number of 0.85. At this condition, the lift coefficient is at its optimum value of 0.5 and the engine *sfc* is 0.018×10^{-3} kg s^{-1} N^{-1}. Using the atmospheric properties data in Table 2:

(i) calculate the total wing area; [3]

(ii) When the aircraft is cruising at an altitude of 9.5 km the lift coefficient is maintained at 0.5. Find the new flight Mach number and using the result derived in part (b) determine the new *sfc*;

Take $T_j / T_a = 1.15$ and $M_j = 1.3$ for both cruise conditions. Assume that the thermal efficiency η_{th} can be treated as a constant; [5]

(iii) Explain why cruising at the higher altitude is preferable. [3]

Assume that for air, $\gamma = 1.4$, $R = 287$ J kg^{-1} K^{-1} and $c_p = 1005$ J kg^{-1} K^{-1}.

Altitude (km)	Temperature (K)	Pressure (kPa)	Density (kg m^{-3})
9.5	226.4	28.5	0.439
12	216.7	19.3	0.311

Table 2

SECTION E *Electrical Engineering*

Answer not more than ***two*** *questions from this section.*

You may refer to the data sheet as necessary at the end of the examination paper.

12 (a) The manufacture of a metal oxide semiconductor transistor (MOST) requires several oxidation steps. Describe the "wet" and "dry" methods of growing high quality silicon dioxide (SiO_2) indicating the material properties of most importance when such a layer is to be used as the gate insulator in high frequency MOSTs. Discuss the problems associated with continuing to use this material for such an application as device dimensions continue to shrink. [8]

(b) Suggest two alternative materials which could replace SiO_2 as the gate insulator in MOSTs of the future. [2]

(c) A layer of SiO_2 is to be grown by thermal oxidation on a 200 μm thick silicon wafer. If a SiO_2 layer of 40 nm is to be grown, estimate the thickness of the silicon left after the oxide is grown. Estimate also the total thickness of the oxide plus silicon. Assume that the molecular weight of Si is 28.9 g mol^{-1} and the density of Si is 2.33 g cm^{-3}, and that the corresponding values for SiO_2 are 60.08 g mol^{-1} and 2.21 g cm^{-3}. [10]

13 (a) (i) Describe what is meant by 'scaling' in CMOS integrated circuits. [1]

(ii) In simple scaling, if a feature length scales by a factor k, how much do: device density, circuit speed, power per chip scale? Make clear any assumptions needed for your answer. [3]

(iii) What is the main factor which is expected to limit scaling? [2]

(iv) What other devices follow a scaling law? [2]

(b) Derive the transit time for an InAs FET with source-drain distance of $0.05\,\mu m$, if the mobility is $3.3\ m^2V^{-1}s^{-1}$ and V_{SD} is 0.5 V. Comment on the result. [6]

(c) Optimise the dopant density so that the transistor will operate with a gate voltage swing of 0.1 V. Assume a dielectric constant for InAs of $1.5\times10^{-10}\,F\,m^{-1}$ and that the channel depth is $0.02\,\mu m$. [6]

14 (a) Explain the difference between a metal, a semiconductor and an insulator in terms of the band structure. [2]

(b) Explain, with the aid of diagrams, how conduction occurs in a doped semiconductor. What controls the number of carriers in a doped semiconductor? [3]

(c) Describe the random walk model of electron scattering that leads to a microscopic expression for the electronic conductivity in terms of the electron effective mass m^* and the time between collisions τ . [7]

(d) The mobility of a semiconductor like silicon is $0.25\ \mathrm{m^2V^{-1}s^{-1}}$ and its scattering limited velocity is $1.25 \times 10^5\ \mathrm{ms^{-1}}$ and its effective mass m^* is 0.2. This material, doped at a donor density of $4 \times 10^{21}\ \mathrm{m^{-3}}$, is used to make a depletion mode FET, operating at a supply voltage of 1 V.

(i) At what source-drain length does the conduction become scattering limited and what is the corresponding transit time? [4]

(ii) Calculate the scattering time between collisions, compare this to the transit time, and comment on the result. [4]

The electronic charge is 1.6×10^{-19} C. Mass of electron = 9.10×10^{-31} kg.

Prepared by Dr Gopal Madabhushi
Modified by Prof. David Cardwell

Sec. H

OK

SECTION F *Information Engineering*

*Answer not more than **two** questions from this section.*

15 (a) Describe the processes that would be required in a photo-editing package in order to approximately correct for the following problems in a digitised colour photograph:

(i) the horizon (which should be level) tilts 3 degrees in a clockwise direction; [4]

(ii) the faces of the human subjects were under-exposed due to a bright sky in the background; [4]

(iii) some noticeable camera noise exists in the lower-lit areas of the image. [4]

You should assume that some limited enlargement of the image is required in order to avoid any visible edge artifacts from the tilt correction. Where possible give mathematical expressions to support your descriptions.

(b) For each of the processes described in (a), discuss briefly any image degradations or trade-offs that need to be considered when deciding the most appropriate parameter values of that process. [8]

16 A commonly used 1D smoothing filter is the Gaussian.

(a) Show that repeated convolution with a series of 1D Gaussians, each with the same standard deviation, is equivalent to a single convolution with a Gaussian of larger variance and derive an expression for this variance. [8]

(b) The first stage in localising edges and blobs at different scales in images is to low-pass filter an image with smoothing filters of different scales. Show how these computations can be performed efficiently. [8]

(c) Explain how blobs are localized in the image and how the appropriate scale is chosen. [4]

17 Consider a model of 10×10 pixel black and white images where the probability of a given pixel being black is $\theta = 0.6$ and all pixels are independent.

(a) What is the most probable image under this model and what is its probability? [4]

(b) Assume that θ is unknown, and that we wish to learn it from data using Bayesian methods. Assume that θ has a uniform prior on the interval [0,1]. What is the posterior distribution of θ after observing only two black pixels and no white pixels? What is the posterior probability that $\theta > 0.5$? [8]

(c) Explain why a model of black and white images where each pixel is independent is a very poor model of real world images, and suggest a better model. [8]

SECTION G *Engineering for the Life Sciences*

*Answer not more than **two** questions from this section.*

18 (a) In the context of reinforcement learning write brief notes on the following:

(i) primary reinforcers;
(ii) secondary reinforcers;
(iii) temporal difference error. [4]

(b) How is the position of the eye within the orbit estimated? Provide experimental evidence for your answer. [4]

(c) Answer the following in the context of inverse model learning.

(i) Describe the direct inverse model learning approach.
(ii) What are the problems of using direct inverse model learning? [4]

(d) Answer the following in the context of human arm stiffness.

(i) What is the stiffness ellipse of the human arm and how can it be estimated experimentally?

(ii) How and why might the stiffness change when using a screwdriver? [4]

(e) Answer the following in the context of Bayes rule.

(i) State Bayes rule as it applies to estimating state based on sensory input.

(ii) Explain why the image in Fig. 6 is perceived as a cube. [4]

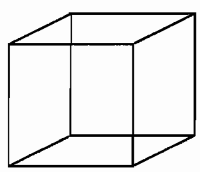


Fig. 6

19 Write brief notes on the following:

(a) The role of successive reversals of the direction of rotation in the flagellar motors for the swimming and navigation of bacteria such as *E. coli.* [4]

(b) The way in which the family of proteins can include not only materials having a very wide range of mechanical properties, but also switches and other machinery components. [4]

(c) The advantages of a general scheme of construction by which the complete prescription for making all of the components of a multi-cellular organism is contained within every individual cell. [4]

(d) The tracing of evolutionary relationships between different plants and animals by means of information about their DNA sequences. [4]

(e) How a single-letter mutation in a DNA sequence can result in a major change in the helical shape of a bacterial flagellar filament. [4]

20 A 35-year old patient presents with a substantial osteochondral defect in the lateral compartment on the tibial plateau of the knee joint following an automobile accident. The physician comes to you, a biomaterials engineer, to discuss the options for repair. You have two options: Implant 1, a metal hemiarthroplasty prosthesis; Implant 2, a tissue-engineered bi-layer construct based on resorbable scaffold technology. Implant 1 is commercially available but Implant 2 is offered as part of a clinical trial.

(a) Describe the composition, structure and basic features of the mechanical response for the tissues that were destroyed in formation of the osteochondral defect. [6]

(b) Describe the composition, structure and functional design of the two implants. [6]

(c) Describe clearly what factors should you take into consideration when making a recommendation in favour of one of these approaches. Is there a clear choice for the 'better' repair option in this case? [8]

(TURN OVER

SECTION H *Manufacturing, Management and Design*

*Answer not more than **two** questions from this section.*

21 A manufacturer of plastic bags is considering two options for developing a new bag made entirely from recycled material: [A] the bag will be made from scrap generated in the existing process; [B] the bag will be made from used bags collected at supermarkets.

Material costs for the present bag are £2 per 1,000 bags, annual output is 100 million bags, direct labour costs £100,000 per year, and management and marketing costs are £100,000 per year. The company rents all required buildings and equipment for an annual charge of £60,000, and has no capital equipment, stocks or other assets.

The price of existing bags is £5 per 1,000. The company estimates that under option A, the price would be unchanged, but 30% of existing output could be replaced by recycled bags saving 25% of existing raw material purchases but incurring a 20% increase in direct labour cost. Under option B, the company estimates that customers would pay up to £10 per 1,000 bags, and the new bags would be sold as a new line, with 20 million bags from recycled material in addition to existing sales. However, collection, cleaning and sorting bags from supermarkets would give a material cost of £7.50 per 1,000 bags and the new line would incur additional direct labour cost of £50,000.

(a) Create profit and loss accounts for the present system and the two options. [12]

(b) The company is uncertain about the price customers will pay for the recycled bags under the two options. If the actual price is 10% lower than expected, calculate the percentage change in profit for the two options. [4]

(c) Discuss what the company could do to reduce its sensitivity to the price uncertainty described in part (b). [4]

22 (a) Describe the elements of the 'Design Mix' and explain how this concept could be used to think about how a cheap writing instrument could be differentiated from its competition. [5]

(b) With the aid of illustrations and examples where appropriate, explain the Kano model. How might the Kano model help in the design of a novel cheap writing instrument? [8]

(c) Explain how you might use different types of prototype to help in the design of such a novel cheap writing instrument. [5]

(d) Comment on the limitations of structured design tools, such as the Kano model in the design process. [2]

23 (a) List the four main types of funding available for a firm. Give an example source for each type. [4]

(b) Give examples of business models based upon:

(i) selling a product;

(ii) selling a service;

(iii) selling a service enabled by a product;

(iv) selling a product with consumables.

What are the strengths and weaknesses of each of these business models? [8]

(c) Discuss the advantages and disadvantages of business models for a product-based start-up firm where the business operations are managed either:

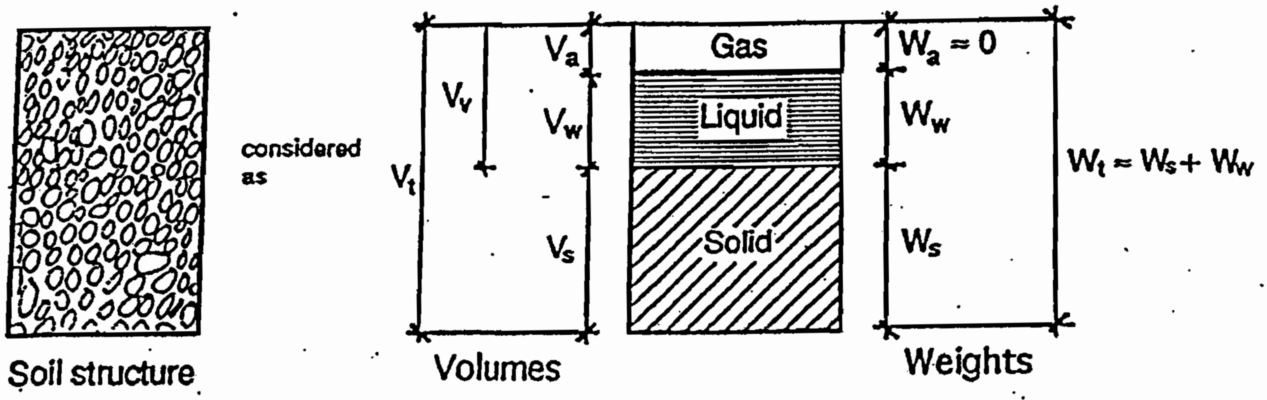
(i) entirely in-house;

(ii) in partnership with other firms. [8]

END OF PAPER

Data sheet – Soil Mechanics

General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w G_s / e)$
Unit weight of water	$\gamma_w = 9.81\ \text{kN/m}^3$ (although we assume $10\ \text{kN/m}^3$)
Unit weight of soil	$\gamma = W_t / V_t = \left(\frac{G_s + S_r e}{1 + e}\right)\gamma_w$
Buoyant (effective or submerged) unit weight	$\gamma' = \gamma - \gamma_w = \left(\frac{G_s - 1}{1 + e}\right)\gamma_w$ (soil saturated)
Unit weight of dry soil	$\gamma_d = W_s / V_t = \left(\frac{G_s}{1 + e}\right)\gamma_w$
Relative density	$I_d = \frac{(e_{max} - e)}{(e_{max} - e_{min})}$

where e_{max} is the maximum voids ratio achievable in the quick tilt test (for sands), and
e_{min} is the minimum voids ratio achievable by vibratory compaction (for sands).

Classification of particle sizes

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D — equivalent diameter of soil particle

D_{10}, D_{60} etc — particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

Stress components

Principle of effective stress (saturated soil):

total stress σ = effective stress $\sigma' +$ pore water pressure u

τ = $\tau' +$ 0

and

σ_v = vertical stress

σ_h = horizontal stresss

τ = shear stress

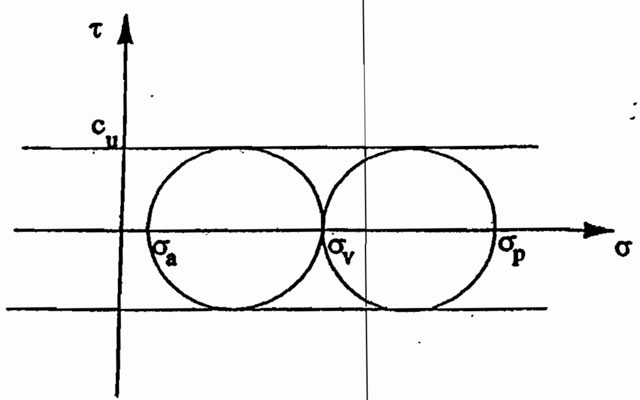
Strength of clays (undrained behaviour only)

Under constant volume (undrained) conditions only, the strength of clays can be characterised by the *undrained shear strength* c_u which is mobilized when the shear stress $\tau = c_u$. This conforms to Tresca's criterion, and the active and passive total horizontal stresses, σ_a and σ_p respectively, are given by

$$\sigma_a = \sigma_v - 2\,c_u$$

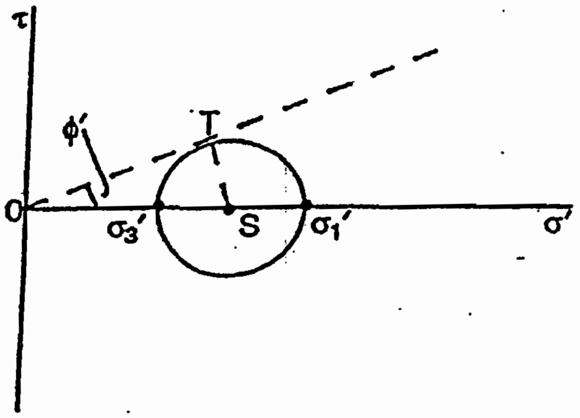
$$\sigma_p = \sigma_v + 2\,c_u$$

where σ_v is the total vertical stress.



Strength of sands

Mobilised angle of shearing ϕ'
where $\tau = \sigma' \tan \phi'$



$$\sin\phi' = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

$$\therefore \phi' = \sin^{-1}\left[\frac{\left(\sigma_1'/\sigma_3'\right) - 1}{\left(\sigma_1'/\sigma_3'\right) + 1}\right]$$

Earth pressure coefficient K: $\sigma_h' = K\sigma_v'$

Active pressure: $\sigma_v' > \sigma_h'$ $\therefore$ $\sigma_1' = \sigma_v'$

$\sigma_3' = \sigma_h'$

$K_a = (1 - \sin\phi')/(1 + \sin\phi')$

Passive pressure: $\sigma_h' > \sigma_v'$ $\therefore$ $\sigma_1' = \sigma_h'$

[We assume principal stresses are horizontal and vertical]

$\sigma_3' = \sigma_v'$

$K_p = (1 + \sin\phi')/(1 - \sin\phi') = \frac{1}{K_a}$

Angle of shearing resistance:

at peak strength ϕ'_{max} at $\left(\sigma_1'/\sigma_3'\right)_{max}$

at critical state ϕ'_{crit} after large strains.

Sand strength data: friction hypothesis

In any shear test on soil, failure occurs when $\phi' = \phi'_{max}$ and

$$\phi'_{max} = \phi'_{crit} + \phi'_{dilatancy}$$

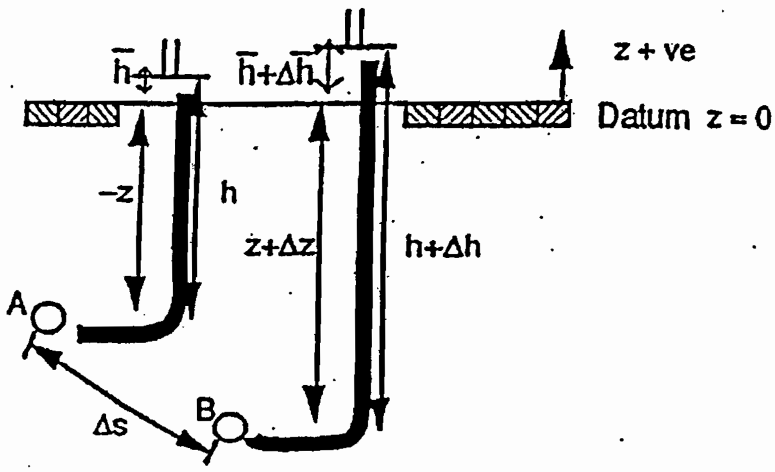
where ϕ'_{crit} is the ultimate angle of shearing resistance of a random aggregate which deforms at constant volume, so the dilatancy, which indicates an increase in volume during shearing, approaches zero $(\phi'_{dilatancy} -> 0)$ as $\phi'_{max} -> \phi'_{crit}$.

ϕ'_{crit} is a function of the mineralogy, size, shape and distribution of particles. For a particular soil it is almost independent of initial conditions. Typical values (±2°):

	ϕ'_{crit}	ϕ'_{max}	
feldspar	40°		
quartz	33°	53°	($I_d = 1$, and mean effective stress OS < 150 kPa)
mica	25°		

Seepage

Excess pore water pressure



Head $h = u/\gamma_w$

Potential $\bar{h} = h + z$

Total pore water pressure head at A: $u = \gamma_w h = \gamma_w(\bar{h} - (-z))$

B: $u + \Delta u = \gamma_w(h + \Delta h) = \gamma_w(\bar{h} + z + \Delta\bar{h} + \Delta z)$

[Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w(\bar{h} + \Delta\bar{h})$]

Hydraulic gradient A–B $i = -\dfrac{\Delta\bar{h}}{\Delta s} = -\dfrac{\Delta\bar{u}}{\gamma_w \Delta s}$

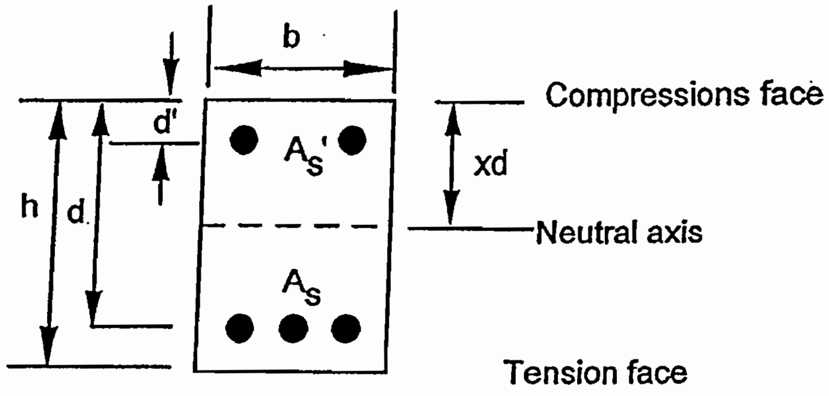
Darcy's law $v = ki$
v = average or superficial seepage velocity
k = coefficient of permeability

Typical permeabilities

$D_{10} > 10$mm	:	non-laminar flow
10 mm > D_{10} > 1μm	:	$k \cong 0.01(D_{10}\text{in mm})^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Design of reinforced concrete

Data sheet for use in Part IB Civil Engineering Elective Course.



Design Stresses

Cube strength for concrete f_{cu}. At failure in bending, stress in concrete $= 0.4f_{cu}$ over whole area of concrete in compression.

Tensile yield stress of steel f_y. At failure in bending, stress in bars in tension $= 0.87f_y$, stress in bars in compression $= 0.75f_y$.

Design Equations

Moment capacity of singly reinforced beam

$$M \leq 0.15 f_{cu} bd^2$$
$$M = 0.87 f_y A_s d(1 - x/2)$$
$$x = 2.175(f_y/f_{cu}).(A_s/bd) \quad (\leq 0.5 \text{ to avoid over reinforcement})$$

Moment capacity of doubly reinforced beam

$$M = 0.15 f_{cu} bd^2 + 0.75 f_y A'_s (d - d')$$
$$0.87 f_y A_s = 0.75 f_y A'_s + 0.2 f_{cu} bd$$

Shear capacity of all beams

Total shear capacity $V = (v_c + v_s)bd$

Where, $v_c = 0.68(100A_s/bd)^{0.33}.(400/d)^{0.25}$ (N/mm^2)

and $v_s = 0.87 f_y A_{sq}/(bs)$

in which s = shear link spacing, A_{sq} is total area of all shear bars in a link and A_s is the total area of effective longitudinal *tension* steel at the section.

Standard bar sizes

Diameter (mm)	6	8	10	12	16	20	25	32	40	50
Area (mm^2)	28	50	78	113	201	314	491	804	1256	1963

Available steel types

Deformed high yield steel $f_y = 460$ N/mm^2

Plain mild steel $f_y = 250$ N/mm^2

Lap and anchorage lengths 40 bar diameters

Density of reinforced concrete: $24\ kN/m^3$

Reinforcement areas per metre width

	Spacing of bars (mm)									
	75	100	125	150	175	200	225	250	275	300
Bar Dia. (mm)										
6	377	283	226	189	162	142	126	113	103	94.3
8	671	503	402	335	287	252	224	201	183	168
10	1050	785	628	523	449	393	349	314	285	262
12	1510	1130	905	754	646	566	503	452	411	377
16	2680	2010	1610	1340	1150	1010	894	804	731	670
20	4190	3140	2510	2090	1800	1570	1400	1260	1140	1050
25	6550	4910	3930	3270	2810	2450	2180	1960	1790	1640
32	10700	8040	6430	5360	4600	4020	3570	3220	2920	2680
40	16800	12600	10100	8380	7180	6280	5580	5030	4570	4190
50	26200	19600	15700	13100	11200	9820	8730	7850	7140	6540

Areas calculated to 3 significant figures according to B.S.I recommendations

Part IB Data Sheet: Electrical Engineering Elective Transistor Design Summary Sheet

Gauss's Theorem

$\varepsilon_0\varepsilon_r E_1 - \varepsilon_0\varepsilon_r E_2$ = charge per unit area enclosed between upper surface 1 and lower surface 2.

FET Design Summary

- $\tau_t \rightarrow$ switching time as 1st approx. (scattering limited transit time).
- $\tau_{eff} = \tau_t + R_{load}C_{eff(output)} \rightarrow$ switching time as 2nd approx.
- $L = v_s \tau_t$ (source-drain spacing).
- $I_{sat} = eNv_s W d_s = eNWLd_s/\tau_t$
- Aspect ratio W/L (technology?).
- $(1/2)eN(d_s)^2/\varepsilon_0\varepsilon_r$ = (Max Gate Voltage)
- $E'_{peak} = eNd_s/\varepsilon_0\varepsilon_r < E_{breakdown}$
- Minimum Drain Source Voltage $\sim E_s L$ (E_s is the field required to reach limiting velocities).

Mutual Conductance

$g_{mo} \sim I_{sat}/V_{gate(max)}$

Mutual conductance reduces with frequency as $g_m(\omega) \approx g_{mo}/(1 + j\omega\tau_t)$;

$$v_{out} = g_m(\omega)\,R(1 + j\omega RC_{eff(out)}) \approx g_{mo}R/[1 + j\omega(\tau_t + RC_{eff(out)})]$$
$$= g_{mo}R/[1 + j\omega\tau_{eff}]$$

Capacitances for FET

Parallel Plate Capacitance: $\varepsilon_0\varepsilon_r$ Area/spacing

Used for rough estimates of parasitic capacitance.

Effective Capacitances for FET

$C_{eff(out)} \rightarrow C_{gate/drain} + C_{drain/source} + C_{load}$;

$C_{eff(in)} \rightarrow MC_{gate/drain} + C_{gate/source\,(proximity)} + C_{gate/source\,(electronic)}$;

$C_{electronic} = g_{mo}\tau_t$; $M = (1 + g_{mo}R_{load})$.

Time Constants for FET

$\mu = e\tau/m^*$ relates mean free time τ and mobility.

Transit time τ_t over distance L and scattering limited velocity v_s are related by $\tau_t = L/v_s$.

$v_{out} \approx g_{mo}R/[1 + j\omega\tau_{eff}] = g_{mo}R/[1 + j\omega/(2\pi f_t)]$

$1/(2\pi f_\tau) = \tau_t + RC_{eff(out)} = \tau_{eff}$ The transition frequency is f_t.

10% to 90% rise time is $T = 2.2\,\tau_{eff} = (2.2/2\pi)(1/f_t) = 0.35/f_t$.

Electrical Engineering Elective:
Tunnel Barrier Design Summary Sheet

Schrodinger's Equation

Complex Wave $\Psi = A\ \exp(-j2\pi f\ t + j2\pi x/\lambda) = A\exp(-j\omega t)\exp(jkx)$;

<momentum> $\Psi = p\Psi = (h/\lambda)\Psi = -j(h/2\pi)\partial\Psi/\partial x$;

<Total energy> $\Psi = E\Psi = hf\Psi = j(h/2\pi)\partial\Psi/\partial t$

$(h/2\pi) \rightarrow \hbar$; $h = 6.625 \times 10^{-34}$ J/s.

Schrodinger's equation:-

$$E\Psi = (1/2m)[-j\hbar\partial/\partial x]^2\Psi + e\phi\Psi$$

Tunneling (Rectangular barrier $\varepsilon\phi$)

Propogating waves outside barrier with incident kinetic energy $U_{incident} = (\hbar k_i)^2/2m^*$

Evanescent waves inside barrier: $-(\hbar k)^2/2m^* = (\hbar k_i)^2/2m^* - e\phi$

Technology Design Summary

Diffusion

Constant Surface Concentration:

$$C(x,t) = C_s\ \text{erfc}\left[\frac{x}{2(Dt)^{1/2}}\right]$$

Constant Total Dopant:

$$C(x,t) = \frac{S}{(\pi Dt)^{1/2}}\ \exp\left[\frac{-x^2}{4Dt}\right]$$

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13.3.06