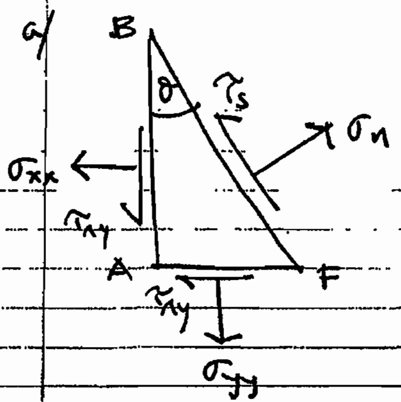
2009 IB PAPER 2 STRUCTURES
SECTION A DR C J BURGOYNE
SECTION B DR M OVEREND

Paper 2 IB/2009/1.1/1

IB 2009

1. a/



- ASSUMING $BF = 1$
  $\therefore$ $AB = \cos\theta$
  AND $AF = \sin\theta$
- ALSO ASSUME UNIT THICKNESS.
- " " COMPLIMENTARY SHEAR STRESSES $\therefore \tau_{xy} = \tau_{yx}$

RESOLVING FORCES NORMAL TO BF:

$$\sigma_n \cdot 1 = \sigma_{xx}\cos^2\theta + \tau_{xy}\sin\theta\cos\theta + \sigma_{yy}\sin^2\theta + \tau_{xy}\cos\theta\sin\theta$$

$$\therefore \sigma_n = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta \quad -(1$$

RESOLVING FORCES PARALLEL TO AB:

$$\tau_s \cdot 1 = -\sigma_{xx}\cos\theta\sin\theta - \tau_{xy}\sin^2\theta + \sigma_{yy}\sin\theta\cos\theta + \tau_{xy}\cos^2\theta$$

$$\therefore \tau_s = (\sigma_{yy} - \sigma_{xx})\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) \quad -(2$$

BUT $\sigma_{xx}\cos^2\theta = \frac{\sigma_{xx}}{2} + \frac{\sigma_{xx}}{2}\cos 2\theta$; $\sigma_{yy}\sin^2\theta = \frac{\sigma_{yy}}{2} - \frac{\sigma_{yy}}{2}\cos 2\theta$

AND $2\tau_{xy}\sin\theta\cos\theta = \tau_{xy}\sin 2\theta$

$\therefore$ EQ. (1) MAY BE RE-WRITTEN AS:

$$\sigma_n = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) + \frac{1}{2}(\sigma_{xx} - \sigma_{yy})\cos 2\theta + \tau_{xy}\sin 2\theta \quad -(3$$

AND (2)

$$\tau_s = \frac{1}{2}(-\sigma_{xx} + \sigma_{yy})\sin 2\theta + \tau_{xy}\cos 2\theta \quad -(4$$

SQUARING BOTH SIDES AND ADDING EQUATIONS (3) AND (4

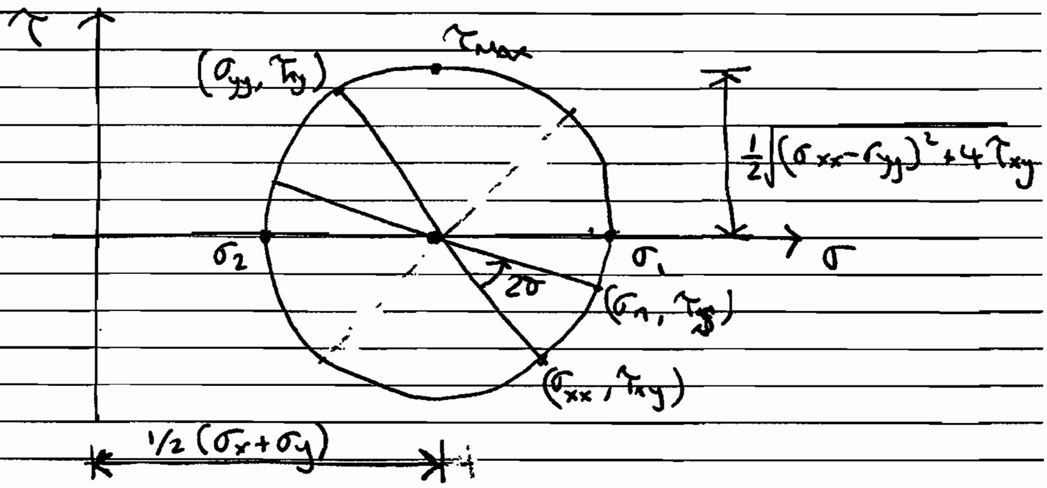
$$\left[\sigma_n - \frac{1}{2}(\sigma_{xx} + \sigma_{yy})\right]^2 + \tau_s^2 = \frac{1}{4}(\sigma_{xx} - \sigma_{yy})^2 + \tau_{xy}^2$$

THIS IS THE EQUATION OF A CIRCLE OF RADIUS:

$$r = \sqrt{\left[\frac{1}{4}(\sigma_{xx} - \sigma_{yy})^2 + \tau_{xy}^2\right]} = \frac{1}{2}\sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}^2}$$

WHOSE CENTRE CO-ORDINATES ARE:

$$\left[\frac{1}{2}(\sigma_x + \sigma_y), 0\right]$$



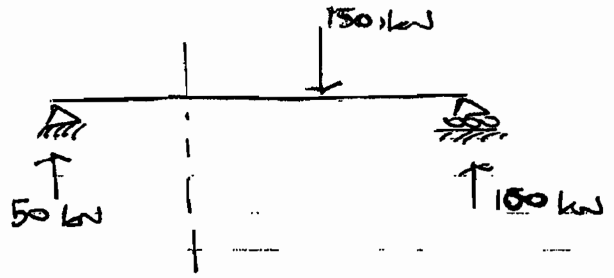
FROM MOHR CIRCLE:

$$\tau_{Max} = \frac{1}{2}\sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}} \quad -(5)$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}} \quad -(6)$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2}\sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}} \quad -(7)$$

2b/



SF = 50 kN
BM = 100 kNm

AT POINT P :

$$\tau_{xy} = \frac{SA\bar{y}}{Ib}$$

$$I = \frac{150 \times 240^3}{12} - \frac{140 \times 200^3}{12} = 79.47 \times 10^6 \text{ mm}^4$$

$$A\bar{y} = (150 \times 20 \times 110) + (50 \times 10 \times 75)$$

$$= 367.5 \times 10^3 \text{ mm}^3$$

$$\therefore \tau_{xy} = \frac{50 \times 10^3 \times 367.5 \times 10^3}{79.47 \times 10^6 \times 10} = 23.12 \text{ N/mm}^2$$

$$\sigma_{xx} = My/I$$

$$= \frac{100 \times 10^6 \times 50}{79.47 \times 10^6}$$

$$= 62.9 \text{ N/mm}^2$$

$$\sigma_{yy} = 0$$

$\therefore$ PRINCIPAL STRESSES

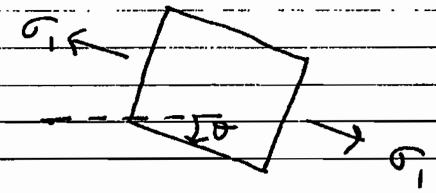
$$\sigma_1 = \frac{62.92}{2} + \frac{1}{2}\sqrt{62.92^2 + (4 \times 23.12^2)}$$

$$= 31.46 + 39.04$$

$$= 70.5 \text{ N/mm}^2$$

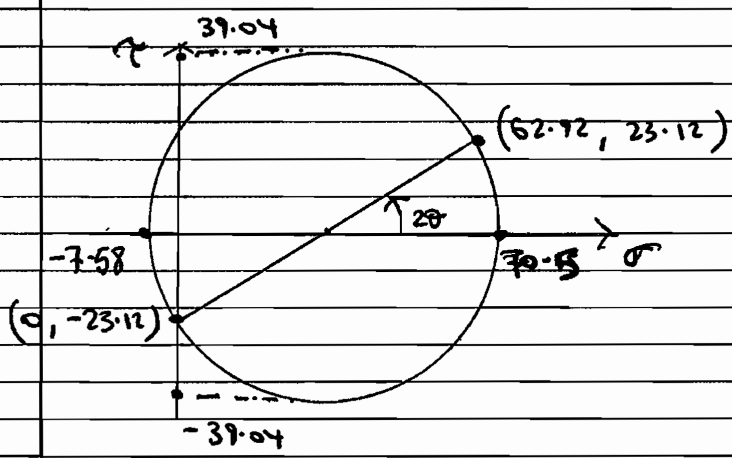
$$\sigma_2 = 31.46 - 39.04$$
$$= -7.58 \text{ N/mm}^2$$

$$\tau_{max} = 39.04 \text{ N/mm}^2$$



$$\theta = \frac{1}{2}\sin^{-1} 23.12/39.04$$
$$= 36.3°$$

Mohr circle:

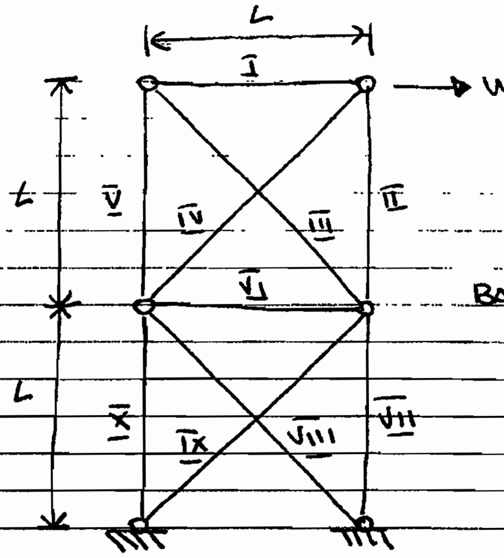


$$2\theta = 72.6°$$

Question 1.
Subject: Elasticity – Mohr's circle for two-dimensional stresses.
This was a popular question. Many candidates where successful in resolving the forces on the wedge element thereby deriving the stresses parallel and perpendicular to 'EF' required in part (a). Several candidates were however unable to rearrange and combine these equations to show that they describe the equations of Mohr's circle. Most candidates were capable of applying Mohr's circle in part (b), but several candidates committed trivial errors such an incorrectly determining the bending moment at the required location and errors in applying the transverse shear equation.

1B/2009/2/1

2



BAR TENSIONS $\underset{\sim}{t} = \begin{bmatrix} t_{I} \\ t_{II} \\ t_{III} \\ t_{IV} \\ t_{V} \\ t_{VI} \\ t_{VII} \\ t_{VIII} \\ t_{IX} \\ t_{X} \end{bmatrix}$

(a) MAXWELL'S EQUATION $s - m = b + r - dj$

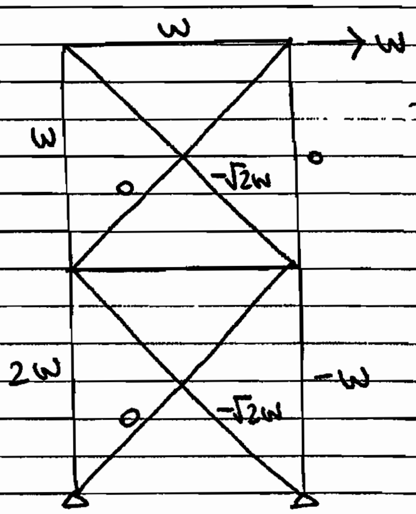
$$s = 10 + 2\times 2 - 2\times 6$$

$$s = 2 \quad \text{STATES OF SELF STRESS (REDUNDANT BARS)}$$

(ALTERNATIVELY 4×2 D.O.F. AND 10 BARS
∴ 2 REDUNDANT BARS)

(b) USING $\overline{IV}$ AND $\overline{IX}$ AS REDUNDANT BARS

(i) PARTICULAR EQUILIBRIUM SOLUTION:

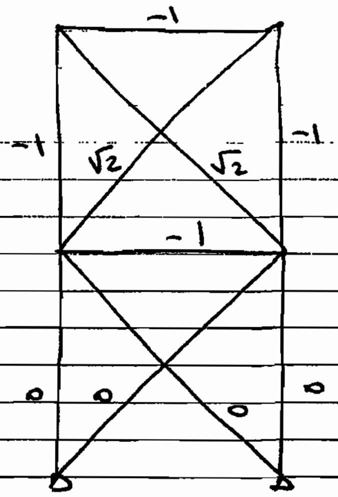


$t_{\overline{IV}} = t_{\overline{IX}} = 0$

$$\therefore \quad \underset{\sim}{t}_0 = \begin{bmatrix} 1 \\ 0 \\ -\sqrt{2} \\ 0 \\ 1 \\ 1 \\ -1 \\ -\sqrt{2} \\ 0 \\ 2 \end{bmatrix} W$$

18/2009/2/2

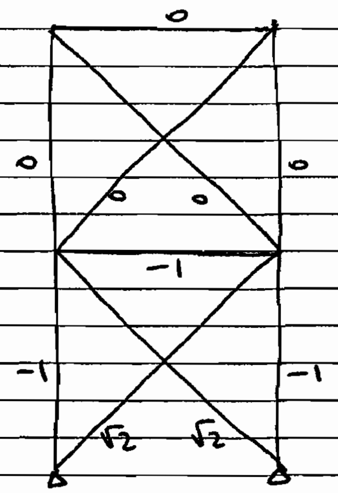
b(ii) STATES OF SELF STRESS



$$t_{\overline{IV}} = \sqrt{2}$$

$$t_{\overline{IX}} = 0$$

$$\therefore \; \underset{\sim}{s_1} = \begin{bmatrix} -1 \\ -1 \\ \sqrt{2} \\ \sqrt{2} \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



$$t_{\overline{IV}} = 0$$

$$t_{\overline{IX}} = \sqrt{2}$$

$$\therefore \; \underset{\sim}{s_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \\ \sqrt{2} \\ \sqrt{2} \\ -1 \end{bmatrix}$$

b(iii) ELASTIC SOLUTION :

GENERAL STATE OF TENSION

$$\underset{\sim}{t} = \underset{\sim}{t_0} + x_1 \underset{\sim}{s_1} + x_2 \underset{\sim}{s_2}$$

1B/2009/2/3

FLEXIBILITY MATRIX $\underset{\sim}{F} = \begin{bmatrix} 1 & & & & & & & & & 0 \\ & 1 & & & & & & & & \\ & & \sqrt{2} & & & & & & & \\ & & & \sqrt{2} & & & & & & \\ & & & & 1 & & & & & \\ & & & & & 1 & & & & \\ & & & & & & 1 & & & \\ & & & & & & & \sqrt{2} & & \\ & & & & & & & & \sqrt{2} & \\ 0 & & & & & & & & & 1 \end{bmatrix} \frac{L}{AE}$

BAR EXTENSIONS $\underset{\sim}{e} = \underset{\sim}{F}\,\underset{\sim}{t}$

$$\therefore \underset{\sim}{e} = \begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \\ 1 \\ 1 \\ -1 \\ -2 \\ 0 \\ 2 \end{bmatrix} \frac{WL}{AE} + \begin{bmatrix} -1 \\ -1 \\ 2 \\ 2 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \frac{x_1 L}{AE} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \\ 2 \\ 2 \\ -1 \end{bmatrix} \frac{x_2 L}{AE}$$

COMPATIBILITY: USING VIRTUAL WORK WITH $\underset{\sim}{S}_1$ AND $\underset{\sim}{S}_2$ AS BAR FORCES WITH ZERO EXTERNAL LOAD TO DETERMINE x_1 AND x_2.

$$\sum F.d = \sum S.e$$

$$\underset{\sim}{S}_1 \underset{\sim}{e} = 0 \Rightarrow \left(-3 - 2\sqrt{2}\right)\frac{WL}{AE} + \left(4 + 4\sqrt{2}\right)\frac{x_1 L}{AE} + \frac{x_2 L}{AE} = 0$$

$$\underset{\sim}{S}_2 \underset{\sim}{e} = 0 \Rightarrow \left(-2 - 2\sqrt{2}\right)\frac{WL}{AE} + \frac{x_1 L}{AE} + \left(3 + 4\sqrt{2}\right)\frac{x_2 L}{AE} = 0$$

RE-ARRANGING:

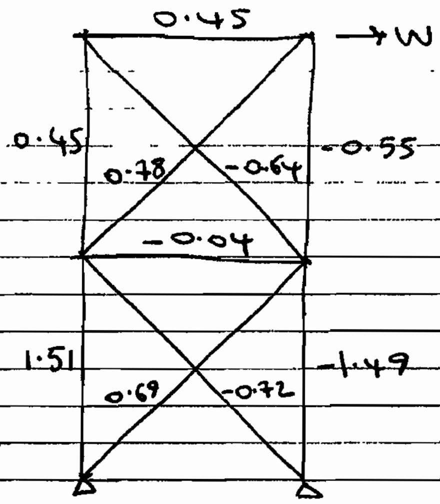
$$-5.83W + 9.66x_1 + x_2 = 0 \quad -(1)$$
$$-4.83W + x_1 + 8.66x_2 = 0 \quad -(2)$$

SOLVING (1) AND (2) FOR x_1 AND x_2:

$x_1 = 0.55W$ AND $x_2 = 0.49W$

BAR TENSIONS ARE : $\underset{\sim}{t_0} + x_1 \underset{\sim}{s_1} + x_2 \underset{\sim}{s_2}$

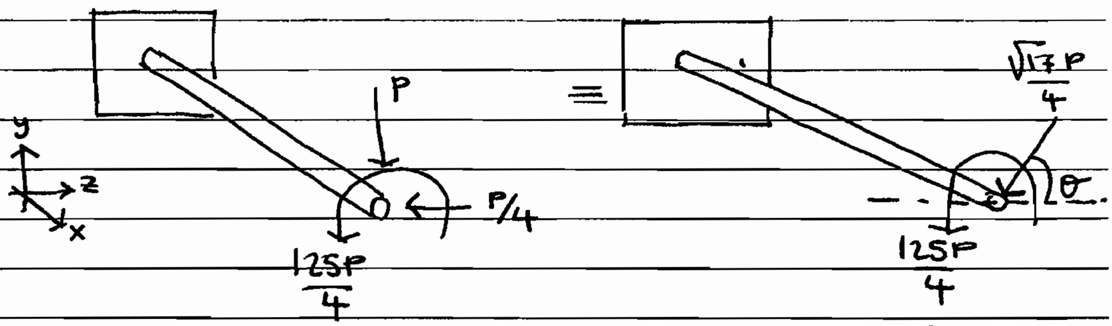
$$\therefore\ t_0 = \begin{bmatrix} 0.45 \\ -0.55 \\ -0.64 \\ 0.78 \\ 0.45 \\ -0.04 \\ -1.49 \\ -0.72 \\ 0.69 \\ 1.51 \end{bmatrix} W$$



Question 2.
Subject: Elasticity – Statically indeterminate structures
This was a very popular and generally well answered question. All candidates were able to show that there were two redundant bars as required in part (a). Most candidates successfully derived the particular equilibrium solution (bi) and the corresponding states of self stress in the structure (bii), but some solutions contained errors arising from incorrect resolution of forces at the joints and inconsistencies in the sign convention. Most candidates used the appropriate method for finding the elastic solution (biii), but most solutions contained several arithmetic and algebraic errors or were incomplete.

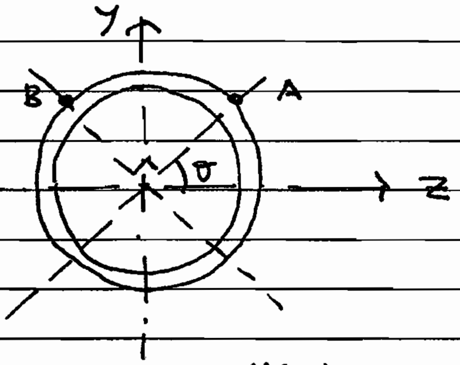
IB / 2009 / 3 / 4

3(a) BRACKET MAY BE SIMPLIFIED TO :



WHERE $\theta = \tan^{-1} 4$
$= 76°$

THEREFORE EXAMINING A CROSS-SECTION AT THE SUPPORTING PLATE END :



$\theta = 76°$

BY INSPECTION → MAXIMUM DIRECT STRESS (TENSION) OCCURS AT A

→ MAXIMUM SHEAR STRESS OCCURS AT B.

DIRECT STRESS σ_{xx} DUE TO BENDING :

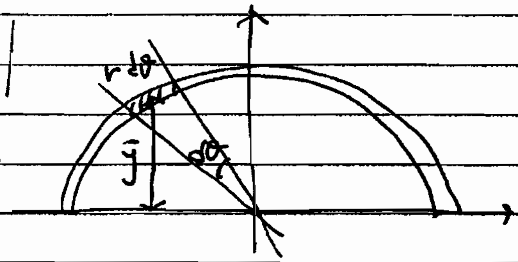
$\sigma_{xx} = My/I$ WHERE $I = \pi r^3 t$

$$\therefore \sigma_{xx} = \frac{\sqrt{17}}{4} P . 250 \times 25 . \frac{1}{\pi . 25^3 . 2} = \underline{\underline{65.56 \times 10^{-3} P}}$$

SHEAR STRESS ~~DUE~~ τ_{xy} DUE TO BENDING:

$$q = \frac{SA\bar{y}}{I}$$

$$\tau_{xy} = \frac{SA\bar{y}}{It}$$



$$\tau_{xy} = \frac{P\frac{\pi}{2} \times (25^2 - 23^2)\frac{2\times 25}{\pi}}{\pi \times 25^3 \times 2 \times 2}$$

$$= 12.2 \times 10^{-3} P \quad -(1)$$

$$\bar{y} = \frac{1}{A}\int y\,dA$$

$$= \frac{1}{A}\int y \,.\, t r\,d\theta$$

$$r^2 = x^2 + y^2$$
$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$\therefore \bar{y} = \frac{2r}{\pi}$$

SHEAR STRESS τ_{xy} DUE TO TORQUE:
(UNIFORM ALONG CIRCUMFERENCE)

$$q = T/2A_e$$

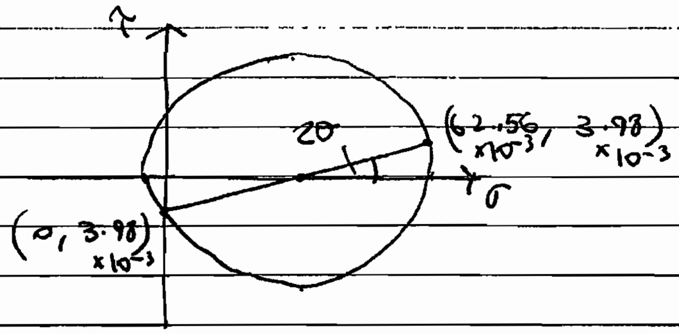
$$\tau = T/2A_e t = \frac{125P}{4} \cdot \frac{1}{2 \times \pi \times 25^2 \times 2}$$

$$= 3.98 \times 10^{-3} P \quad -(2)$$

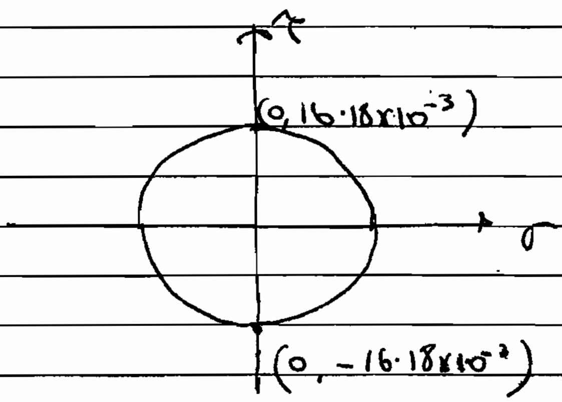
$\therefore$ TOTAL SHEAR STRESS τ_{xy} AT B $= (1) + (2)$

$$= \underline{\underline{16.18 \times 10^{-3} P}}$$

(b) MOHR CIRCLE AT A:



MOHR CIRCLE AT B:



∴ LOCATION A IS CRITICAL.

DESIGN STRESSES AT A (SAFETY FACTOR = 2):

$\sigma_{xx} = 62.56 \times 10^{-3} P \times 2$

$= 125.12 \times 10^{-3} P$

$\tau_{xy} = 3.98 \times 10^{-3} P \times 2$

$= 7.96 \times 10^{-3} P$

FROM MOHR CIRCLE:

$$\sigma_1 = \frac{125.12}{2} + \sqrt{\left(\frac{125.12}{2}\right)^2 + (3.98)^2}$$

$\therefore \sigma_1 = 125 \cdot 25 \times 10^{-3} P$

$$\sigma_2 = \frac{125 \cdot 12}{2} \sqrt{\left(\frac{125 \cdot 12}{2}\right)^2 + (3 \cdot 98)^2}$$

$\therefore \sigma_2 = -1 \cdot 2 \times 10^{-4} P$

$\sigma_3 = 0$

VON MISES:

$$2Y^2 = \left[\left(125 \cdot 25 \times 10^{-3} + 1 \cdot 2 \times 10^{-4}\right) + \left(125 \cdot 25 \times 10^{-3}\right)^2 + \left(-1 \cdot 2 \times 10^{-4}\right)^2\right] P^2$$

$\therefore Y = 0 \cdot 125 P$

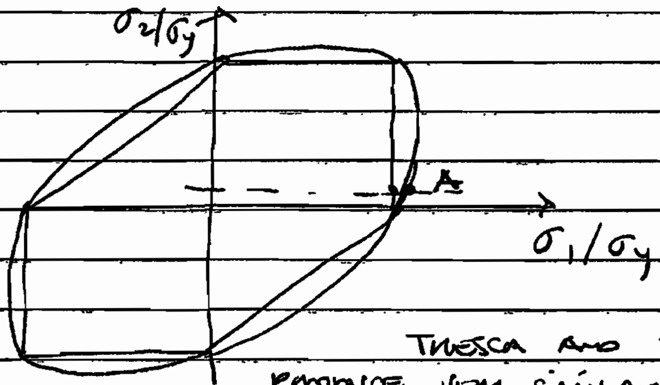
$\underline{\underline{P < 2200 N}}$

TRESCA:

$Y = 125 \cdot 25 \times 10^{-3} P$

$\underline{\underline{P < 2196 N}}$

(c)

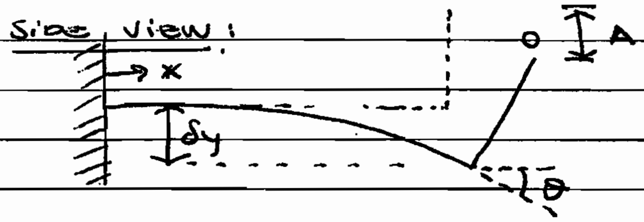


TRESCA AND VON MISES PRODUCE VERY SIMILAR RESULTS HOWEVER AT LOCATION A $\sigma_{xx} \gg \tau_{xy}$ AND $\sigma_{yy} = 0$ THIS RESULTS IN $\sigma_1 \gg \sigma_2$ $\therefore$ TRESCA AND VON MISES RESULTS ARE NEARLY IDENTICAL.

(d) ASSUMPTIONS:

- IGNORE SHEAR DEFORMATIONS CAUSED BY FLEXURE
- IGNORE SHORTENING OF VERTICAL SECTION
- ASSUME STIFFNESS OF 90° BEND IS EQUIVALENT TO OTHER LOCATIONS ALONG BRACKET.

SIDE VIEW:

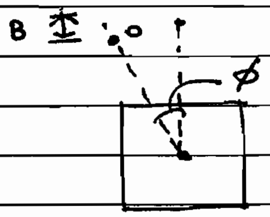


$l_1 = 250$ mm

$l_2 = 125$ mm

FLEXURAL DEFORMATION A $= \delta_y + l_2(1-\cos\theta)$

FRONT VIEW:



TORSIONAL DEFORMATION B $= l_2(1-\cos\phi)$

∴ FLEXURAL DEFORMATION A:

$$\delta_y = Wl^3/3EI = 2200\times 250^3 / 3\times 210\times 10^3 \times \pi \times 25^3 \times 2$$

$$= 0.56 \text{ mm}$$

$$\theta = \int_0^l \frac{M}{EI}\,dx = \frac{1}{EI}\int_0^l Wx\,dx = \left.\frac{Wl^2}{2EI}\right]$$ ALSO IN DATA BOOK.

$$\therefore \theta = 2200\times 250^2 / 2\times 210\times 10^3 \times \pi \times 25^3 \times 2$$

$$= 0.00333 \text{ radians.}$$

$\therefore A = 0.561 \text{ mm}$

TORSIONAL DEFORMATION B:

$$\phi = \frac{TL}{GJ} \qquad \left(\tau = \frac{Gr\phi}{L} = \frac{T}{2\pi r^2 t} \right.$$

$$\therefore \phi = \frac{TL}{\underbrace{G 2\pi r^3 t}_{J}} \left. \right)$$

$$\phi = \frac{125 \times 1200}{4} \cdot 250 \cdot \frac{1}{81 \times 10^3 \times 196349} \qquad J = 196349 \text{ mm}^4$$

$= 0.00108$ radians.

$$\therefore B = 125 \left(1 - \cos 0.00108\right)$$
$$= 7.28 \times 10^{-5} \text{ mm}$$

$\therefore$ TOTAL VERTICAL DEFLECTION AT O

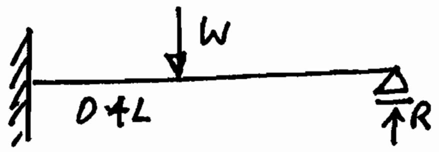
$= 0.561 \text{ mm} + 7.28 \times 10^{-5} \text{ mm}$

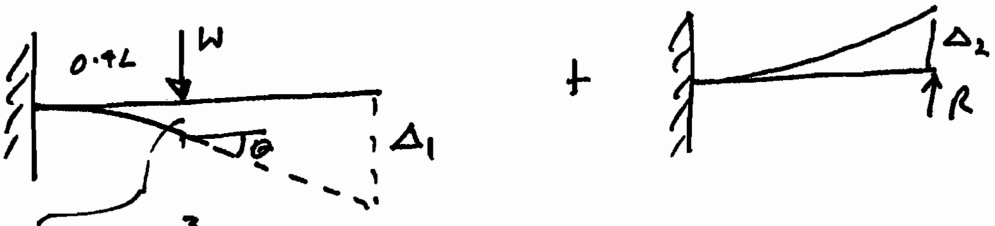
$= \underline{\underline{0.561 \text{ mm}}}$

Question 3.
Subject: Elasticity – Yield criteria
This was an unpopular and poorly answered question. Very few candidates realised that they could simplify the structure considerably in part (a) by using the resultant of the applied forces thereby transforming the problem to a relatively simple cantilever with a single point load and a single torque applied at the free end. Most candidates who attempted this question either ignored one of the applied forces altogether or used a rather lengthy process of superposition to account for both forces. A substantial proportion of the candidates also failed to account for the two components of shear stress (i.e. flexure and torsion). Most candidates were able to apply the Tresca and von Mises yield criteria required in parts (b) an and (c). Most candidates calculated the flexural deflection component required in part (d), but several failed to account for the torsional deformation.

1B/2009/4/1

4 (a)





$$\delta = \frac{W\cdot(0.4L)^3}{3EI} \qquad \Delta_2 = \frac{RL^3}{3EI}$$

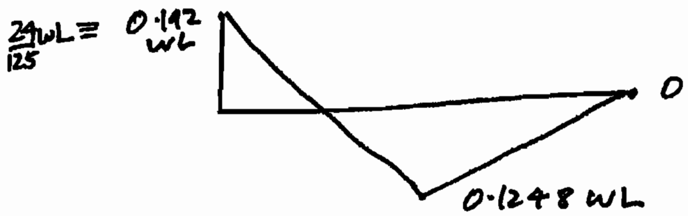
$$\theta = \frac{W\cdot(0.4L)^2}{2EI}$$

$$\therefore \Delta_1 = \frac{WL^3}{EI}\left[\frac{(0.4)^3}{3} + \frac{(0.4)^2}{2}\cdot 0.6\right]$$

$$= 0.069\dot{3}\dot{3}\,\frac{WL^3}{EI}$$

$$\Delta_1 = \Delta_2 \qquad \therefore \quad \underline{R = 0.208W} \quad \left(\equiv \frac{26}{125}W\right)$$

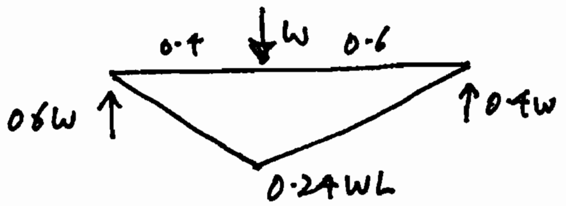
$\therefore$ B.M diag



Assumes. (1) Linear elasticity
(2) Supports level (or stress free when built)
(3) No dead load.
(4) Small deflections

1B/2009/4/2

(b) Free BM diagram

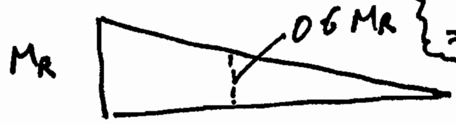


Alternative forms possible

&

but would give same answer by similar procedure.

Reactant BM diag



Common mistake $= 0.5M_R$

Any combination of these diagrams will give a lower bound provided total B.M less than M_p everywhere

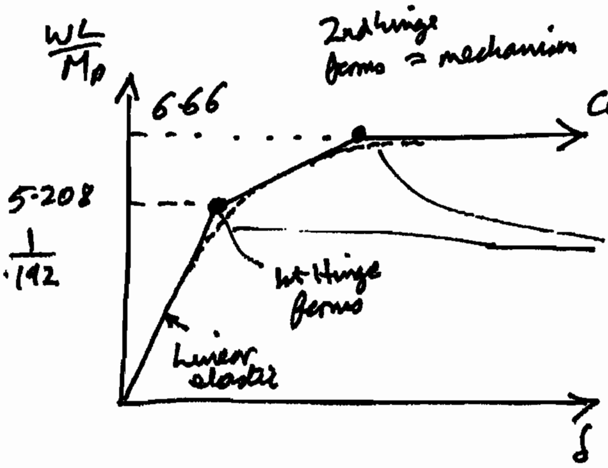
To get highest LB.

$M_R = M_p$ & $0.24WL - 0.6M_R = M_p$

$\Rightarrow 0.24WL = 1.6M_p$

$\Rightarrow \underline{WL = 6.66M_p}$

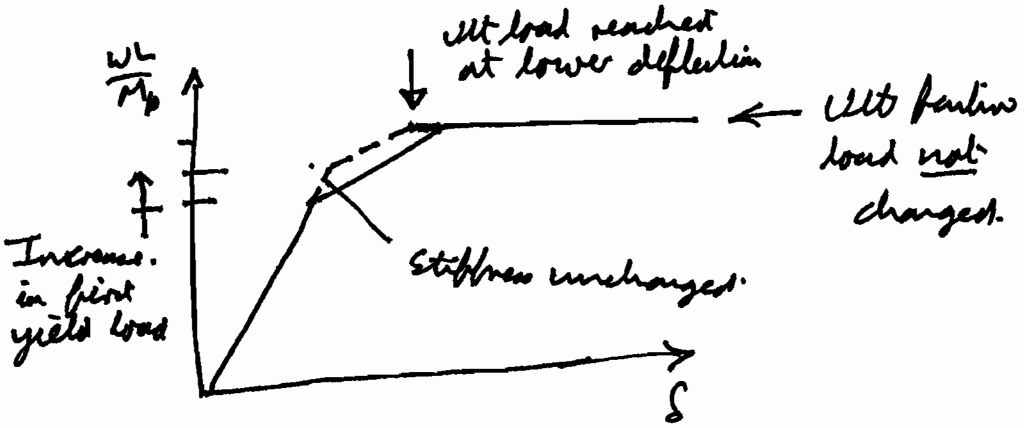
(c)



Bonus for stating that real beam will show elasto-plastic behaviour, avoiding sharp kinks

18/2009/4/3

(d) Effect of raising the tip will be to cause a sagging bending moment. ∴ Unless the lifting is very large it will have the effect of reducing the hogging moment at the support. This will delay the onset of first yield, so the diagram will be modified.



This question was tackled by most of the candidates, but very few attempted all parts. The difficulty appeared to be that the question mixed things from different parts of the course. The easiest way to do the elastic analysis was to use data book coefficients for a cantilever under two different point loads, but it can also be tackled correctly assuming it is fully fixed with a moment release at one end. However, their ingenuity in combining random data book formulae to get the wrong answer was almost limitless! Only one candidate used virtual work (which is a bit tedious in this example), but got it right. Very few attempted seriously to state their assumptions, and only a tiny proportion stated that the beam was unstressed when unloaded. The lower bound analysis was where most of the marks were picked up. The biggest surprise was an almost complete inability to do a load-deflection plot. "What's a Load deflection plot?" Surely the clue is in the question? Of the small proportion who did draw a plot with load on one axis and deflection on the other, none showed a change in stiffness at the load when the first hinge forms and many did not appear to realise that the same structure could exhibit both elastic and plastic portions. The final part asked them to consider the effect of raising slightly the propped support. Most correctly stated that the plastic solution was unaffected, and although many stated that the elastic solution was altered, most said that the induced prestress would reduce the elastic load capacity, when it actually increases it since it delays the onset of the first hinge. No one associated the lack-of-fit introduced here with the assumptions they had made in part (a).

IB/2009/5/1

5(a) Sway mechanism ABDE will mean that load at C does no work.

See top diagram on velocity diag sheet attached

Joint angles

A	θ
B	1.4142θ
D	1.4142θ
E	θ
	4.8284θ

Work done by H at D $= 0.7070 R\theta . H$

$\therefore$ Collapse will occur when

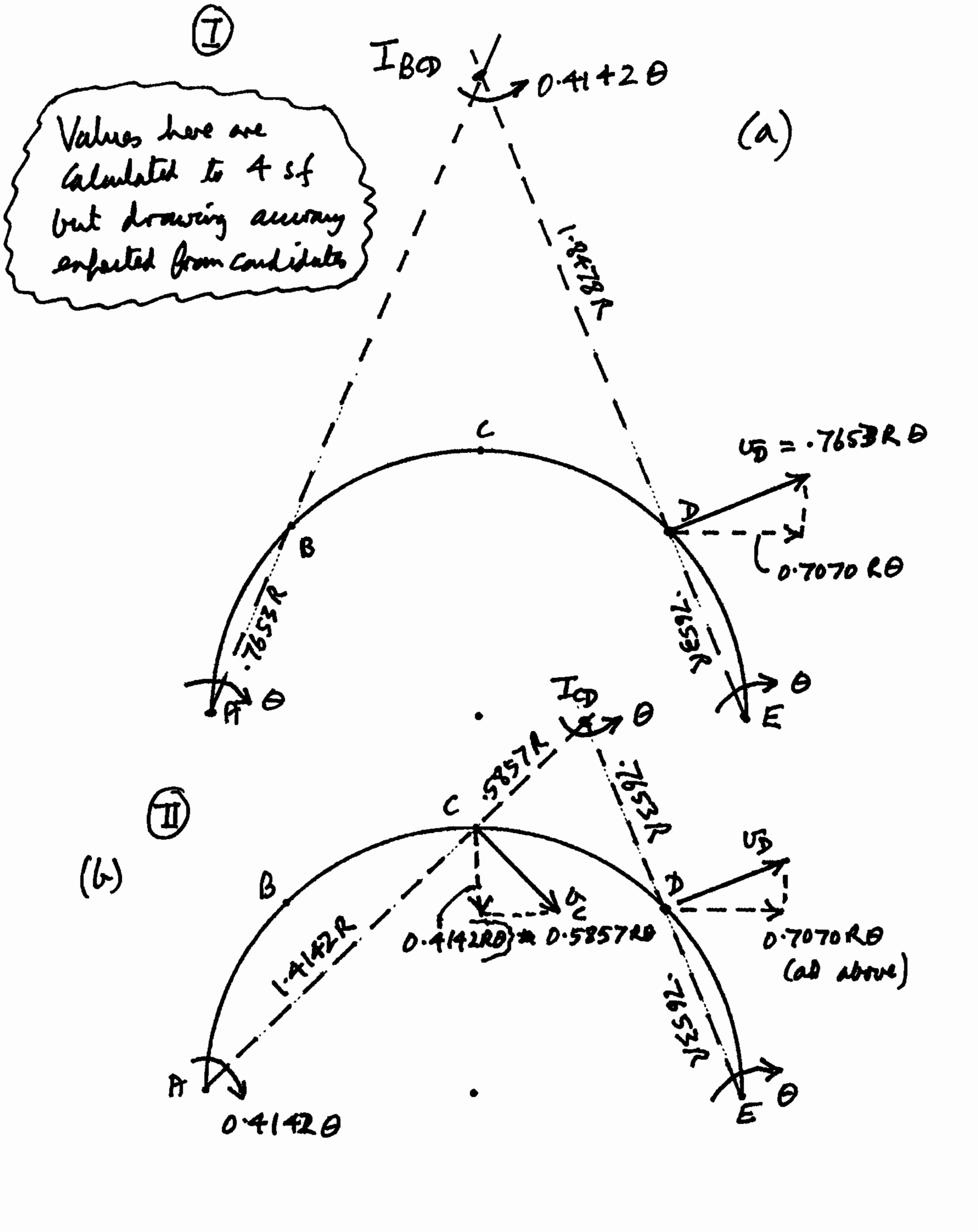
$$4.8284\,\theta . M_p = 0.7070\, R\theta H$$

$$\Rightarrow H = 6.8294 \frac{M_p}{R}$$

V will have no effect.

Done well by many candidates. Most drew an Instantaneous Centre diagram and got the right answer, but some tried to draw the displaced form of the mechanism and they almost invariably got it wrong. Some assumed that because, in the sway mechanism, the vertical load did no work, then the top element didn't rotate. Only a few realised that there was a symmetrical collapse mechanism but that this required 5 hinges so was not allowed.

1B/2009/5/2


I
Values here are calculated to 4 s.f but drawing accuracy expected from candidates
I_{BCD}
0.4142 θ
(a)
1.8478 R
C
v_D = .7653 R θ
D
B
0.7070 R θ
.7653 R
.7653 R
θ
A
θ
E
I_{CD}
θ
.5857 R
.7653 R
II
(b)
C
B
v_D
D
v_C
1.4142 R
0.4142 R θ
0.5857 R θ
0.7070 R θ
(as above)
.7653 R
A
0.4142 θ
θ
E

(d) Mechanism ACDE will involve work being done by V. (see v.d. sheet)

Joint Angles

A	$0.4142\,\theta$
C	$1.4142\,\theta$
D	2θ
E	θ
	$4.8284\,\theta$

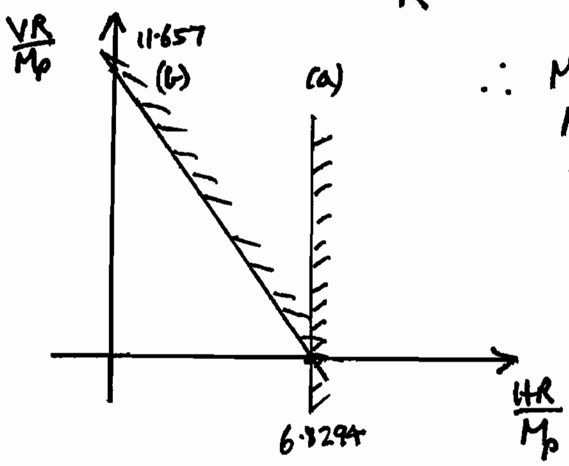
$$\therefore\; 4.8284\, M_p \cancel{\theta} = 0.4142\, R\cancel{\theta}.V + 0.7070\, R\cancel{\theta}.H$$

$\therefore$ Collapse when

$$H + 0.5859\,V = 6.8294\,\frac{M_p}{R}$$

(when $H=0$, $V = 11.657\,\frac{M_p}{R}$

$V=0$, $H = 6.8294\,\frac{M_p}{R}$



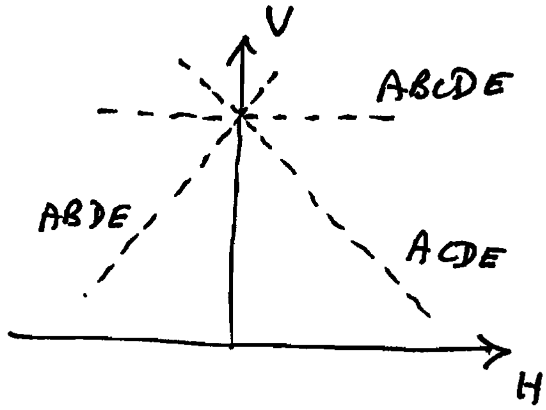
$\therefore$ Mechanism (b) ACDE will occur for all +ve values of V

(c) Mechanism that is symmetrical and gives vertical displacement at C will be

But this will involve 5 hinges

Collapse will have occurred by one of the unsymmetric mechanism ACDE or ABCE first.

Interaction diagram will look like

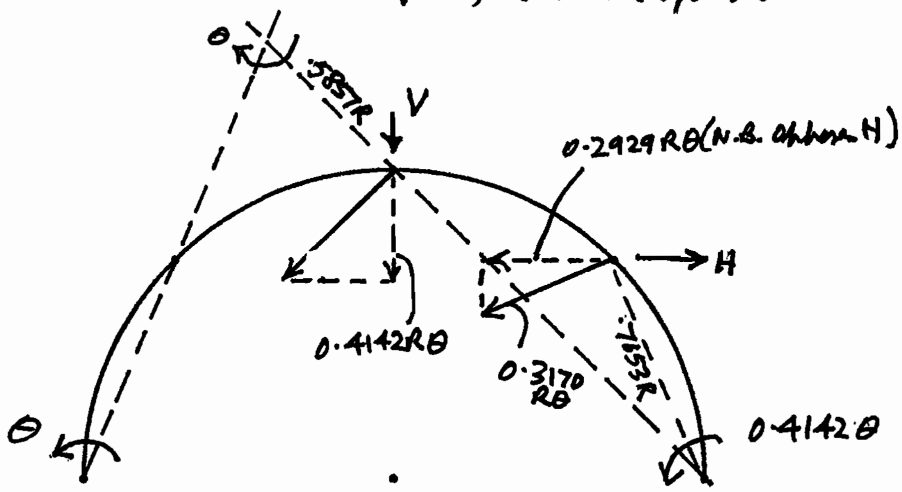


Imperfections will mean that symmetry is never absolute so the 5 hinge mechanism will never be the most favoured.

There are three other mechanisms that involve 4 hinges. These are kinematically admissable. Candidates were not expected to do more than 1 mechanism and were given full credit if they did one of these.

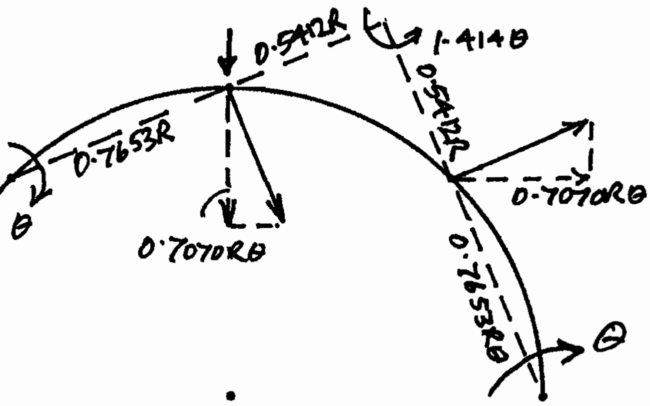
III

No hinge at D



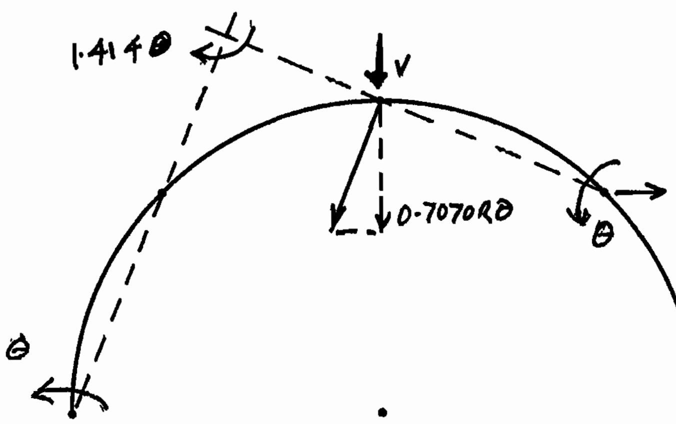
IV

No hinge at A



V

No hinge at E



NB H does no work.

1B/2009/5/6

For Mechanism III

NB. H does negative work. Several candidates wrote + here

$$0.4142\,R\theta.V - 0.2929\,R\theta\,H = (\theta + 2\theta + 1.4142\theta + 0.4142\theta)M_p$$

$$\Rightarrow V = 11.657\frac{M_p}{R} + 0.7071\,H$$

For Mechanism IV

$$0.7070\,R\theta.V + 0.7070R\theta.H = M_p(\theta + 2.4142\theta).2$$

$$\Rightarrow V + H = 9.658\,M_p/R$$ (Critical at low +ve values of H)

For Mechanism V

N.B. H does no work.

$$0.7070R\theta V = M_p.(\theta + 2.4142\theta).2$$

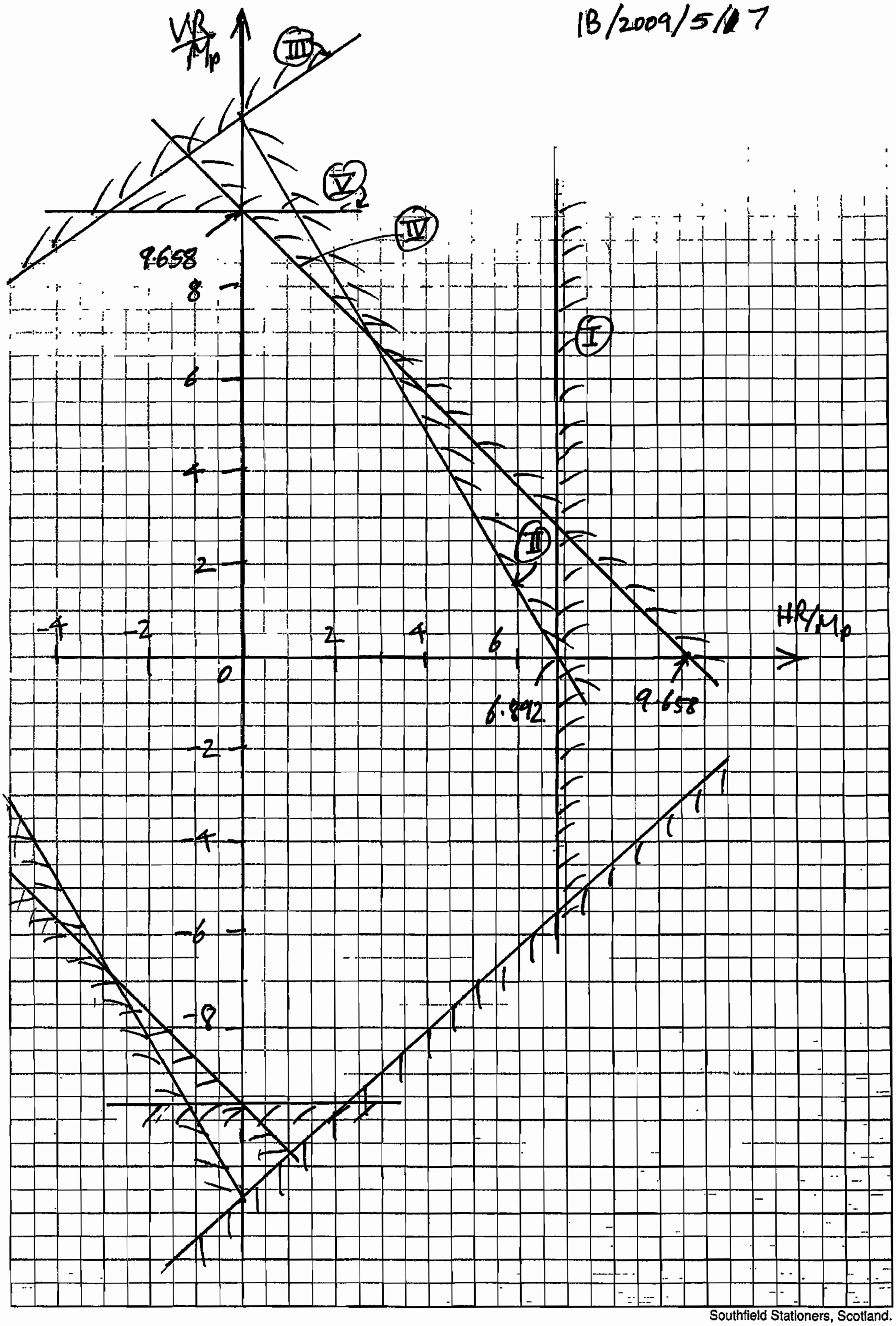
$$\Rightarrow V = 9.658\frac{M_p}{R}$$

See complete interaction diagram on next sheet.

Note that the diagram is <u>not</u> symmetrical about the VR/M_p axis. A negative value of H (at D) is not the same as a force to the left applied at B.

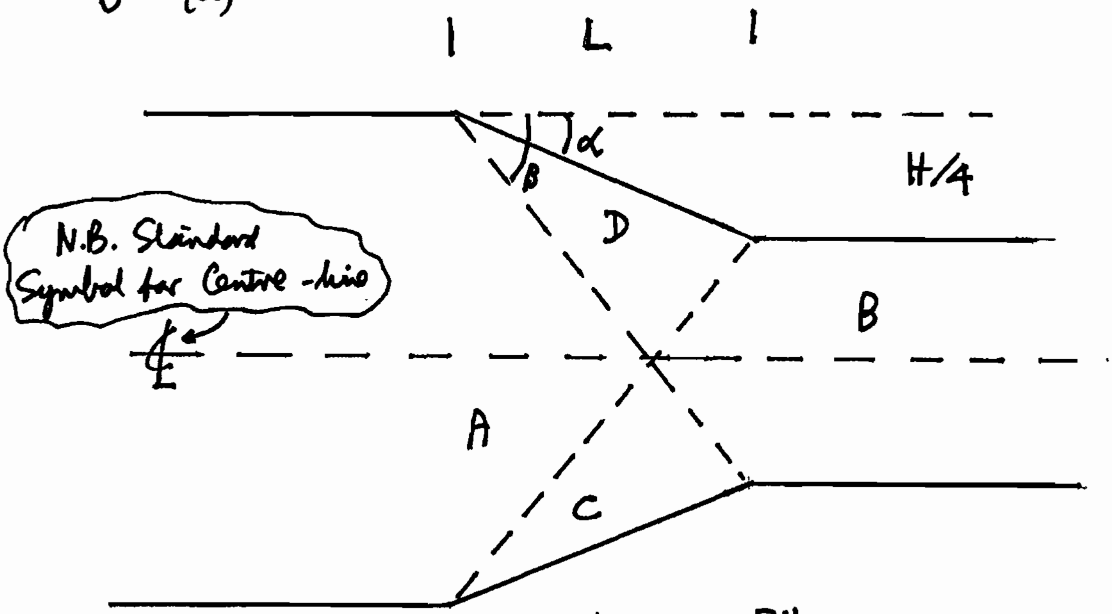
The candidates were not expected to cover the material on these last 3 sheets - it is included here for completeness and to aid revision!

IB/2009/5/7



Southfield Stationers, Scotland.

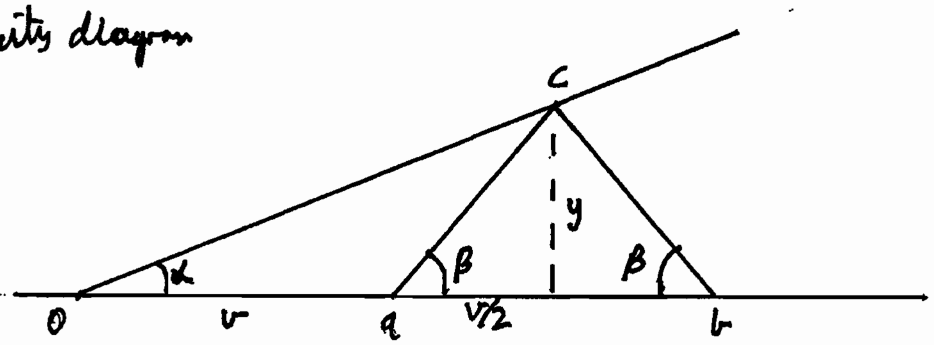
6 (a)



$\tan\alpha = \dfrac{H}{4L}$ $\qquad \tan\beta = \dfrac{3H}{L}$

Conservation of material means $v_{out} = v/2$

Velocity diagram



$$\tan\alpha = \frac{H}{4L} = \frac{y}{3v/2} \qquad \therefore\ y = \frac{H}{4L}\cdot\frac{3v}{2} = \frac{3Hv}{8L}$$

$$\therefore\ v_c = \sqrt{\left(\frac{3v}{2}\right)^2 + \left(\frac{3Hv}{8L}\right)^2} = \frac{3v}{2}\left(1 + \frac{H^2}{16L^2}\right)^{1/2}$$

Q.E.D.

(b) velocity $v_{ac} = \sqrt{\left(\frac{v}{2}\right)^2 + \left(\frac{3Hv}{8L}\right)^2}$

$$= \frac{v}{2}\left(1 + \left(\frac{3H}{4L}\right)^2\right)^{1/2}$$

velocity $v_{cB} = v_{ac}$ so we only need the total length of the slip planes, which is $\sqrt{L^2 + \left(\frac{3H}{4}\right)^2}$

$$= H\left(\left(\frac{L}{H}\right)^2 + \frac{9}{16}\right)^{1/2}$$

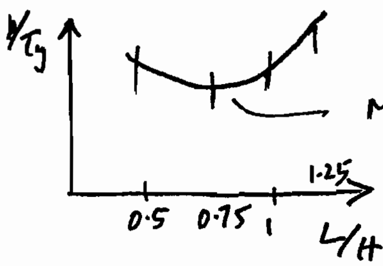
Equating work done

$$pvH = \underbrace{\tau_y \cdot 2}_{\uparrow\ \text{2 planes}} \cdot \frac{v}{2}\underbrace{\left(1 + \left(\frac{3H}{4L}\right)^2\right)^{1/2}}_{v_{ac}/v_{ab}} \cdot \underbrace{H\left(\left(\frac{L}{H}\right)^2 + \frac{9}{16}\right)^{1/2}}_{\text{length of slip planes}}$$

$$\therefore \frac{p}{\tau_y} = \left(1 + \left(\frac{3H}{4L}\right)^2\right)^{1/2}\left(\left(\frac{L}{H}\right)^2 + 9/16\right)^{1/2}$$

Values

L/H	p/τ_y
0.5	1.625
0.75	1.5
1.0	1.5625
1.25	1.7



Minimum at $L/H = 0.75$

or, by calculus.

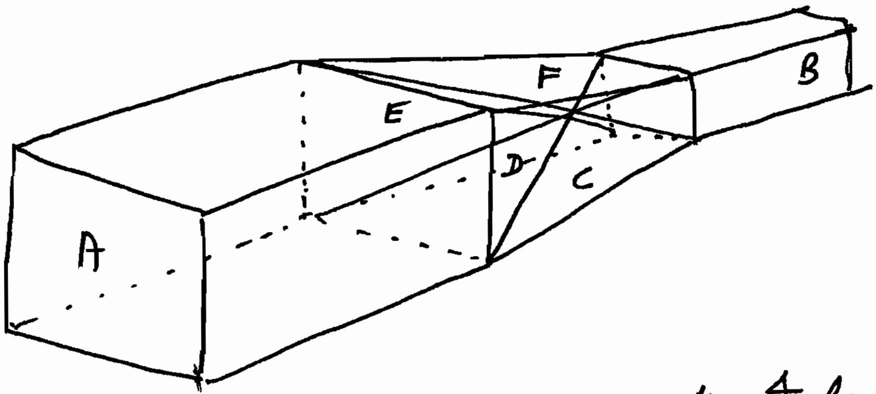
minimising $p/\tau_y \equiv$ minimising $\frac{p^2}{\tau_y^2} = \lambda$ $\qquad \frac{L}{H} = r$

$$\lambda = \left(1 + \frac{9}{16r^2}\right)\left(r^2 + \frac{9}{16}\right) = \frac{18}{16} + r^2 + \frac{81}{256r^2}$$

$$\therefore \frac{d\lambda}{dr} = 2r - \frac{2.81}{256} \cdot \frac{1}{r^3} = 0 \text{ when}$$

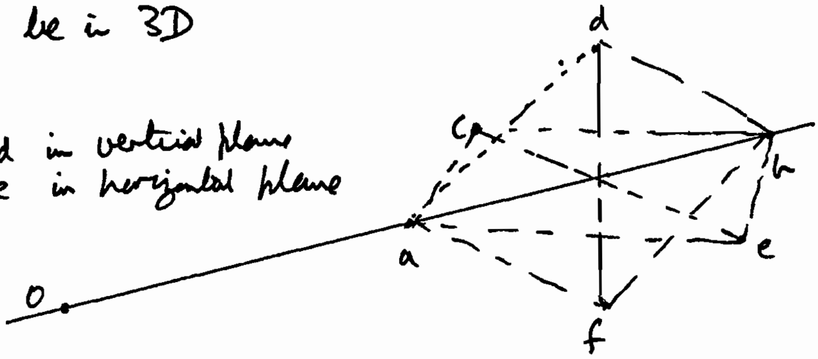
$$r^4 = \frac{81}{256} \Rightarrow r = 3/4$$

(c) Now a 3D problem



There will be regions in contact with 4 faces of the die (C, D, E & F) and the velocity diagram will be in 3D

abfd in vertical plane
abce in horizontal plane



Lengths will be replaced by areas

Least popular plasticity question; tackled either as a last resort (getting virtually no marks) or by people who knew what they were doing and got quite good marks. Very little nonsense in the answers from the second group, which was heartening although some unwillingness to simplify complex equations or to substitute numbers; "you expect me to plot that?"; Yes.

Answers

1. (b)(i) 70.5 MPa, -7.58 MPa, 39.04 MPa (36.3° clockwise from horizontal axis)

2. (b)(i) [1, 0, -√2, 0, 1, 1, -1, -√2, 0,2]W
   (ii) [-1, -1, √2, √2, -1, -1, 0, 0, 0, 0]W; [0, 0, 0, 0, 0, -1, -1, √2, √2, -1]W
   (iii) [0.45, -0.55, -0.64, 0.78, 0.45, -0.04, -1.49, -0.72, 0.69, 1.51]W

3. (a)(i) 0.065P (ii) 0.016P
   (b) 2200 N, 2196 N
   (d) -0.56 mm

4. (a) 0.192WL
   (b) $6.66M_P/L$

5. (a) $H = 6.83M_P/R$

6. L/H=0.75

C J Burgoyne
19th June 2009

This sheet should be used for the answer to Q5. It should be handed in with your script.
Additional copies are available from the invigilator.