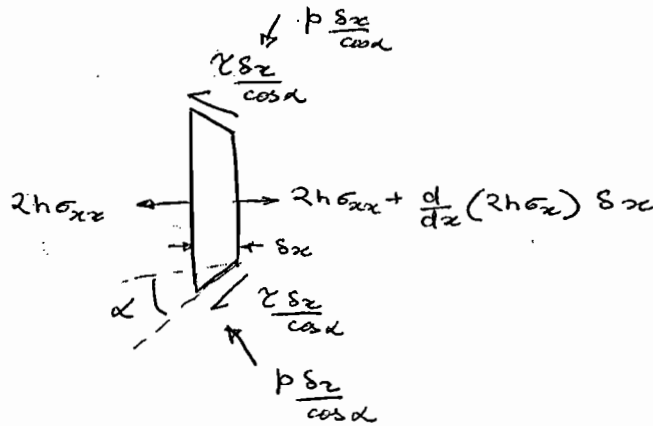


C.R.I.B Paper 3 (Part IB: 2009)

①

(a)



(b) Force equilibrium x in x -direction gives

$$2p \frac{\delta x}{\cos \alpha} \sin \alpha + \frac{2\tau \delta x}{\cos \alpha} \cos \alpha = \delta x \frac{d}{dx} (2h\sigma_{xx})$$

$$\Rightarrow p \tan \alpha + \tau = \frac{d}{dx} (h\sigma_{xx})$$

(c) $\tau = \mu p$; $h \rightarrow \bar{h}$ & $p = \gamma - \sigma_{xx}$

The equilibrium eq. becomes

~~(\gamma - \sigma_{xx})~~ $p \tan \alpha + \mu p \tan \alpha = \bar{h} \frac{d\sigma_{xx}}{dx}$

$$(\gamma - \sigma_{xx})(\mu + \tan \alpha) = \bar{h} \frac{d\sigma_{xx}}{dx}$$

$$\frac{\bar{h}}{\mu + \tan \alpha} \frac{d\sigma_{xx}}{dx} + \sigma_{xx} = \gamma$$

$$\sigma_{xx} = \tau + Ae^{-x/L} \quad \text{where } L = \frac{\bar{h}}{h + \tan \alpha} \quad (2)$$

$$\text{at } x = l \quad \sigma_{xx} = 0$$

$$\Rightarrow 0 = \tau + Ae^{-l/L}$$

$$A = -\tau e^{l/L}$$

$$\sigma_{xx} = \tau \left(1 - e^{(l-x)/L} \right)$$

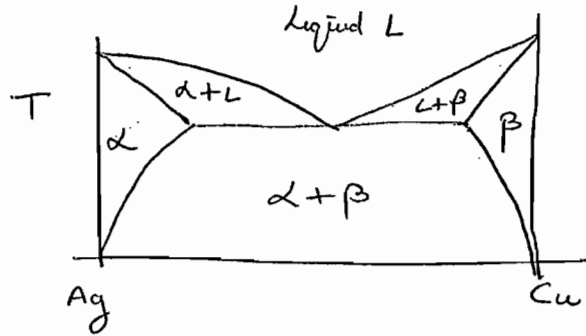
$$(d) \quad F = -2\bar{h} \sigma_{xx} \Big|_{x=0} = -2\bar{h} \tau \left[1 - e^{l/L} \right]$$

$$\Rightarrow F = 2\bar{h} \tau \left[e^{(h + \tan \alpha)l/\bar{h}} - 1 \right]$$

Q2

(3)

(a)

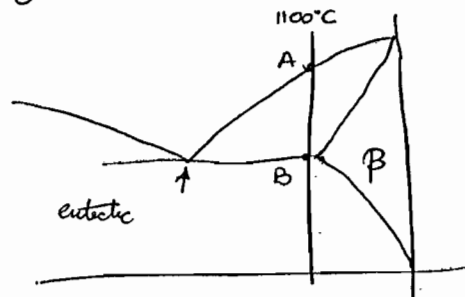


α -phase - silver with Cu in solid solution

β -phase - copper with silver in solid solution

(b)

Alloy of 40 wt% Ag & 60 wt% Cu



Slow cool. At 1100°C liquid of 60 wt% Cu. At B

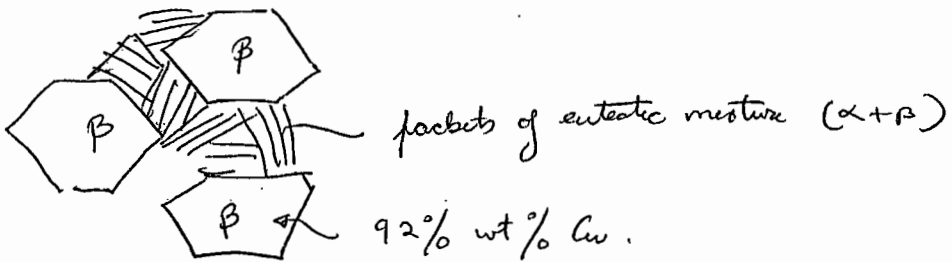
have a mixture of β (92 wt% Cu) & eutectic (28.1 wt%

Cu).

At B weight fraction of eutectic from lever rule is

$$\frac{92-60}{92-28.1} = 0.5 = \text{wt \% of } \beta$$

(4)



Eutectic mix: made up of $\alpha + \beta$ phases.

α phase is of composition: 8.8 wt% Cu

β phase is of composition: 92 wt% Cu

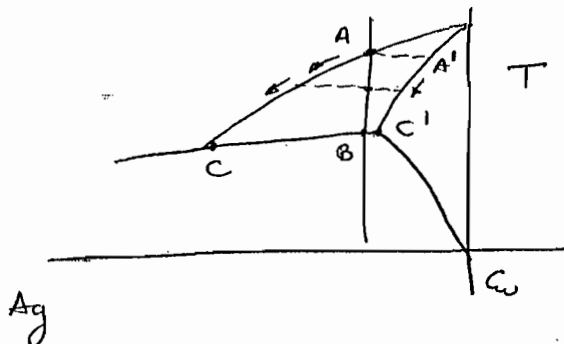
weight fraction of α in eutectic mixture

$$= \frac{92 - 28.1}{92 - 8.8} = 0.768$$

weight fraction of β in eutectic mixture

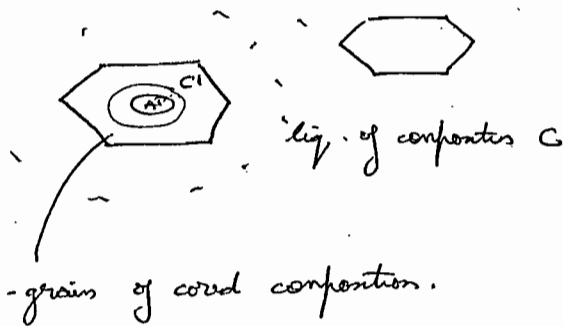
$$= \frac{28.1 - 8.8}{92 - 8.8} = 0.232$$

c)



At A first solid to form is of composition A'. The remaining liquid becomes richer in Ag. As cooling proceeds solid of composition in the range A' → C' forms & the remaining liquid is of composition C (eutectic composition).

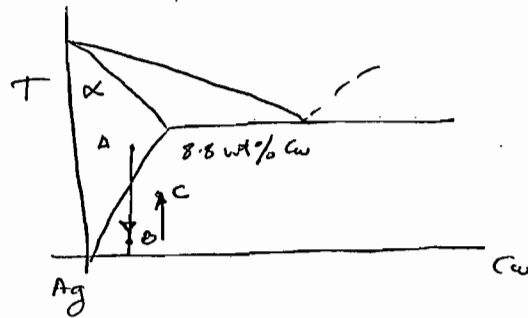
At T just above T_E ,



When T is decreased through the eutectic temp. T_E , the remaining liquid of eutectic comp. transforms to the eutectic mixture.

(d)

(6)



Choose an alloy of composition $\sim 8 \text{ wt\% Cu}$.

(i) solution treat to $\sim 780^\circ\text{C}$ to dissolve all the Cu in a solid solution of α

(ii) quench in water to room temp: the Cu remains in solution

(iii) Age harden by ~~the~~ re-heating to 100°C . Hold for $\sim 1 \text{ hr}$ then air cool. Fine precipitates of β in α appear.

(7)

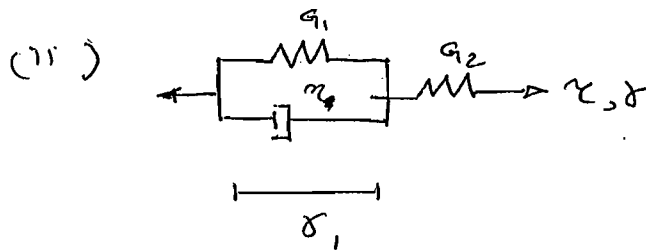
Q3.

(a) The temperature at which bonds between the main chains (van der Waals or hydrogen bonds) melts is T_g . E drops by $\sim 10^3$ at T_g .

The value of T_g is controlled by bond strength.

(b)

(i) Viscous dissipation associated with chains sliding between entanglement points.



$$\tau = G_1 \dot{\gamma}_1 + \tau \dot{\gamma}_1, \quad \dot{\gamma}_1 = \dot{\gamma} - \frac{\tau}{G_2}$$

$$\dot{\gamma}_1 = \dot{\gamma} - \frac{\tau}{G_2}$$

$$\tau = G_1 \left[\dot{\gamma} - \frac{\tau}{G_2} \right] + \tau \left[\dot{\gamma} - \frac{\tau}{G_2} \right]$$

$$\frac{\tau \tau}{G_2} + \left(\frac{G_1 + G_2}{G_2} \right) \tau = \tau \dot{\gamma} + G_1 \dot{\gamma}$$

(8)

(iii) At $t = 0^+$ $\gamma = \frac{\tau_0}{g_2}$ ~~with the initial condition~~

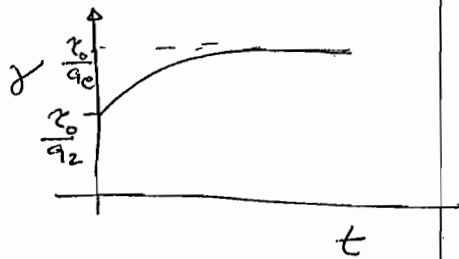
~~$\frac{\tau_0}{g_2}$~~ $\infty \delta_1 = 0$ but $\dot{\delta}_1 = \frac{\tau_0}{\tau}$

(iv) $\tau \dot{\gamma} + g_1 \gamma = \left(\frac{g_1 + g_2}{g_2} \right) \tau_0$

$$\frac{d\gamma}{\left(\frac{g_1 + g_2}{g_2} \right) \tau_0 - g_1 \gamma} = \frac{dt}{\tau}$$

$$\Rightarrow \gamma = \frac{\tau_0}{g_e} - \left[\frac{\tau_0}{g_e} - \frac{\tau_0}{g_2} \right] \exp \left[-\frac{g_1 t}{\tau} \right]$$

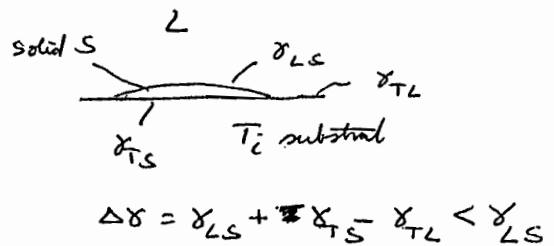
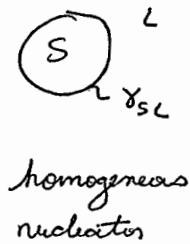
where $\frac{1}{g_e} = \frac{1}{g_1} + \frac{1}{g_2}$



Q4

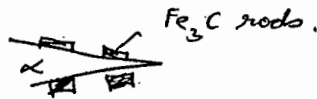
(9)

(a) The Ti powder ~~acts as~~ produces sites for heterogeneous nucleation, lowering the energy barrier.



(b)

Furnace cool produces bainite (probably upper bainite)



Bainite similar to pearlite: soft tough ferrite α with reinforcing particles or rods of the ceramic Fe_3C

Water quench produces martensite α' . This is a distorted tetragonal structure with interstitial C atoms blocking dislocation motion: structure is hard & brittle.

(c) High loss rubbers on tread increases friction & (10)
dissipation & hence grip whereas the walls require a
low loss rubber to minimise energy loss due to
cyclic deformation.

(d) T_g for PET $\sim 100^\circ\text{C}$. Bottle made by
blow moulding with molecular alignment within the
walls. In as processed state the crystals of PE
are smaller than the wavelength of light & it is
transparent.

When heated to 150°C the van der Waals bonds
melt & the structure relaxes. Also the crystals
grow resulting in dispersion of light which gives
the whitening effect.

(e) Grains nucleate where there is a concentration of
plastic deformation because of the energy of dislocations. To
provide a nucleus, the ~~new~~ material must be deformed by
plastic deformation & then annealed.

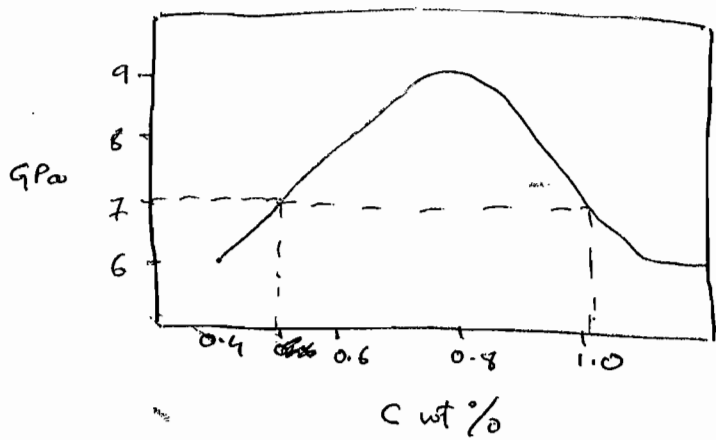
(i) To make a small grain sized material the mtl ⁽ⁱ⁾
should be heavily deformed, ^{& then annealed} so that the dislocations
form walls & hence are close together & there
exist many nuclei & grain size is small. ~~the~~

(ii) Deform mtl by a small amount & then anneal.
Requires a minimum amt of deformation so that there
are atleast a few nuclei.

Q5

(12)

(a) Carburizing is used in the production of bearing races, gears, bushings etc, i.e. in situations where a high-carbon hard case & a low carbon tough core is desirable.



(b)

(i) Requires carbon content in range 0.5 to 1.0 wt%

(ii) Thus $C_s = 1.02$

require $C = 0.5$ at $x = 0.2$ mm

$$0.5 = (1.02 - 0.2) \left[1 - \operatorname{erf} \frac{x}{2\sqrt{Dt}} \right] + 0.2$$

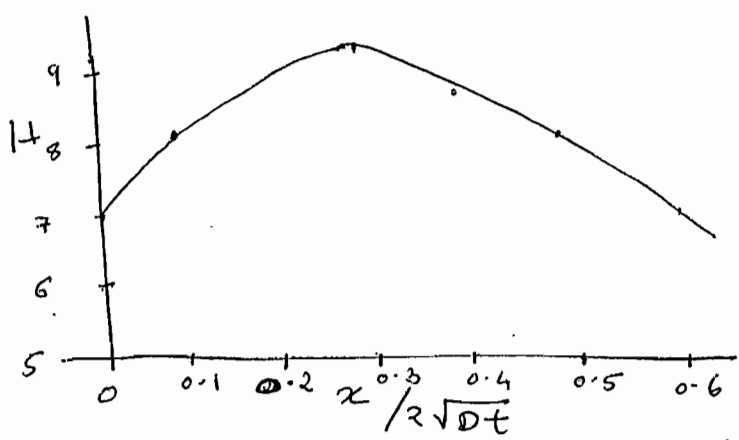
$$\operatorname{erf} \frac{x}{2\sqrt{Dt}} = 1 - \frac{0.3}{0.82} = ~~0.63~~ 0.63$$

$$\therefore \frac{x}{2\sqrt{Dt}} \approx ~~0.6~~ 0.6 + \frac{0.007}{0.078} = ~~0.64~~ 0.64$$

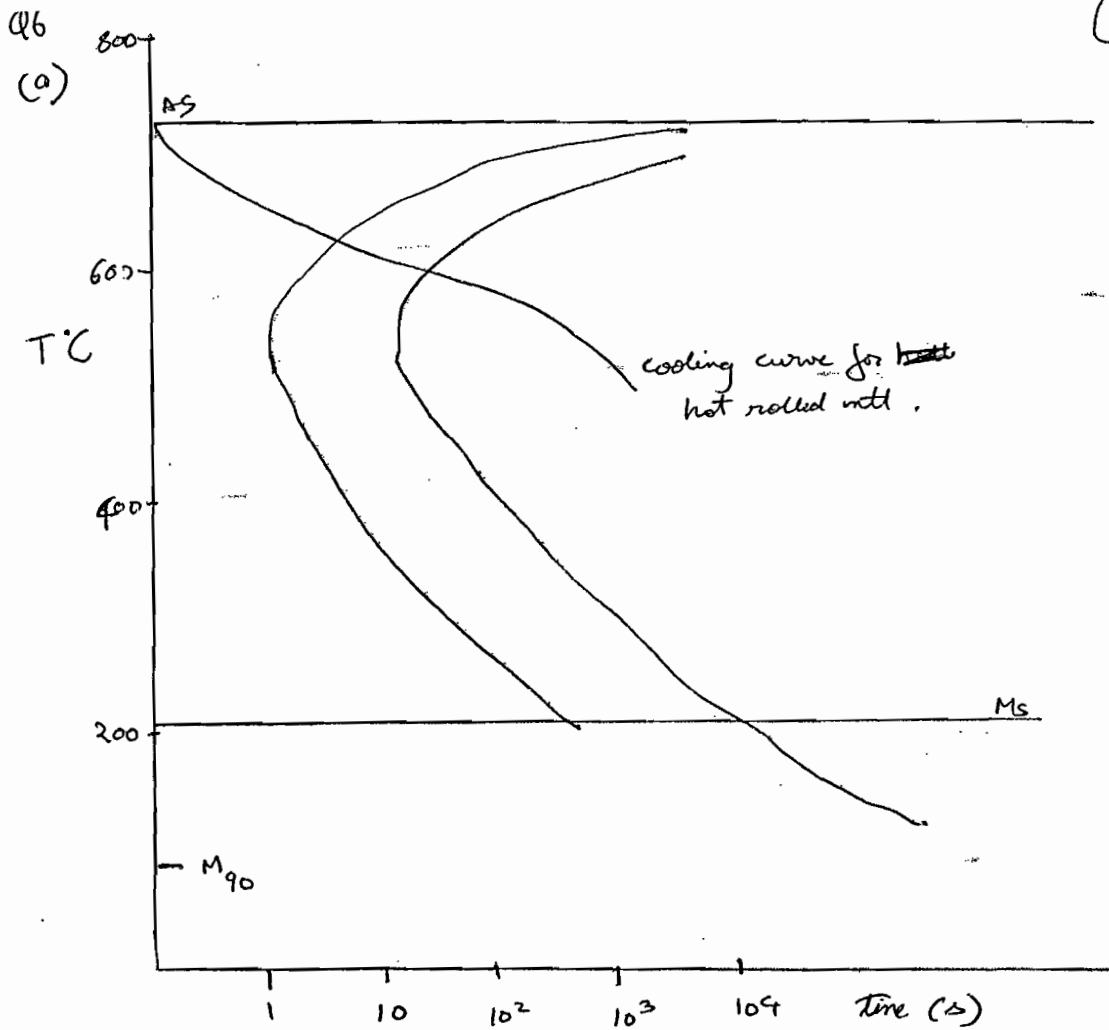
$$t = \left(\frac{2 \times 10^{-4}}{2 \times 0.64} \right)^2 / 4 \times 10^{-11}$$
$$= 10.18 \text{ mins.}$$

(iii)

$\frac{x}{2\sqrt{Dt}}$	C	H
0	1.02	7
0.1	0.92	8
0.2	0.84	8.8
0.3	0.75	9
0.4	0.67	8.5
0.5	0.59	8
0.6	0.52	7
0.7	-	-



Peak at $\frac{x}{2\sqrt{Dt}} \sim 0.3$ i.e. at $x = 94 \text{ km.}$



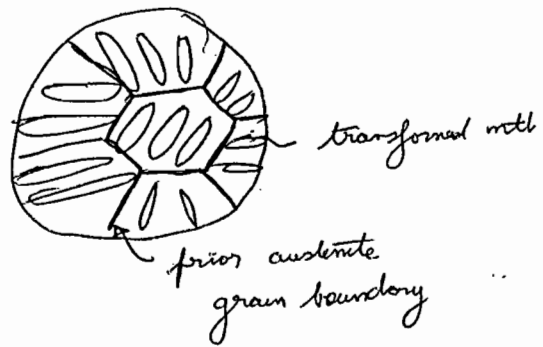
The C-shape of the lines above M_s results from the combined effect of nucleation & growth of one phase within another. At high T diffusion is fast but the driving force for both nucleation & growth is low. As the T is decreased the driving force increases but rate of diffusion increases. Thus, there is an optimum.

(b)

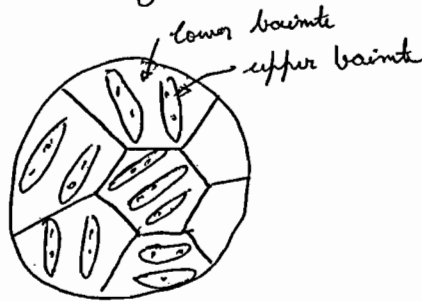
Hot rolled steel is 100% pearlite, which is lamellar structure of ferrite & iron carbide

(c)

(i) Martensite



(ii) Mixture of upper & lower bainite



(d) Cracks form on the surface due to thermal cycling of the brittle ~~2000~~ ~~2000~~ ~~2000~~ M_s . They do not penetrate much.

(i) The intensity of thermal cycling reduces with depth

(ii) The bainite below the surface is tougher than the ~~near~~ surface martensite.

Numerical answers to Paper 3 (2009)

1. (c) $\sigma_x = Y \left[1 - \exp\left(\frac{\ell - x}{L}\right) \right]$, where $L = \frac{\bar{h}}{\mu + \tan \alpha}$
- (d) $F = 2\bar{h}Y \left[\exp\left(\frac{(\mu + \tan \alpha)\ell}{\bar{h}}\right) - 1 \right]$
2. (b) Fraction of eutectic mix = 0.5
 Weight fraction of α in eutectic mixture = 0.768
 Weight fraction of β in eutectic mixture = 0.232
3. (b)(ii) $\frac{\dot{\nu}\eta}{G_2} + \left(\frac{G_1 + G_2}{G_2}\right)\tau = \eta\dot{\gamma} + G_1\gamma$
- (b)(iii) $\gamma = \frac{\tau_0}{G_2}$
- $$\gamma = \frac{\tau_0}{G_e} - \left[\frac{\tau_0}{G_e} - \frac{\tau_0}{G_2} \right] \exp\left(-\frac{G_1 t}{\eta}\right),$$
- (b)(iv)
 where $\frac{1}{G_e} = \frac{1}{G_1} + \frac{1}{G_2}$
5. (b)(i) Requires carbon in the range 0.5 to 1.02 wt%
- (b)(ii) time = 10.18mins.
- (b)(iii) peak at $x = 94 \mu\text{m}$