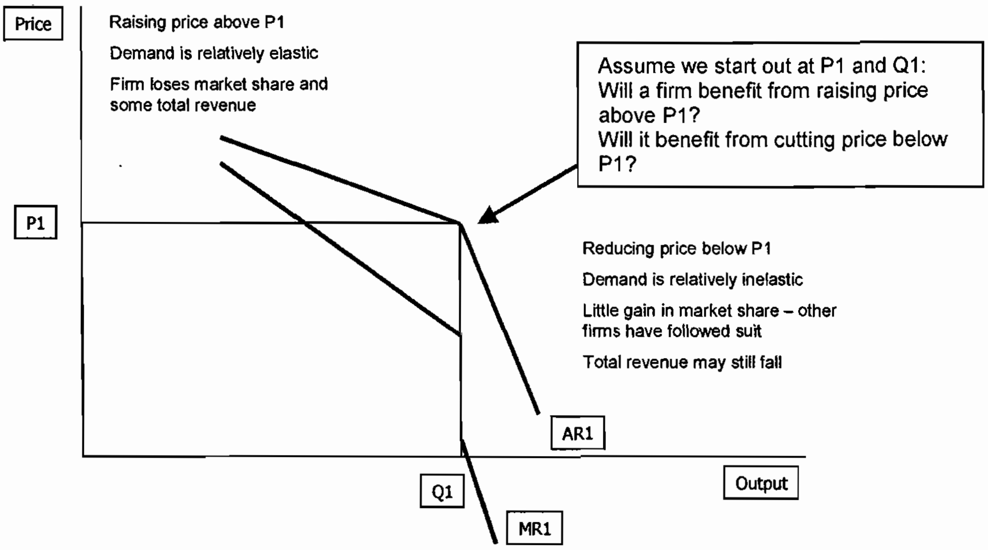
Section A *Introductory Business Economics*

1 (a) Using an appropriate diagram, describe the kinked demand curve theory. [6]

With the kinked demand theory, as shown in the diagram below, firms believe that their competitors would follow them if they were to reduce their price below the prevailing market price, and that it is therefore not possible to gain much in the way of extra sales by reducing price (i.e. demand is relatively inelastic below the market price). Also, as shown below, firms believe that their competitors would not follow them if they were to raise their price above the prevailing market price (i.e. demand is relatively elastic above the market price).



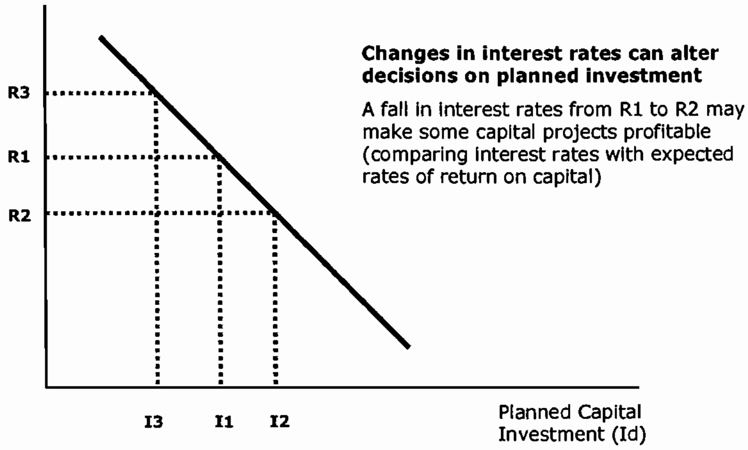
(b) Under what circumstance, might monopolies be beneficial for consumers? [5]

The potential benefits of monopolies for consumer include the following. First, the ability to exploit economies of scale (this can be illustrated with a diagram) which can lead to lower prices. This can be discussed with examples including the potential benefits from natural monopolies. Second, the ability to undertake significant research and development spending which can lead to higher levels of innovation and better products and services for consumers.

(c) What impact would the following have on the level of investment in the macroeconomy:

i; a fall in the rate of interest; [3]

As shown in the diagram below, a decrease in the interest rate will increase the amount of autonomous investment in the macro economy. This is because the downward sloping marginal efficiency of capital curve indicates that there are more profitable investment opportunities at lower interest rates.



ii; a rise in uncertainty of consumers; [3]

A rise in uncertainty is likely to lead to fall in consumption which is a major component of aggregate demand. The Accelerator model suggests that this can then lead to a fall in investment. Net capital investment is the amount by which the required capital stock changes. It follows that the amount of investment depends on the size of the expected change in output. If the latter is decreasing, the desired size of the capital stock needs to be reduced and investment will fall.

iii; a Government subsidy on investment. [3]

A Government subsidy on investment will reduce the cost and should increase the level of investment. Candidates can refer to examples such as the R&D tax credit and may observe that such subsidies may create inefficiencies as investment will not solely be determined by market forces.

2 (a) Explain the concept of profit maximisation? [4]

Profits are maximised when the next unit produced and sold, adds as much to total revenue as it does to total cost. Profit maximisation occurs when marginal revenue = marginal cost (MR=MC). If MR exceeds MC profit can be increased by increasing production, if MC exceeds MR profit can be increased by cutting back on production.

(b) Under what circumstances may firms not seek to maximise profits? [6]

Insufficient Knowledge
Profit maximisation requires knowledge of cost and revenue conditions in the market so that MR and MC can be found – this may not be possible under conditions of uncertainty.

Managerial Capitalism (the separation of ownership and control)
Profit maximisation assumes that owners control the management of the business: where these roles are separated then the mangers of firms may have other objectives such as sales maximisation. Sales maximisation focuses on behaviour of manager-controlled businesses where salaries and other benefits more closely correlated with sales.

Satisficing Theories
Traditional economic theory assumes there is a single goal but behavioural economists argue differently. Any corporation is an organization with various groups (employees, managers, shareholders, customers) and each group may have different objectives/goals The dominant group at any moment in time can give greater emphasis to their own objectives but maximising behaviour may be replaced by satisficing (satisficing = satisfy + suffice) – i.e. setting minimum acceptable levels of achievement for conflicting objectives.

(c) Using the Keynesian Consumption Function model, explain the potential impact of cutting indirect taxes (such as VAT) on the level of aggregate demand. [5]

In the Keynesian case, current consumption (C) is determined by real current personal disposable income (Y). This reduction in indirect taxes (such as VAT) should lead to a reduction in prices and hence a rise in real current income which will lead to an to increase in consumption and thus a rise in aggregate demand. This may lead to a rise in output or inflation (or possibly both) depending on the level of economic activity in the economy.

(d) Using either the Life Cycle model or the Permanent Income model, explain the potential impact of cutting direct taxes (such as VAT) on the level of aggregate demand. [5]

With both the Life Cycle model and the Permanent Income model use a notion of long-run or normal income. In both, the consumer's consumption is based on his/her long run average income. Thus, temporary income changes will have less impact on current consumption than permanent income changes. Thus the impact of the tax cut will depend on whether it is considered to be temporary (with little impact on consumption) or permanent (when it will lead to rise in consumption). That said, even a temporary change in expenditure tax may encourage the bringing forward of purchases of products that generate utility in the future (such as durables).

P8, Q13

SECTION - B

A city council is considering constructing a metro system comprising two lines. A river runs through the city in a north-south direction, and a metro line is being considered on each side of the river. To the east of the river the ground conditions can be summarised as a stiff clay extending from the ground surface with an undrained shear strength of 150 kN/m^2 at a depth of 12.5m and 250 kN/m^2 at a depth of 25m. To the west of the river the ground conditions can be summarised as 15m of sand overlying a soft clay with an undrained shear strength of approximately 30 kN/m^2 constant with depth. On both sides of the river there are many historic masonry buildings and the water table is close to the ground surface.

You are to make a presentation of a feasibility study for the metro to the city councillors. The scheme is likely to comprise bored tunnels between stations constructed by the cut-and-cover techniques. Give answers to the following points:

(a) What would be suitable construction techniques for the bored tunnels on each side of the river if the axes of the tunnels were at depths of 12.5m and 25m? Illustrate your answers, where appropriate, by consideration of the stability ratio for the tunnels. Under what circumstances could sprayed concrete linings be used? [40%]

(b) What would be key considerations in construction of the stations on each side of the river? Suggest how the walls supporting the soils could be constructed, giving two alternatives? [30%]

(c) If settlements are likely to be significant, why might buildings be damaged and what measures could be taken to prevent this? [30%]

P8 Q1 solutions

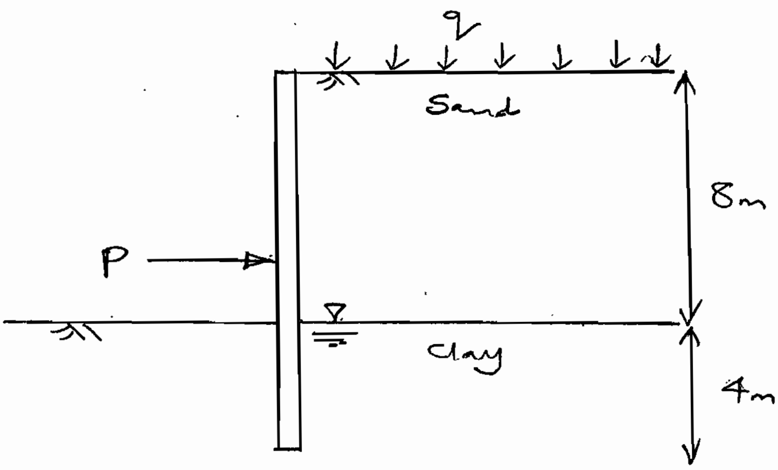
(a) On the east side of the river, the stability ratio for the tunnels at 12.5m and 25m depth (assuming the unit weight γ of the soil is approximately 20 kN/m^3) would be (12.5 x 20)/150 = 1.67 and (25 x 20)/250 = 2.0. These stability ratios are low enough to ensure that open face tunnelling (i.e. no support to the tunnel face during excavation) would be safe. On the west side of the river, the stability ratio for the tunnel at 25m depth (assuming the unit weight γ of the soils is approximately 20 kN/m^3) would be (25 x 20)/30 = 16.7. This is extremely high – if it exceeds about 5 it means that it would be impossible to construct the tunnel under open face conditions, because the tunnel face would not be stable. Closed face tunnelling machines would be necessary, using either slurry machines or earth pressure balance machines to provide support to the tunnel face continuously during excavation. The tunnel at a depth of 12.5m on the west side would be in the sand (stability ratio not relevant) and measures would be needed to prevent water entry – if open face tunnelling, permeation grouting could be undertaken to achieve this – but it would probably be more economic to adopt closed face tunnelling machines. Sprayed concrete linings could only be used on the east side of the river, ie in the stiff clay. [40%]

(b) On the east of the river, the ground is a uniform stiff clay extending from the ground surface. The clay is low permeability and therefore leakage of water through the walls during construction is not an issue. On the west side of the river the top 15m is highly permeable sand which will mean potential leakage into the station could be an issue if there are defects in the walls. The walls could be constructed by the following methods:

- reinforced concrete diaphragm walls, installed in slurry-supported trenches cut in the ground (first the trench is cut – typical plan dimensions 4m x 1m - then a reinforcement cage is lowered into the trench, then concrete is poured via a 'tremie' tube inserted to the bottom of the trench, displacing the slurry)
- reinforced concrete secant pile wall in which alternate 'hard' and 'soft' bored piles are constructed – the 'soft' piles, comprising bentonite-cement are constructed first, and then the 'hard' reinforced concrete piles cutting into the 'soft' piles
- steel sheet piles, driven or jacked into the ground [30%]

(c) Settlement can potentially damage masonry buildings if there is *differential* settlement. This can cause *tensile strains* in the masonry which leads to cracking. Differential settlement can be minimised during tunnel construction by compensation grouting, comprising installation of horizontal steel tubes in the ground between the tunnel and the building foundations. Cement grout is injected from the tubes during tunnelling to prevent the building experiencing damaging differential settlements. Instrumentation is a key part of this process, because monitoring of ground and building movements is essential to inform decisions about when and where to grout. [30%]

Q. 4



(a) Short - term, undrained conditions in clay, $q = 0$

$K_p = \frac{1+\sin\phi'}{1-\sin\phi'}$ for sand. (K_p = passive coefficient)

$\phi' = 30°$ $\Rightarrow$ $K_p = \frac{1+\sin 30°}{1-\sin 30°} = \frac{1.5}{0.5} = 3.0$

At depth of 8m,

$\sigma_h = K_p \sigma_v' = K_p \gamma z$

(Pore pressure, $u = 0$)

$\therefore \sigma_h = 3 \times 18 \times 8 = 432$ kPa

At top of clay,

Passive pressure

$\sigma_h = \gamma z + 2 S_u$

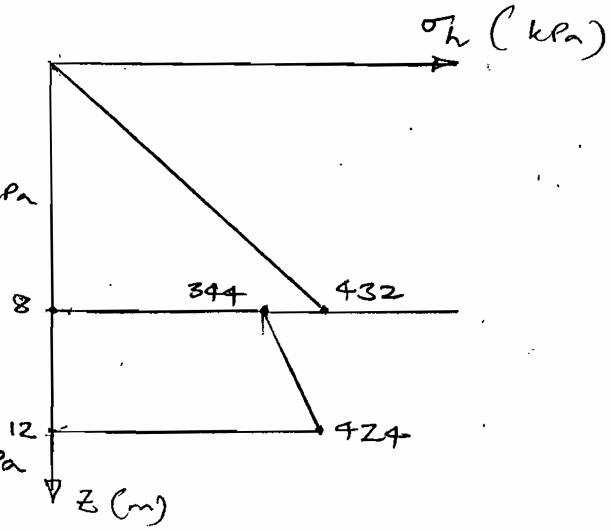
$= (18 \times 8) + 2 \times 100$

$= 144 + 200 = 344$ kPa

At bottom of wall

$\sigma_h = 144 + (4 \times 20) + 2 \times 100$

$= 424$ kPa



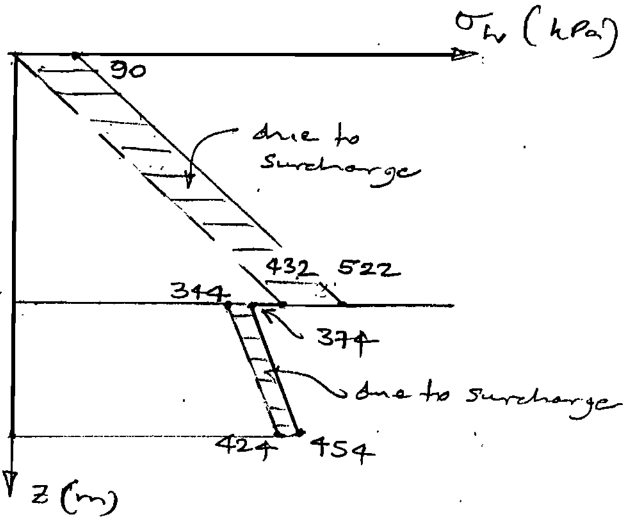
$\therefore P = \left(\frac{1}{2} \times 432 \times 8\right) + \frac{1}{2}(344 + 424) \times 4$ kN/m

$= 1728 + 1536 = \underline{\underline{3264 \text{ kN/m}}}$

②

4(b) Short-term, undrained conditions in clay and $q = 30\ kN/m^2$

Additional passive $\Delta\sigma_h$ pressure caused by surcharge:



In sand

$\Delta\sigma_h = K_p . q$

$= 3 \times 30 = 90\ kPa$

In clay

$\sigma_h = \gamma z + q + 2 s_u$

$\therefore \Delta\sigma_h = q = 30\ kPa$

∴ additional passive resistance

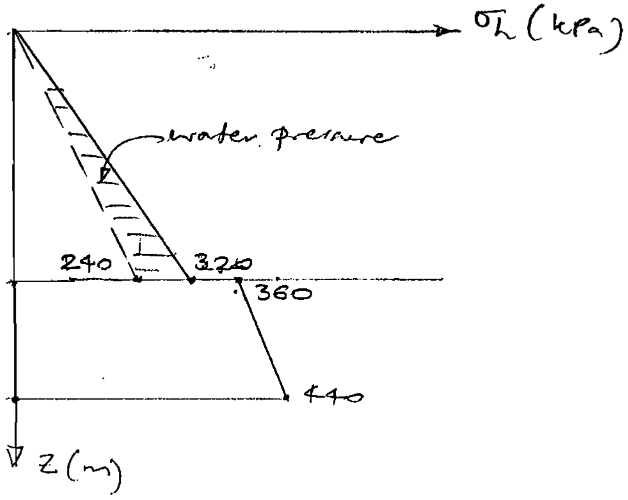
$\Delta P = (90 \times 8) + (30 \times 4)\ kN/m$

$= 720 + 120 = 840\ kN/m$

Total $P = 3264 + 840 = \underline{\underline{4104\ kN/m}}$
(from (a))

4(c) Inundated sand, undrained conditions in clay, $q = 0$

passive $\sigma_h' = K_p \sigma_v'$ in sand



at $z = 8$, $\sigma_v = 20 \times 8 = 160\ kPa$

$u = 10 \times 8 = 80\ kPa$

$\therefore \sigma_v' = 160 - 80 = 80\ kPa$

$\sigma_h' = 3 \times 80 = 240\ kPa$

$\sigma_h = \sigma_h' + u = 240 + 80 = 320\ kPa$

passive pressure at top of clay $= \gamma z + 2 s_u$

$= (20 \times 8) + 2 \times 100$

$= 160 + 200 = 360\ kPa$

At bottom of wall

$\sigma_h = 160 + (4 \times 20) + 2 \times 100$

$= 440\ kPa$

Total $P = \left(\frac{1}{2} \times 320 \times 8\right) + \frac{1}{2}\left(360 + 440\right) \times 4\ kN/m$

$= 1280 + 1600 = \underline{\underline{2880\ kN/m}}$

4(d) Dry sand, long-term conditions in clay, $q = 0$ ③

Passive pressures in sand same as for (a)

In clay, $K_p = \frac{1 + \sin 20°}{1 - \sin 20°}$

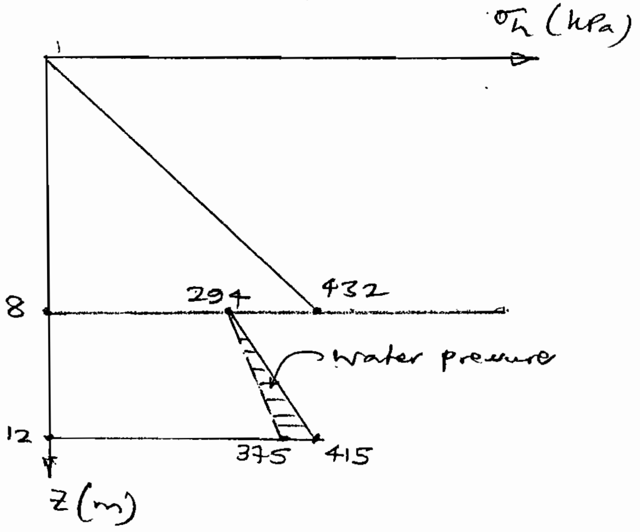
$K_p = \frac{1.342}{0.658} = 2.04$

At top of clay, $u = 0$

Passive $\sigma_h = K_p \sigma_v' = K_p \sigma_v$

$\sigma_v = \gamma z = 18 \times 8 = 144$ kPa

$\sigma_h = 2.04 \times 144 = 294$ kPa



At bottom of clay

$\sigma_v = 144 + (20 \times 4) = 224$ kPa

$u = 4 \times 10 = 40$ kPa

$\sigma_v' = 224 - 40 = 184$ kPa

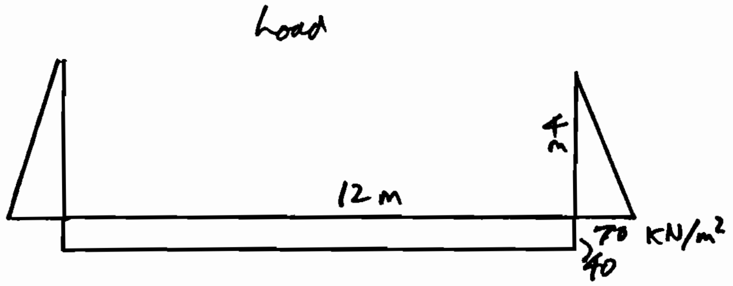
$\sigma_h' = 2.04 \times 184 = 375$ kPa

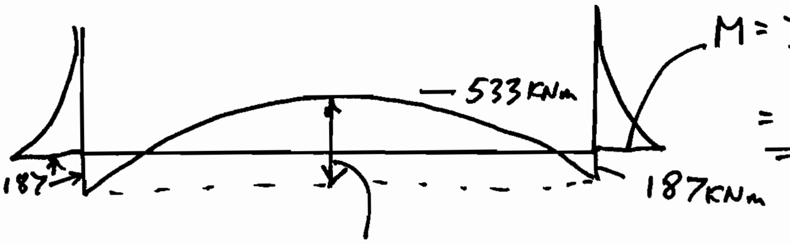
$\sigma_h = \sigma_h' + u = 375 + 40 = 415$ kPa

$P = \underset{\text{(as for (a))}}{1728} + \frac{1}{2}(294 + 415) \times 4$ kN/m

$= 1728 + 1418 = \underline{\underline{3146 \text{ kN/m}}}$

5 (a)





$M = \frac{70}{2} . 4 . \frac{4}{3}$

$= 187$ kNm

$= 40 . \frac{12^2}{8} = 720$ kNm

5 (b) Maximum moment = 533 kNm

∴ $0{\cdot}15\ f_{cu}\ bd^2 = 533$ kNm

$0{\cdot}15 . 40 . 1000 . d^2 = 533 . 10^6$ (Nmm)

$\Rightarrow d = 298$ say 300 mm

∴ Allow 15 mm for ½ bar + 40 mm cover

Overall slab thickness typically 350 → 360 mm

5 (c) Rebar Use data sheet.

At centre of slab

For $d = 300$ mm $x = 0.5$

$$\therefore \quad 0.87 \cdot \underset{\overset{\uparrow}{460}}{f_y} \cdot A_s \underset{\overset{\uparrow}{300}}{d} \underbrace{\left(1 - \frac{x}{2}\right)}_{0.75} = 533.10^6 \text{ mm}$$

$$\Rightarrow A_s = 5919 \text{ mm}^2/\text{m}$$

(Normally would iterate here but since d is very close to minimum value no point).

32 mm bars at 125 crs = 6430 mm²/m

(≈ 2% steel – quite high).

At corners, moment = 187 kNm.

Guess $x = 0.3$

$$0.87 \cdot f_y \cdot A_s \cdot d \left(1 - \frac{0.3}{2}\right) = 187.10^6$$

$$\Rightarrow A_s = 1832 \text{ mm}^2/\text{m}$$

Calculate x $\quad x = 2.175 \cdot \underbrace{\frac{f_y}{f_{cu}}}_{\frac{460}{40}} \cdot \underbrace{\frac{A_s}{bd}}_{\frac{1832}{300.000}} = 0.152$

Recalculate A_s = 1685 mm²/m

16 mm bars @ 125 crs = 1610 ← too small (just) ($x = 0.134$,

@ 100 crs = 2010 ← spacing doesn't match main steel

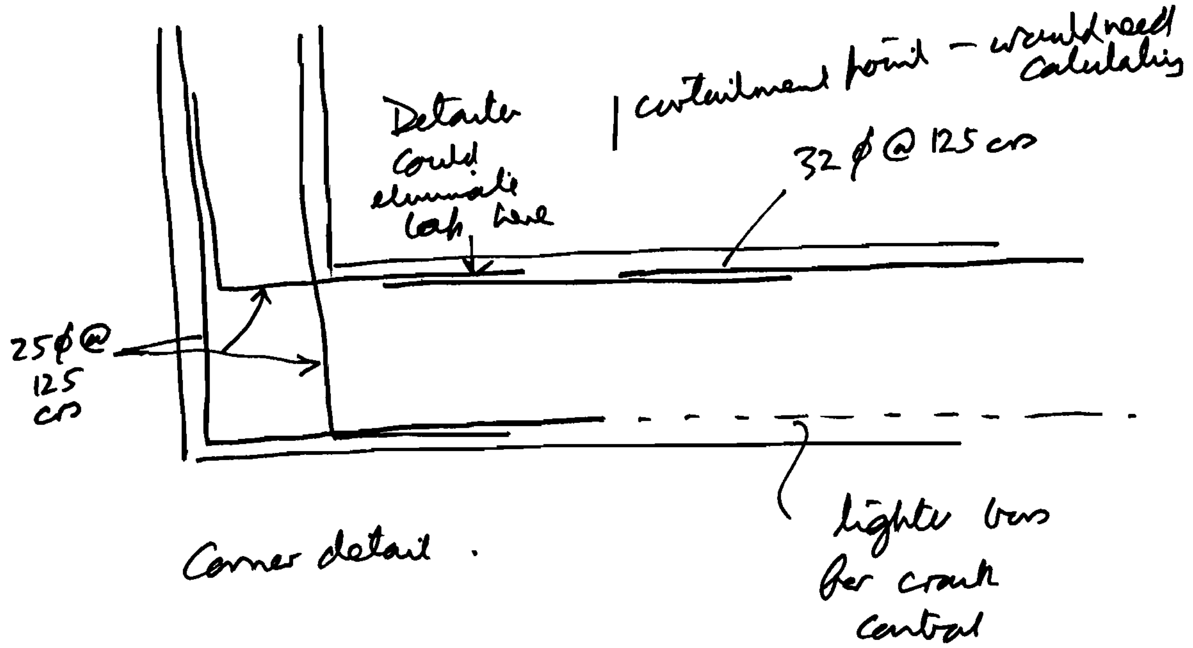
25 mm bars @ 125 crs = 2510 ← Probably used despite larger area of steel

5(d) No point in double reinforcing, since

(i) Peak moment not over a short distance

(ii) Section is already fairly congested

Details



Corner detail.

Part IB Paper 8 Selected Topics June 2009
Section B Civil Engineering Elective

Answers

Q 3a) East side at 12.5m and 25m depths N = 1.67 & 2 respectively (open face tunnelling is possible)
West side at 12.5m depth, sand so closed face tunnelling; at 25m depth N=16.67 so closed face tunnelling is required

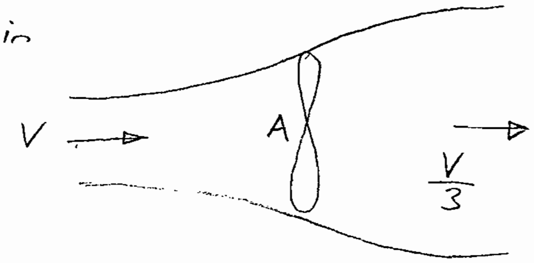
Q 4a) $P = 3264$ kN/m
4b) $P = 4104$ kN/m
4c) $P = 2808$ kN/m
4d) $P = 3146$ kN/m

Q 5a) Max BM at the corner = 187 kNm; Max BM at the centre = 533 kNm
5b) $d = 360$ mm (including cover)
5c) $A_s = 5919$ mm^2/m at the centre; $A_s = 1685$ mm^2/m
5d) no point in double reinforcement as peak BM is not over a short distance and the section is already congested.

SPGM

Section C Mechanics, Materials + Design

6.(a)(i) Betz Limit is the optimum efficiency for a wind turbine. For a turbine of swept area A in the optimal power is 59% of $\frac{1}{2}\rho V^3 A$



(ii) $$5000 = 0.59 \times \frac{1}{2} \times 1.3 \times \frac{\pi 5^2}{4} \times V^3$$

$$\therefore V = \underline{\underline{8.7 \ ms^{-1}}}$$

(iii) From data sheet, thrust $= \frac{4}{9}\rho V^2 A$

$$= \frac{4}{9} \times 1.3 \times 8.7^2 \times \frac{\pi 5^2}{4}$$

$$= \underline{\underline{861 \ N}}$$

(b) (i) Wind turbines have three blades for various reasons. Cost and weight (why have more when three will do?) Minimum thrust (less obstacle to flow). Windmills have many blades because this gives maximum torque at low speed, needed for startup. Two blades is not good because of uneven gyroscopic effects when yawing.

6. (b)(ii) large wind turbines spin slowly (tip speed cannot become supersonic) but generators are efficient at high rpm. Below 5m diameter the spin speed can be fast enough. For example Iskra 5m turbine has no gearbox, runs at 240 rpm

(iii) Consider a 1MW turbine rated to produce 1MW at 14 ms^{-1} wind speed. Unfortunately the wind is not 14 ms^{-1} all the time and since Power $\propto V^3$ a small drop in wind speed is significant. Also, during high wind (storms) the turbine cannot run. Down time for maintenance is another factor. All this transpires that a typical wind turbine generates only around 30% of its rated power aveaged over a year.

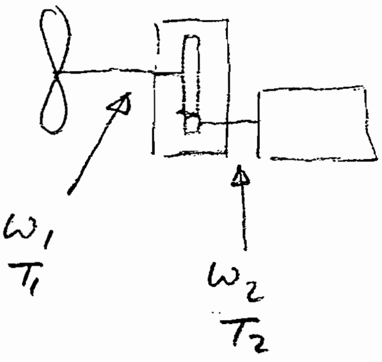
(c)(i) Power = Torque × ω

87 rpm $\equiv$ $\omega_1 = 9.11$ rad s^{-1}

501 rpm $\equiv$ $\omega_2 = 52.5$ rad s^{-1}

$$T_1 = \frac{5 \times 10^5}{9.11} = 55 \text{ KNm}$$

$$T_2 = \frac{5 \times 10^5}{52.5} = 9.5 \text{ KNm}$$



6(c)(ii) Gear ratio $= \frac{501}{87} = \frac{167}{29}$ (ratio of primes)

so suppose gears have 167 and 29 teeth

rotor speed $= \frac{87}{60} = 1.45$ Hz

generator speed $= \frac{501}{60} = 8.35$ Hz

twelvepole generator produces $6 \times 8.35 = 50.1$ Hz voltage

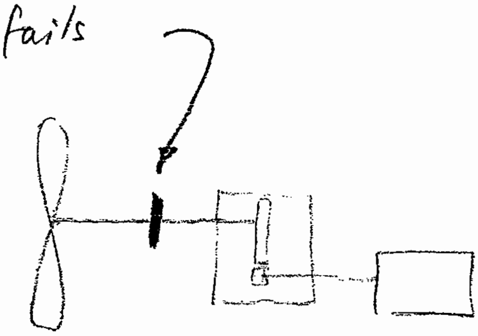
but current peaks at 100.2 Hz *

tooth mesh frequency $= 167 \times 1.45$

$= 242$ Hz *

The two marked * are plum in the audible range

(iii) Emergency brake must stop the rotor even if the gearbox fails



(It is more effective to put a brake near the generator because a smaller torque is required but not acceptable for emergency)

7. (a) Mass flow rate in the annular control volume:

$$\delta\dot{m} = 2\pi r\,\delta r\,\rho\,V_0(1-a)$$

(b) (i) Equate the change in axial momentum in the control volume with the forces acting on the blades

$$\delta N = \delta\dot{m}\left[V_0 - V_0(1-2a)\right]$$
$$= 2\pi r\,\delta r\,\rho\,V_0(1-a)\left[2aV_0\right]$$
$$= 4\pi r\rho V_0^2 a(1-a)\,\delta r$$
$$= B F_N\,\delta r$$

(ii) Consider the change in angular momentum, and equate it to the torque acting on the blades.

$$\delta T = \delta\dot{m}\times 2\omega r a' \times r$$
$$= 2\pi r\,\delta r\,\rho\,V_0(1-a)\times 2\omega r a'\times r$$
$$= 4\pi r^3\rho V_0(1-a)\,\omega a'\,\delta r$$
$$= rBF_T\,\delta r$$

(c) The wind turbine power may be found from the total torque acting on the blades.

Total power $\quad P = \omega T$

$$= \omega\int dT$$
$$= \omega\int_{r=0}^{r=R} 4\pi r^3\rho V_0(1-a)a'\omega\,dr$$
$$= 4\pi\rho V_0(1-a)a'\omega^2\int_0^R r^3\,dr$$
$$= \pi\rho V_0(1-a)a'\omega^2 R^4$$

__Note:__
a and a′
are assumed
to be constant
over the length
of the blade

(c) (cont.)

Coefficient of performance $C_P = \dfrac{P}{\frac{1}{2}\rho V_0^3 \pi R^2}$

$$\therefore C_p = \frac{\pi \rho V_0 (1-a)\, a' \omega^2 R^4}{\frac{1}{2}\rho V_0^3 \pi R^2}$$

$$= 2\frac{\omega^2}{V_0^2}(1-a)\, a' R^2 = 2(1-a)\, a' \left(\frac{\omega R}{V_0}\right)^2$$

$$= 2(1-a)\, a' \lambda^2 = 2(1-0.2)\times 0.005 \times 8^2$$

$$= 0.512$$

$\Bigg\{$ Alternatively $P = V_0(1-a)\int \delta N = V_0(1-a)\int_0^R 4\pi\rho V_0^2 a(1-a)\, r\, dr$

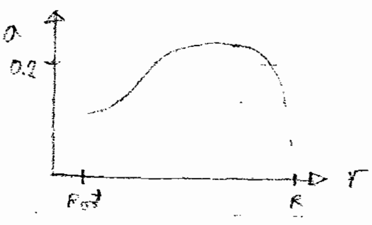
$$= 4\pi\rho V_0^3 a(1-a)^2 \frac{R^2}{2}$$

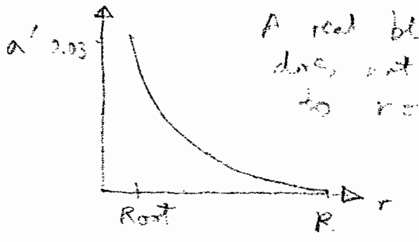
$$\therefore C_p = \frac{2\pi\rho V_0^3 a(1-a)^2 R^2}{\frac{1}{2}\rho V_0^3 \pi R^2} = 4a(1-a)^2$$

$$= 4\times 0.2\times 0.8^2 = 0.512 \Bigg\}$$

(d) The value obtained for $C_p = 0.512 < 0.59 = \frac{16}{27}$ which is the theoretical "Betz" limit. So this is a high, but theoretically possible efficiency.

The assumptions that a and a' are constant along the length of the blades are not realistic. Typical plots for a and a' are:



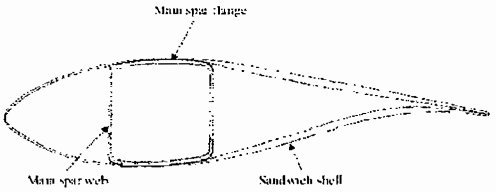


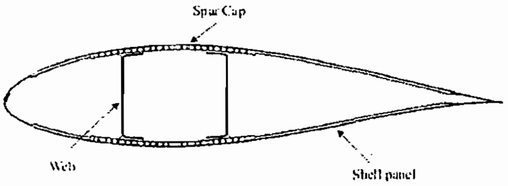
A real blade also does not continue to r = 0.

8. (a) [This answer longer than expected.] Structural design needs to give bending stiffness and strength along with possible centrifugal and self weight loading. Due to scaling the self-weight tends to only come in at larger scales. Moreover stiffness tends to dominate over strength at large scales.

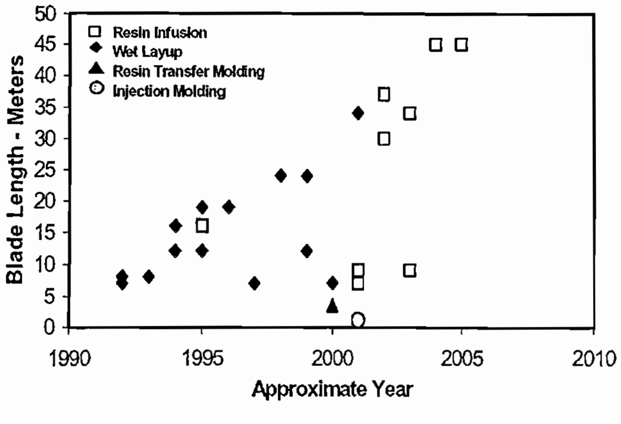
The envelope of the structure is strongly influenced by aerodynamic considerations, so that the outer shall has a typical aerofoil section. At the same time the role of stiffness and strength, particularly for larger blades, dictates that an optimum shape tends to be tapered, keeping a straight section would lead to excessive root moments and deflections due to the large load at the tip. The actual shape is a compromise. [Compare with traditional windmills on a smaller scale, where the stiffness is presumably not so important, so that the blades tend to be straight.]

To achieve appropriate stiffness in bending, use a shape-efficient structure, with the middle hollow and with stiff spars (of unidirectional material) running along the length of the blade. For torsional stiffness the shell needs to be sufficiently thick and made of biaxial or triaxial layup (if composite material). Also the shell needs to be strong and stiff enough to accommodate local loads and not fail. Typical designs for large blades have a form as shown, combining the spar and shell elements, with a central box section or web taking shear. The bigger the blade the more room for manoeuvre in terms of more sophisticated structural design of the various elements.

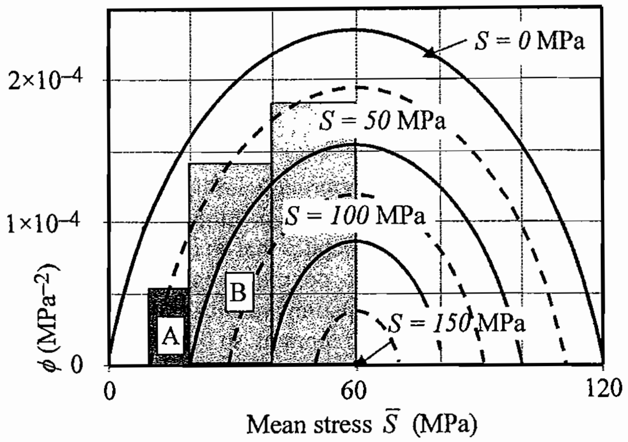




Manufacturing routes depend on the material chosen and the construction type, see figure below taken from the notes. For composites (whether natural or manmade) generally used for blades there are a number of routes typically used as detailed in the figure. Sketches of the various routes are in the lecture notes and standard text books. The difficulty and cost of using closed moulds for resin transfer moulding makes this generally unattractive. For large blade sizes wet layup is very time consuming, resin infusion (where the dry fabric is laid up by machine and resin is forced through the dry lay-up under vacuum) is faster and of better quality.



(b)(i) Alternating stress could arise from either the alternate flapwise bending load due to self weight of the blade or due to variable wind speeds. Meanwhile mean stress could be due to the constant component of wind speed, or due to centrifugal forces. In any case the variation seen in these loads is due to the intermittent nature of the wind giving a variation in loading.



(b)(ii) Need to draw up a table of the probability of being in a range of mean and alternating stresses. One possible way of doing this is to add intermediate curves for S=25, 75 and 125, then add height bars representing the mean height over the corresponding interval. The probability p of being in the range 0<S<50 and $0<\bar{S}<20$ is then the height times the width of box A times the length of the box (50 MPa), while for 0<S<50 and $20<\bar{S}<40$ the corresponding area of B. Note that the sum of the probabilities should sum to 1, in this case it sums to 0.98 which is within tolerance.

Probability p

	0<S<50	50<S<100	100<S<150
$0<\bar{S}<20$	10×50×0.6×10⁻⁴	0	0
$20<\bar{S}<40$	20×50×1.4×10⁻⁴	10×50×0.5×10⁻⁴	0
$40<\bar{S}<60$	20×50×1.8×10⁻⁴	20×50×1.0×10⁻⁴	10×50×0.3×10⁻⁴
$60<\bar{S}<80$	20×50×1.8×10⁻⁴	20×50×1.0×10⁻⁴	10×50×0.3×10⁻⁴
$80<\bar{S}<100$	20×50×1.4×10⁻⁴	10×50×0.5×10⁻⁴	0
$100<\bar{S}<120$	10×50×0.6×10⁻⁴	0	0

For the first element in the table $p = 0.03$

Convert stress range S to zero mean stress equivalent values S_{eq} using Goodman

$$S = S_{eq}\left(1 - \frac{\bar{S}}{\sigma_{ts}}\right)$$

For the first element S = 25 MPa, $\bar{S}$ = 10 MPa, S_{eq} = 25/0.96=25.4 MPa

Number to failure using $N_{fi} = \left(\frac{S_0}{Seq}\right)^m = \left(\frac{600}{Seq}\right)^{12}$ $N_{fi} = 2.43\times10^{16}, p = 0.03$

Miner's rule: find lifetime used up by one month ($N_i = p\times2\times10^6$)

For the first element in the table $N_i/N_{fi} = 2.47\times10^{-12}$

Total life =1/(12α)=<u>12 years</u>, with $\alpha = \sum_i \frac{N_i}{N_{fi}}$.(summing up over all the elements in the table, factor of 12 to convert to years).

(iii) Errors likely to be caused by inaccuracies in material properties (this will be a key problem), variability in wind load, difficulty in estimating stresses in structure, problems with defects in structure.

(9) a) 'Natural' force scale $A p_{02}$, 'natural' velocity scale $\sqrt{\gamma R T_{02}} \propto \sqrt{C_p T_{02}}$

$$\Pi_4 = \frac{\text{Momentum flux}}{\text{Force}} = \frac{\dot{m}\sqrt{C_p T_{02}}}{A p_{02}}$$

$$\Pi_5 = \frac{\text{Fuel energy flux}}{\text{Force} \times \text{velocity}} = \frac{\dot{m}_f \, LCV}{A p_{02} \sqrt{C_p T_{02}}}$$

[4

b) The nozzle should be choked but shock free and the fuel flow must be neglected $\dot{m}_f \ll \dot{m}_{air}$. constant gas properties $(C_p, \gamma \ldots)$ [

c) The net thrust is that transmitted to the airframe

$F_N = (\dot{m}_{air} + \dot{m}_f) V_{jet} - \dot{m}_{air} V_{aircraft}$

$F_G =$ "

~~$F_G = F_N$ for static rig test~~

using $F_G + p_a A$ means dimensionless group wholly determ by conditions inside the engine.

[4]

d) i) $$\left(\frac{\dot{m}_{air}\sqrt{C_p T_{02}}}{p_{02} A}\right)_T = \frac{70 \sqrt{1005 \times 288}}{(101.3 \times 10^3)(0.175)} = 2.12$$

T – Test

$$\left(\frac{F_G + p_a A_N}{p_{02} A}\right)_T = \frac{50 \times 10^3 + (101.3)(0.175)^{\times 10^3}}{(101.3 \times 10^3)(0.175)} = 3.82$$

$$\dot{m}_{air,c} = \left(\frac{2.12 \quad p_{02} A}{\sqrt{C_p T_{02}}}\right)_C$$

C – cruise

$M = 0.8$

$$T_{02c} = 217\left(1 + \frac{\gamma-1}{2} 0.8^2\right) = 244.8\text{ K}$$

$$p_{02c} = 22.7\left(1 + \frac{\gamma-1}{2} 0.8^2\right)^{\gamma/(\gamma-1)} = 34.6\text{k}$$

$$\dot{m}_{air,c} = \frac{(2.12)(34.6 \times 10^3)(0.7)}{\sqrt{(1005)(244.8)}} = \frac{49602.56}{496} = 103$$

[3]

d)(ii)

$$\left(\frac{F_G + P_a A_N}{P_{02} A}\right)_c = 3.82 \qquad F_G = 3.82\,P_{02}A - P_a A_N$$

$$= (3.82)(34.6\times10^3)(0.7) - 22.7\times10^3(0.7)$$

$$= 77\text{kN}$$

$$F_{N_c} = F_{G_c} - (\dot{m}V)_c$$

$$V = 0.8\sqrt{\gamma RT} = \cancel{25}\ 236.3 \text{ m/s}$$

$$F_{N_c} = 77\times10^3 - (103)\,236.3 = 52 \text{ kN}$$

[3]

d)(iii)

$$\left(\frac{\dot{m}_f\,LCV}{A_N P_{02}\sqrt{C_p T_{02}}}\right)_c = \left(\frac{\dot{m}_f\,LCV}{A_N P_{02}\sqrt{C_p T_{02}}}\right)_T$$

$$\frac{\dot{m}_{fc}}{F_{Nc}} = \frac{A_{NC}}{A_{NT}}\ \frac{P_{02C}}{P_{02T}}\sqrt{\frac{T_{02C}}{T_{02T}}}\ \frac{\dot{m}_{fT}}{F_{NT}}\ \frac{F_{NT}}{F_{NC}}$$

$$SFC_c = 4\ \frac{0.396}{1.013}\sqrt{\frac{244.8}{288}}\ 1.25\ \frac{50}{52} = 1.51 \text{ kg/h/kg}$$

[3]

d)(iv)

$$\left(\frac{Nr}{\sqrt{\gamma R T_{02}}}\right)_T = \left(\frac{Nr}{\sqrt{\gamma R T_{02}}}\right)_c$$

$$\frac{20000\,r_T}{\sqrt{288}} = \frac{N_c r_c}{\sqrt{244.8}}$$

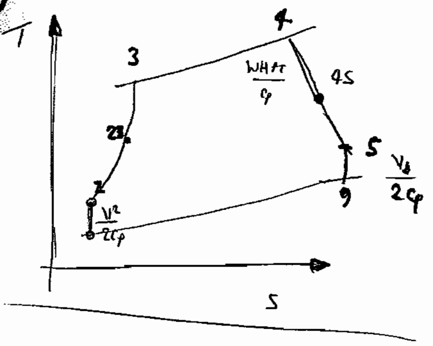
$$N_c = 2\times10^4\ \frac{\sqrt{244.8}}{\sqrt{288}}\ \frac{r_T}{r_c} = 2\times10^4\sqrt{\frac{244.8}{288}}\ 0.5$$

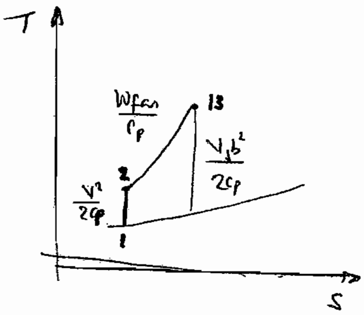
$$= \underline{9216} \text{ rpm}$$

[4]

①

a)





[4]

b) $M = 0.85$ $\quad T_{02} = T_A\left\{1 + \frac{\gamma-1}{2}M^2\right\} = 259.5\,K$

$$p_{02} = \underset{0.287\,bar}{p_A}\left\{1 + \frac{\gamma-1}{2}M^2\right\}^{\gamma/\gamma-1} = 0.46\ bar.$$

[2]

c) $T_{023} = T_{02} + \Delta T_{0f} = 295.5 + 41.44 = 300.9\,K$

$$T_{03} - T_{023} = T_{023}\left\{\frac{25^{\frac{\gamma-1}{\gamma}} - 1}{0.9}\right\} = 504.3\,K$$

(Temp drop in core turbine)

$T_{03} = 300.9 + 504.3 = 805.2\,K$

[3]

d) $W_{T,HPT} = W_{c,HPC} \rightarrow c_p(T_{04} - T_{045}) = c_p(T_{03} - T_{023})$

$$T_{045} = T_{04} - (T_{03} - T_{023}) = 1450 - 504.3 = 945.7\,k$$

[2]

For core turbine

$$\Delta T_0 = 504.3 = \eta_t \underset{1450}{T_{04}}\left\{1 - \left(1/r_t\right)^{\frac{\gamma-1}{\gamma}}\right\}$$

Rearranging gives $r_t = 5.52$

$p_{03} \simeq p_{04}$, $\quad p_{03} = \overset{(1.6)(25)}{40}\, p_{02}$, $\quad p_{04} = 40\, p_{02}$

$$p_{045} = \frac{p_{04}}{r_t} = \frac{40\,p_{02}}{r_t} = \frac{(40)(0.46\times10^5)}{5.52} = 0.33\ MPa$$

[2]

e) $T_{045} - T_{05} = 361\,K = \eta_t \underset{945.7}{T_{045}}\left\{1 - \left(\frac{1}{r_{LPT}}\right)^{\frac{\gamma-1}{\gamma}}\right\}$

Rearranging gives $r_{LPT} = 6.9 = \frac{p_{045}}{p_{05}}$ $\quad p_{05} = \frac{p_{045}}{6.9} = \frac{0.33\times10^6}{6.9} = 48.21$ kPa

$$T_{05} = T_{045} - 361\,k = 945.7 - 361 = 584.7\,K$$

[2]

$$V_{jc}^2 = 2c_p(T_{05} - T_9) = 2c_p T_{05}\left(1 - \frac{T_9}{T_{05}}\right)$$

$$= \underbrace{2c_p T_{05}}_{8119\times10^3}\left\{1 - \left(\frac{p_9}{p_{05}}\right)^{\frac{\gamma-1}{\gamma}}\right\}$$

98.26×10^3

$$V_{jc} = 402.7 \text{ M/s}$$

[1]

$$\eta_p = \frac{2V}{V_j + V} = \frac{(2)\cdot256.6}{402.7 + 256.6} = 0.778$$

$$V = M\sqrt{\gamma R T_a} = 256.6$$

[1]

---

f) $T_{013} = T_{023} = 300.9 \text{ K}$

LP power balance

$$\cancel{\dot{m}_c c_p}\underbrace{(T_{045} - T_{05})}_{LPT} = \cancel{\dot{m}_c c_p}\underbrace{(T_{023} - T_{02})}_{\text{Fan}\to\text{core}} + \cancel{\dot{m}_c}\,BPR\,\cancel{c_p}\underbrace{(T_{013} - T_{02})}_{\text{Fan to BP jet}}$$

$$(T_{045} - T_{05}) = (1 + BPR)(T_{023} - T_{02})$$

$$BPR = \frac{T_{045} - T_{05}}{T_{023} - T_{02}} - 1 = \frac{945.7 - 584.7}{41.4} - 1$$

$$= \frac{361}{41.4} - 1$$

$$= 7.7$$

[2]

---

g) Assuming $V_{jc} = V_{jb}$ has little influence on global parameters like thrust and specific fuel consumption but the detailed parameters ~~with~~ are more strongly sensitive.

[1]

(11). a) $p_0^{1/\gamma} = c\rho_0$ where p_0 is pressure at sea level and c is a constant. Hence

$$dp = -\frac{p^{1/\gamma}}{c} g\,dz \qquad \int_{p_0}^{p_{\tilde{z}}} \frac{1}{p^{1/\gamma}}dp = -\frac{g}{c}\int_0^{\tilde{z}} dz$$

$$\left[\frac{p^{1-1/\gamma}}{\frac{\gamma-1}{\gamma}}\right]_{p_0}^{p_{\tilde{z}}} = -\frac{g\tilde{z}}{c}$$

$$\left[p^{\frac{\gamma-1}{\gamma}}\right]_{p_0}^{p_{\tilde{z}}} = -\left(\frac{\gamma-1}{\gamma}\right)\frac{g\tilde{z}}{c}$$

$$p_{\tilde{z}}^{\frac{\gamma-1}{\gamma}} - p_0^{\gamma-1/\gamma} = -\left(\frac{\gamma-1}{\gamma}\right)\frac{g\tilde{z}}{c}$$

$$(p_{\tilde{z}})^{\frac{\gamma-1}{\gamma}} = -\left(\frac{\gamma-1}{\gamma}\right)\frac{g\tilde{z}}{c} + p_0^{\gamma-1/\gamma}$$

$$\left(\frac{p_{\tilde{z}}}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = p_0^{-\frac{\gamma-1}{\gamma}}\left(\frac{\gamma-1}{\gamma}\right)\frac{g\tilde{z}}{c} + 1$$

Eliminate $c = p_0^{1/\gamma}/\rho_0$

$$\left(\frac{p_{\tilde{z}}}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{1}{p_0^{\frac{\gamma-1}{\gamma}}}\left(\frac{\gamma-1}{\gamma}\right)\frac{g\tilde{z}}{c} = 1 - \frac{1}{p_0^{\frac{\gamma-1}{\gamma}}}\left(\frac{\gamma-1}{\gamma}\right)\frac{g\tilde{z}}{p_0^{1/\gamma}/\rho_0}$$

$$= 1 - \left(\frac{\gamma-1}{\gamma}\right)\frac{g\hat{z}\rho_0}{p_0}$$

$$p_0/\rho_0 = RT_0$$

$$\frac{p_{\tilde{z}}}{p_0} = \left(1 - \frac{(\gamma-1)}{\gamma}\frac{g\tilde{z}}{RT_0}\right)^{\frac{\gamma}{\gamma-1}} \qquad [7]$$

b)

$$\frac{\partial\left(\frac{ML}{D}\right)_F}{\partial C_L} = F + 2GC_L = 0$$

$$C_L = -\frac{F}{2G} = \frac{+51}{(2)(473)} = 0.54 \qquad [2]$$

c) $C_L = \dfrac{W}{\frac{1}{2}\rho V^2 A} = \dfrac{W}{\frac{1}{2}\rho M^2 \frac{\gamma}{\rho} p A} = \dfrac{W}{\frac{1}{2}\gamma M^2 p A}$

Hence $A = \dfrac{mg}{\frac{1}{2}\gamma M^2 C_L p}$

$\dot{M} = 500 \times 10^3$ kg, $M = 0.83$ (part (b)), $C_L = 0.54$ (part (c))

$$A = \frac{(500 \times 10^3)(9.81)}{(\frac{1}{2})(1.4)(0.83)^2(0.54)p} = \frac{18.8 \times 10^6}{p}$$

$$p = (1.013 \times 10^5)\left\{1 - \left(\frac{0.2857(9.81)9500}{(287.3)(288.5)}\right)\right\}^{3.5}$$

$p = 0.263$ bar

$A = 713.5 \text{ m}^2$ [4]

d) For constant C_L, $\dfrac{W}{\frac{1}{2}\rho V^2 A} = \dfrac{W}{\frac{1}{2}\rho M^2 \gamma \frac{p}{\rho} A} = \text{constant}$

i.e. $\dfrac{W}{p} = \text{const}$ $\quad \dfrac{p_2}{p_1} = \dfrac{W_2}{W_1} = \dfrac{375}{500}$

$p_2 = \frac{3}{4} p_1 = 0.2$ bar

$$1 - \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{gz}{RT_0}\right) = \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$$

$$z = \frac{RT_0}{g}\frac{\gamma}{\gamma - 1}\left(1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)$$

$$z = \frac{(287.3)(288.5)(3.5)}{9.81}\left\{1 - \left(\frac{0.2}{1.013}\right)^{0.2857}\right\}$$

$z = 11$ km [3]

e) Flying close to tropopause where T should be about constant. The equation used here to model atmospheric conditions drastically underestimates T with increasing z. Hence, T at start of cruise probably not used for calculating range.

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \qquad T = T_0\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = 288.5\left(\frac{0.263}{1.013}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T = 196\ K$$

We know $\frac{ML}{D} = 1.2 + 51\,C_L - 47.3\,C_L^2 = 15$

$$\frac{V_\infty L}{D} = \frac{ML}{D}\,a = \frac{ML}{D}\sqrt{\gamma RT} = 15\sqrt{(196)(1.4)287.3}$$

$$= 4.2\times10^3$$

$$s = -\frac{(4.2\times10^3)(3600)}{(9.81)\,0.5}\ln\left(\frac{3}{4}\right) = 900\ km$$

[3]

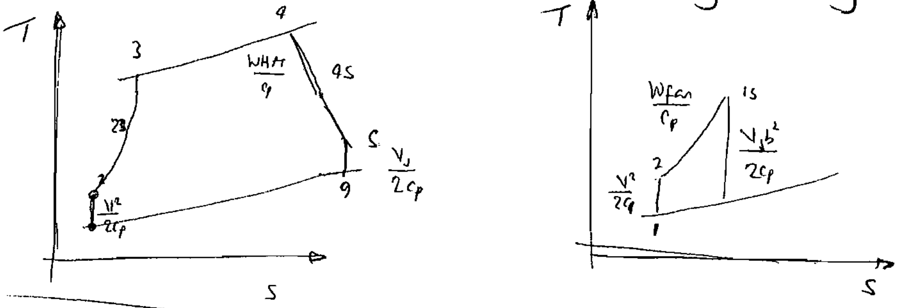
f) $\left(\frac{VL}{D}\right)_{max}$ gives the maximum range. Hence plane will try to fly with this value constant.

The fuel consumption at take off and landing are assumed negligible.

[1]

⑲ SECTION D Aerothermal Engineering

a)



[4]

b) $M = 0.85$ $T_{02} = T_A\left\{1 + \frac{\gamma-1}{2}M^2\right\} = 259.5\,K$

$p_{02} = \underset{0.287bar}{p_A}\left\{1 + \frac{\gamma-1}{2}M^2\right\}^{\gamma/\gamma-1} = 0.46$ bar. [2]

c) $T_{023} = T_{02} + \Delta T_{0f} = 295.5 + 41.44 = 300.9k$

$T_{03} - T_{023} = T_{023}\left\{\frac{25^{\frac{\gamma-1}{\gamma}} - 1}{0.9}\right\} = \underset{\uparrow}{504.3\,K}$

(Temp drop in core turbine)

$T_{03} = 300.9 + 504.3 = \underline{805.2\,K}$ [3]

d) $W_{T,HPT} = W_{c,HPC} \rightarrow c_p(T_{04} - T_{045}) = c_p(T_{03} - T_{023})$

$T_{045} = T_{04} - (T_{03} - T_{023}) = 1450 - 504.3 = 945.7k$

[2]

For core turbine

$\Delta T_0 = 504.3 = \eta_t \underset{1450}{T_{04}}\left\{1 - \left(\frac{1}{r_t}\right)^{\frac{\gamma-1}{\gamma}}\right\}$

Rearranging gives $r_t = 5.52$

$p_{03} \simeq p_{04}$, $p_{03} = 40\,p_{02}$, $p_{04} = 40\,p_{02}$

$p_{045} = \frac{p_{04}}{r_t} = \frac{40\,p_{02}}{r_t} = \frac{(40)(0.46 \times 10^5)}{5.52} = \underline{0.33\,MPa}$ [2]

e) $T_{045} - T_{05} = 361\,K = \eta_t \underset{945.7}{\underset{\uparrow}{T_{045}}}\left\{1 - \left(\frac{1}{r_{LPT}}\right)^{\frac{\gamma-1}{\gamma}}\right\}$

Rearranging gives $r_{LPT} = 6.9 = \frac{p_{045}}{p_{05}}$ $p_{05} = \frac{p_{045}}{6.9} = \frac{0.33 \times 10^6}{6.9} = 48.26$ kPa

$T_{05} = T_{045} - 361\,k = 945.7 - 361 = \underline{584.7\,k}$

[2]

$$V_{jc}^2 = 2c_p (T_{05} - T_9) = 2c_p T_{05} \left(1 - \frac{T_9}{T_{05}}\right)$$

$$= \underbrace{2c_p T_{05}}_{8119 \times 10^3} \left\{ 1 - \left(\frac{p_9}{\underbrace{p_{05}}_{98.26 \times 10^3}}\right)^{\frac{\gamma-1}{\gamma}} \right\}$$

$$V_{jc} = 402.7 \quad \text{m/s}$$

[1]

$$\eta_p = \frac{2V}{V_j + V} = \frac{(2)\,256.6}{402.7 + 256.6} = 0.778$$

$$V = M\sqrt{\gamma R T_a} = 256.6$$

[1]

---

g) $T_{013} = T_{023} = 300.9$ K

LP power balance

$$\underbrace{\cancel{\dot m_c c_p} (T_{045} - T_{05})}_{LPT} = \underbrace{\cancel{\dot m_c c_p} (T_{023} - T_{02})}_{\text{Fan} \to \text{core}} + \underbrace{\cancel{\dot m_c}\, BPR \,\cancel{c_p} (T_{013} - T_{02})}_{\text{Fan to BP jet}}$$

$$(T_{045} - T_{05}) = (1 + BPR)(T_{023} - T_{02})$$

$$BPR = \frac{T_{045} - T_{05}}{T_{023} - T_{02}} - 1 = \frac{945.7 - 584.7}{300.9 - 259.5} - 1$$

$$= \frac{361}{41.4} - 1$$

$$= 7.7$$

[2]

---

h) Assuming $V_{jc} = V_{jb}$ has little influence on global parameters like thrust and specific fuel consumption but the detailed parameters ~~will~~ are more strongly sensitive.

[1]

11 a) $p_0^{1/\gamma} = k\rho_0$ where p_0 is pressure at sea level and c is a constant. Hence

$$d\rho = -\frac{p^{1/\gamma}}{k} g\,dz \qquad \int_{p_0}^{p_{\tilde{z}}} \frac{1}{p^{1/\gamma}}dp = -\frac{g}{c}\int_0^{\tilde{z}} dz$$

$$\left[\frac{p^{1-1/\gamma}}{\frac{\gamma-1}{\gamma}}\right]_{p_0}^{p_{\tilde{z}}} = -\frac{g}{c}\tilde{z}$$

$$\left[p^{\frac{\gamma-1}{\gamma}}\right]_{p_0}^{p_{\tilde{z}}} = -\left(\frac{\gamma-1}{\gamma}\right)\frac{g\tilde{z}}{c}$$

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$$(p_{\tilde{z}})^{\frac{\gamma-1}{\gamma}} = -\left(\frac{\gamma-1}{\gamma}\right)\frac{g\tilde{z}}{c} + p_0^{\gamma-1/\gamma}$$

$$\left(\frac{p_{\tilde{z}}}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \frac{-}{p_0}\left(\frac{\gamma-1}{\gamma}\right)\frac{g\tilde{z}}{c} + 1$$

Eliminate $c = p_0^{1/\gamma}/\rho_0$

$$\left(\frac{p_{\tilde{z}}}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{1}{p_0^{\frac{\gamma-1}{\gamma}}}\left(\frac{\gamma-1}{\gamma}\right)\frac{g\tilde{z}}{c} = 1 - \frac{1}{p_0^{\frac{\gamma-1}{\gamma}}}\left(\frac{\gamma-1}{\gamma}\right)\frac{g\tilde{z}}{p_0^{1/\gamma}/\rho_0}$$

$$= 1 - \left(\frac{\gamma-1}{\gamma}\right)\frac{g\tilde{z}\rho_0}{p_0}$$

$p_0/\rho_0 = RT_0$

$$\frac{p_{\tilde{z}}}{p_0} = \left(1 - \frac{(\gamma-1)}{\gamma}\frac{g\tilde{z}}{RT_0}\right)^{\frac{\gamma}{\gamma-1}}$$ [7]

b)

$$\frac{\partial\left(\frac{ML}{D}\right)_F}{\partial C_L} = F + 2GC_L = 0$$

$$C_L = -\frac{F}{2G} = \frac{-51}{(2)(47\cdot3)} = 0\cdot54$$ [2]

c) $C_L = \dfrac{W}{1/2\rho V^2 A} = \dfrac{W}{1/2 \rho M^2 \frac{\gamma}{\rho} p A} = \dfrac{W}{1/2 \gamma M^2 p A}$

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$$p = (1.013 \times 10^5)\left\{ 1 - \left(\frac{0.2857\,(9.81)\,9500}{(287.3)(288.5)}\right)\right\}^{3.5}$$

$p = 0.263$ bar

$A = 713.5 \ m^2$ [3]

d) For constant C_L, $\dfrac{W}{1/2\rho V^2 A} = \dfrac{W}{1/2 \rho M^2 \gamma \frac{p}{\rho} A}$ = constant

So $\dfrac{W}{p}$ = const $\quad \dfrac{p_2}{p_1} = \dfrac{W_2}{W_1} = \dfrac{375}{500}$

$p_2 = 3/4 \, p_1 = 0.2$ bar

$$1 - \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{gz}{RT_0}\right) = \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$$

$$z = \frac{RT_0}{g}\frac{\gamma}{\gamma - 1}\left(1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right)$$

$$z = \frac{(287.3)\ (288.5)(3.5)}{9.81}\left\{ 1 - \left(\frac{0.2}{1.013}\right)^{0.2857}\right\}$$

$z = 11$ M [2]

e) Flying close to tropopause where T should be about constant. The equation used here to model atmospheric conditions effectively underestimates T with increasing z. Hence, T at start of cruise probably best used for calculating range.

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \qquad T = T_0\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = 288.5\left(\frac{0.263}{1.013}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T = 196\text{ K}$$

We know $\frac{ML}{D} = 1.2 + 51\,C_L - 47.3\,C_L^2 = 15$

$$\frac{V_\infty L}{D} = \frac{ML}{D}a = \frac{ML}{D}\sqrt{\gamma RT} = 15\sqrt{(196)(1.4)287.3}$$

$$= 4.2\times10^3$$

$$s = -\frac{(4.2\times10^3)(3600)}{(9.81)\,0.5}\ln\left(\frac{3}{4}\right) = 900\text{ km}$$

[3]

f) $\left(\frac{VL}{D}\right)_{max}$ gives the maximum range. Hence plane will try to fly with this value constant.

The fuel consumption at take off and landing are assumed negligible.

12a) 'Natural' force scale $A p_{02}$, 'natural' velocity scale $\sqrt{\gamma R T_{02}} \propto \sqrt{C_p T_{02}}$

$$\Pi_4 = \frac{\text{Momentum flux}}{\text{Force}} = \frac{\dot{m}\sqrt{C_p T_{02}}}{A p_{02}}$$

$$\Pi_5 = \frac{\text{Fuel energy flux}}{\text{Force} \times \text{velocity}} = \frac{\dot{m}_f \, LCV}{A p_{02}\sqrt{C_p T_{02}}}$$

[4]

b) The nozzle should be choked but shock free and the fuel flow must be neglected $\dot{m}_f \ll \dot{m}_{air}$. [2

c) The net thrust is that transmitted to the airframe

$F_N = (\dot{m}_{air} + \dot{m}_f) V_{jet} - \dot{m}_{air} V_{aircraft}$

$F_G = \quad "$

$F_G = F_N$ for static rig test

using $F_G + p_a A$ means dimensionless group wholly determin by conditions inside the engine. [4]

d)
i) $$\left(\frac{\dot{m}_{air}\sqrt{C_p T_{02}}}{p_{02} A}\right)_T = \frac{70\sqrt{1005 \times 288}}{(101{\cdot}3 \times 10^3)(0{\cdot}175)} = 2{\cdot}12$$

T ~ Test

$$\left(\frac{F_G + p_a A_N}{p_{02} A}\right)_T = \frac{50 \times 10^3 + (101{\cdot}3)(0{\cdot}175)\times 10^3}{(101{\cdot}3 \times 10^3)(0{\cdot}175)} = 3{\cdot}82$$

$$\dot{M}_{air,c} = \left(\frac{2{\cdot}12 \; p_{02} A}{\sqrt{C_p T_{02}}}\right)_c$$

c ~ cruise

$M = 0{\cdot}8$

$$T_{02c} = 217\left(1 + \frac{\gamma-1}{2} 0{\cdot}8^2\right) = 244{\cdot}8 \text{ K}$$

$$p_{02c} = 22{\cdot}7\left(1 + \frac{\gamma-1}{2} 0{\cdot}8^2\right)^{\gamma/\gamma-1} = 34{\cdot}6 \text{k}$$

$$\dot{M}_{air,c} = \frac{(2{\cdot}12)(34{\cdot}6 \times 10^3)(0{\cdot}7)}{\sqrt{(1\,005)(244{\cdot}8)}} = \frac{49602{\cdot}56}{496} = 103$$

[3]

d)(ii)

$$\left(\frac{F_G + P_a A_N}{P_{02} A}\right)_c = 3 \cdot 82 \qquad F_G = 3{\cdot}82\, P_{02} A - P_a A_N$$

$$= (3{\cdot}82)(34{\cdot}6 \times 10^3)(0{\cdot}7) - 22{\cdot}7 \times 10^3 (0{\cdot}7)$$

$$= 77 \text{kN}$$

$$F_{N_c} = F_{G_c} - (\dot{m} V)_c$$

$$V = 0{\cdot}8 \sqrt{\gamma R T} = \cancel{25}\ 236{\cdot}3 \text{ m/s}$$

$$F_{N_c} = 77 \times 10^3 - (103)\, 236{\cdot}3 = 52 \text{ kN}$$

[3]

d)(iii)

$$\left(\frac{\dot{m}_f \, LCV}{A_N P_{02} \sqrt{C_p T_{02}}}\right)_c = \left(\frac{\dot{m}_f \, LCV}{A_N P_{02} \sqrt{C_p T_{02}}}\right)_T$$

$$\frac{\dot{m}_{fc}}{F_{NC}} = \frac{A_{NC}}{A_{NT}} \frac{P_{02C}}{P_{02T}} \sqrt{\frac{T_{02C}}{T_{02T}}} \frac{\dot{m}_f}{F_{NT}} \frac{F_{NT}}{F_{NC}}$$

$$SFC_c = 4 \; \frac{0{\cdot}396}{1{\cdot}013} \sqrt{\frac{244{\cdot}8}{288}} \; 1{\cdot}25 \; \frac{50}{52} = 1{\cdot}51 \text{ kg/h/kg}$$

[3]

d)(iv)

$$\left(\frac{N r}{\sqrt{\gamma R T_{02}}}\right)_T = \left(\frac{N r}{\sqrt{\gamma R T_{02}}}\right)_c$$

$$\frac{20000 \, r_T}{\sqrt{288}} = \frac{N_c r_c}{\sqrt{244{\cdot}8}}$$

$$N_c = 2 \times 10^4 \frac{\sqrt{244{\cdot}8}}{\sqrt{288}} \frac{r_T}{r_c} = 2 \times 10^4 \sqrt{\frac{244{\cdot}8}{288}} \; 0{\cdot}5$$

$$= \underline{9216} \text{ rpm}$$

[1]

SECTION E *Electrical Engineering*

12 (a) An electric field applied between 2 transparent conducting (typically Indium Tin Oxide-coated) plates control the polarization rotation when polarised light is passed through the liquid crystal cell. The electric field places a torque on the liquid crystal molecules trying to orient them parallel to the field, i.e. perpendicular to the cell walls. Although the molecules in the center will retain their parallel order, the majority of the molecules in the center will turn parallel to the light path. Thus a large part of the polarization-rotation vanishes and the cell goes to the opposite transmission state – i.e. black for a normally white cell and clear for a normally black cell.

This principle led to the production of the liquid crystal display (LCD) . This is known as a **passive display** as the action is rather like a light valve where by application of a field across the Liquid crystal layers, light from a **backlight** is allowed to pass through or is prevented from passing through the structure.→

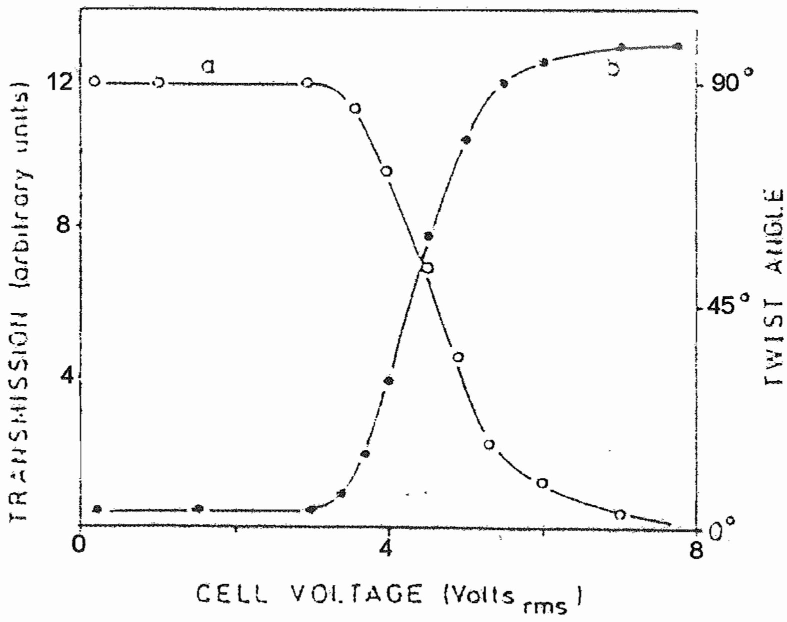
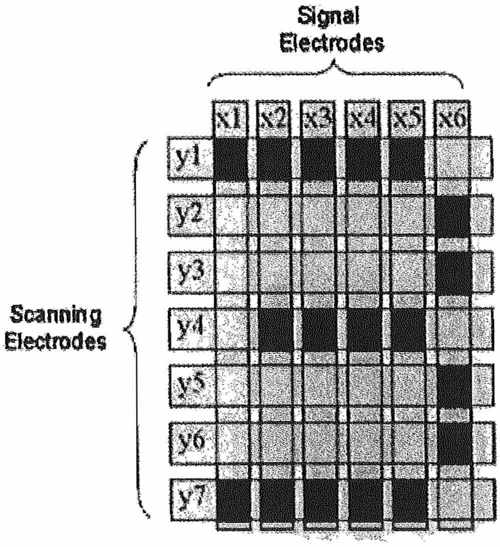


Fig. X Transmission-voltage characteristics of a twisted nematic cell. The right ordinate shows: (a) the angle of rotation of polarization effected by the cell. The left axis shows (b) the intensity of transmited light in the case of parallel polarisers (normally-black mode).

PASSIVE Matrix Addressed LCD

The simplest type of liquid crystal display is the passive matrix addressed LCD, where the front and back cell wall are covered with stripes of conducting material (typically Indium Tin Oxide -ITO). In order to bring a selected pixel into the on-state a voltage is applied to one of the row electrodes and a voltage of opposite polarity onto the column. If the voltages are appropriately chosen only the pixel at the crossing of the electrodes will be selected- see below. (5)



(Passive Matrix LCD)

Passive Matrix Addressed LCD

(b) 3:1 Dynamic Addressing Scheme

Basically as shown in figure X there is a threshold voltage below which the liquid crystal will stay in its "off-state".

In order to switch it "on" a rms voltage above this level will have to be applied. Fig. Y shows a commonly used address scheme for a 3 row-4 column display.

This is the 3:1 scheme where it can be seen that during the first 3 time intervals the 3 rows are addressed one after the other by a voltage of + 2V. When a row is selected the desired "on-state" pixels get a voltage of –V applied to the column electrode. Those which are to remain "off" get +V applied to the column, i.e. for the pixels selected to be on, 3V is applied across the cell and for those to remain off, V volts (i.e. below the threshold voltage) are applied.

3.1 addressing scheme. The top three graphs on the left side show the time dependence of the row electrode voltages which are the same for any selected pattern. The bottom four graphs contain the column voltages which are necessary to generate the select pattern displayed on the right.

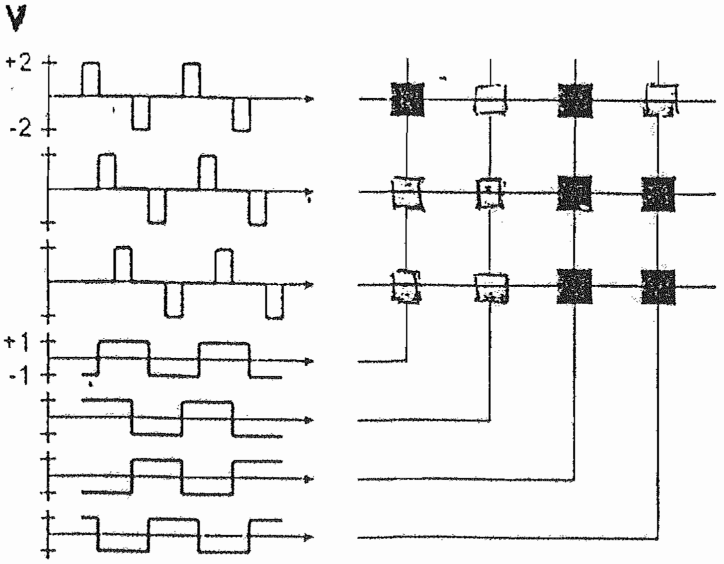


Fig. Y

Since a pixel is selected only during one time interval in three, the rms voltage on the selected pixel is

$$(V^2 + V^2 + (3V)^2)^{1/2} = (11/3)^{1/2} = 1.91 \text{ V}$$

For all pixels not selected the V remains as V. (6)

After the first 3 time intervals the sequence is repeated (but inverted to ensure a zero DC component).

Thus by using a 3:1 scheme a rms voltage which is 91% greater than the "off" voltage is applied to each which we want to be in the "on" state.

It is easy to see if we increase the number of rows significantly (i.e. in any useful display) then the relative voltage difference between selected and unselected pixels will get smaller and smaller – dynamic addressing this way thus becomes impracticable as the contrast differences will become lower and lower (from fig. X). (3)

This led to the use of active matrix addressing.

(c) AMLCD

Here each display pixel is equipped with its own TFT switch. Thus the glass substrate contains a matrix of TFTs whose gates are connected to the row lines and drain electrodes are connected to the column lines. The pixels are not constructed as the intersection of row and column lines (as in the case of passive matrix addressing), but rather by individual electrode regions connected to the sources of the TFTs. The counter electrode extends over the whole pixel area of the opposite glass substrate. See below. The addressing scheme is very straight forward – see Fig. The TFTs simply act as switches. The rows are opened sequentially and when open, the voltage on the column is transferred to the liquid crystal cell. The charge needed to maintain the electric field can be stored in a capacitor located on every pixel. (6)

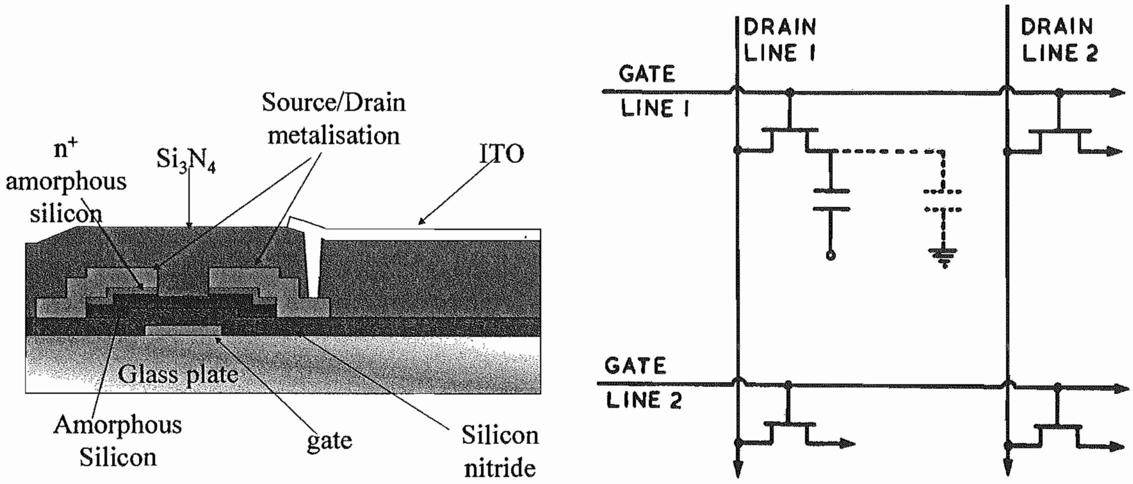


Fig. Z Cross section of the (a-Si:H) TFT in the active matrix and schematic of TFT at each pixel

Q13(a) Scale down by k (k>1)
Density k^2 speed k, power 1.

Other devices - HDDs

(b) $\sigma = ne\mu$ from flux of charge along a pipe.

(c) $\mu = 6.7 \times 10^{-3}$ $m^2/V.s$

(d) $V = L^2/\mu t$,

$I = WdveN = Wd(V/L)\mu eN = (W/L)\ V\mu eN$

So
$P = V.I = V^2\ (W/L)\ \mu eN$

Putting in numbers, V = 5 volts.

So $L^2 = 5\text{x}10^{-12} \text{ x } 0.1 \text{ x } 5 = 2.5\ 10^{-12}$ $L = 1.58\ 10^{-6}$ m.

Q14
Quantum – photoelectric effect,

(b) $E = hc/\lambda = 4.95 \times 10^{-19}$ J = 3.09 eV

(c) $E = n^2\ (\hbar\pi)^2/(2mL^2)$

$E = 1.5\ 10^{-24}$ J = 9.4×10^{-6} eV

PAPER 8 SECTION F SOLUTIONS
2008/2009

15 (a) Interpolation is useful when one needs to calculate pixel values at locations that are different from the sampling points in the input image. These input points are usually on a regular grid.

Typical image processing operations that require interpolation are resizing and rotation. Resiz requires a new sampling grid to be created which is either more or less dense than the original on Each location on the new grid which is not also on the old grid requires the pixel to be interpolate at that location. Similarly rotation requires the sampling grid to be rotated about the centre of the image by the required rotation angle, and the new sampling points will not in general coincide with the old ones. Morphing or any other non-linear distortion of an image also require interpolation.

(b) Linear interpolation requires x_p to equal x_a when $p=a$, & to equal x_b when $p=b$, and to be a linear function of p at intermediate locations.

Hence $f(a) = 0$ and $f(b) = 1$

& $f(p) = mp + c$

$$\therefore \quad ma + c = 0$$
$$mb + c = 1$$
$$\therefore \quad m(b-a) = 1 - 0 = 1 \qquad \therefore \quad m = \frac{1}{b-a}$$
$$\therefore \quad c = -ma = -\frac{a}{b-a}$$
$$\therefore f(p) = \underline{\underline{\frac{p-a}{b-a}}} \qquad \text{so } \underline{\underline{x_p = \frac{b-p}{b-a} \cdot x_a + \frac{p-a}{b-a}}}.$$

(c) To perform bi-linear interpolation on 2-D data we can first interpolate in the vertical direction & then in the horizontal direction. Hence, vertically:

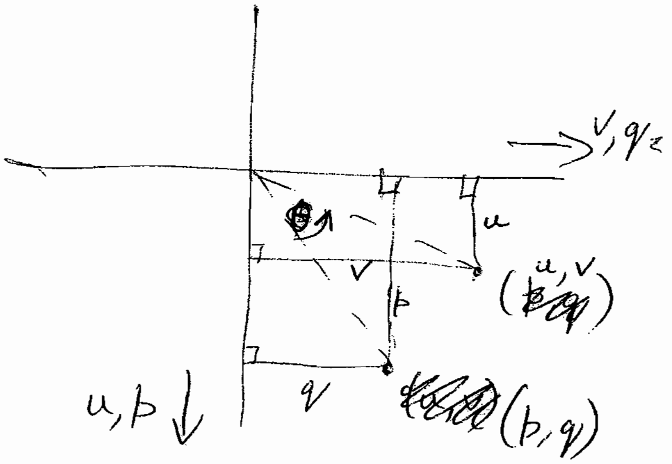
$$x(p,c) = \frac{(b-p)x_{a,c} + (p-a)x_{b,c}}{b-a}$$
$$x(p,d) = \frac{(b-p)x_{a,d} + (p-a)x_{b,d}}{b-a}$$

& then horizontally

$$x(p,q) = \frac{(d-q)x_{p,c} + (q-c)x_{p,d}}{d-c}$$
$$= \frac{(d-q)[(b-p)x_{a,c} + (p-a)x_{b,c}] + (q-c)[(b-p)x_{a,d} + (p-a)x_{b,}}{(b-a)(d-c)}$$

(d) (p, q) is a location in the input image & (u, v) is the corresponding location in the output image, both with respect to the image centre.

Hence for an anticlockwise rotation by θ



Note u & p increase from top to bottom of image, so positive vertical direction is downwards.

For the direction of rotation to be as shown on the diagram:

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

so that $p > u$ & $q < v$ when θ is smal and positive, & p, q, u, v are all positive as in the diagram.

16. Feature detection

(a) (i) The first stage of most edge detection algorithms is to smooth the image $I(x,y)$ by convolution with a 2D Gaussian kernel $G_\sigma(x,y)$:

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\ \exp - \left(\frac{x^2+y^2}{2\sigma^2}\right)$$

This is a low pass filter to reduce the effect of additive image noise before differentiation. The latter amplifies high frequencies. The size of the filter determines the scale of finding edges. The larger the kernel the lower the low pass filter cut-off spatial frequency. [20%]

(ii) The intensity of a smoothed pixel is computed by discrete convolution:

$$S(x,y) = \sum_{i=-n}^{n}\sum_{j=-n}^{n} G_\sigma(i,j) I(x-i, y-j)$$

The 2D convolution can be decomposed into two 1D convolutions as follows:

$$G_\sigma(x,y) * I(x,y) = \sum_{i=-n}^{n}\sum_{j=-n}^{n} g_\sigma(i)\ g_\sigma(j)\ I(x-i, y-j) = g_\sigma(x) * [g_\sigma(y) * I(x,y)]$$

where $g_\sigma(x)$ is a 1D discrete approximation to the Gaussian kernel:

$$g_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$ [20%]

(iii) The next step is to find the gradient of the smoothed image $S(x,y)$ at every pixel. This can be achieved by convolving $S(x,y)$ with the kernel $[1\ \ -1]$ in the x and y directions. The resulting gradient estimate applies half way between the pair of pixels being convolved. [10%]

(b) (i) Band-pass filtering by convolution with the Laplacian of Gaussian or its approximation the Difference of Gaussian (DOG)

$$\nabla^2 G_\sigma \simeq G_{K\sigma} - G_\sigma$$

(ii) Look for maxima of $\nabla^2 G$ in both image and across scale σ. Produce scale-space representation by smoothing images by family of discrete values of σ - Done efficiently by an image pyramid. Blob is detected if local maxima in image and across neighbouring scale.

(iii) Image matching of features eg. localisation of phone image; matching images to images in database.

17 (a) the likelihood is:

$$p(S|p) = \prod_{n=1}^{N}\prod_{m=1}^{M} p_m^{x_{nm}}(1-p_m)^{(1-x_{nm})}$$

log likelihood is

$$\mathcal{L}(p) = \ln p(S|p) = \sum_n\sum_m x_{nm}\log p_m + (1-x_{nm})\log(1-p_m)$$

solve

$$\frac{\partial\mathcal{L}(p)}{\partial p_m} = \frac{\sum_n x_{nm}}{p_m} - \frac{\sum_n(1-x_{nm})}{1-p_m} = 0$$

$$\frac{\sum_n x_{nm}}{p_m} = \frac{N-\sum_n x_{nm}}{1-p_m}$$

$$\sum_n x_{nm} - \cancel{p_m\sum_n x_{nm}} = Np_m - \cancel{p_m\sum_n x_{nm}}$$

$$\boxed{p_m = \frac{1}{N}\sum_n x_{nm} \qquad \text{for all } m}$$

(b)

$$P(S) = \int p(S|p)\,P(p)\,dp$$

$$= \int \prod_m \prod_n p_m^{x_{nm}}(1-p_m)^{(1-x_{nm})}\,dp$$

$$= \prod_m \int p_m^{\sum_n x_{nm}}(1-p)^{N-\sum_n x_{nm}} \qquad \text{let } a_m = \sum_n x_{nm}$$

$$= \boxed{\prod_m \frac{a_m!\,(N-a_m)!}{(N+1)!}}$$

The marginal probability of the data is the probability of the data averaging over all possible parameter values. It can be used for model comparison.

17 (c) Here is a possible (good) answer to this question.

We define "fits better with" as "has higher probability of being generated with".

Then we evaluate and compare $p(x,S)$ and $p(x,S')$ which is easy to do given the solution to part (b). For example

$$p(x,S) = \int p(x,S \mid p)\, p(p)\, dp$$

Let $a_m = \sum_{n \in S} x_{nm}$ and $a_m' = \sum_{n \in S'} x_{nm}$

then $$p(x,S) = \prod_m \frac{(a_m + x_m)!\,(N - a_m + 1 - x}{(N+2)!}$$

for uniform $p(p)$. Similarly for $p(x,S')$.

We need to be careful to normalise for the probability of S and S', thus we divide by $p(S)$ and $p(S')$.

Algorithm:

1) Compute $c = \dfrac{p(x,S)\, p(S')}{p(x,S')\, p(S)}$

2) if $c > 1$, x "fits better with" S
otherwise if $c < 1$, x "fits better with" S'

SECTION G *Engineering for Life Sciences*.

18(a)

Darwin's idea of a "tree of life", originating with a single primordial ancestor and branching successively into different life forms -- bacteria, plants, fungi, flies, worms, vertebrates etc.-- over very long time scales provided a unifying picture of biology as a whole. Thus, botany, zoology, mycology (the study of fungi) were no longer separate subjects since all living things are related to a common ancestor, and use similar machinery at a cellular level. Darwin proposed a mechanism for the branching of the tree by means of small, naturally occurring differences in individual organisms which could, in extreme conditions (e.g. of temperature) give a few specimens an advantage, thereby enabling them to survive while most perished. Darwin was not aware of Mendel's work on the inheritance of characteristics; which makes his work all the more remarkable. Modern molecular biology has amply substantiated Darwin's ideas—e.g. the discovery that all forms of life use the same DNA and the same genetic code.

(b)

A virus consists of a specific sequence of DNA or RNA, enclosed in a shell-like container built from specific protein-molecule building blocks. Viruses which attack bacteria typically have machinery for injection the DNA/RNA through the bacterial cell wall. Once inside, the DNA/RNA takes over the cell's machinery and makes many copies of itself, complete of course with the protein coat which is programmed by the DNA/RNA. The bacterium's interior becomes crowded with virus particles, and the cell's membrane eventually ruptures—enabling the many new viruses to attack other bacteria.

(c)

The DNA of a gene is a long string of "letters" A,T,C and G. These letters stand for four different nucleic acids which link together in pairs (A-T and G-C) forming a double-helical structure with complementary strands. When the gene is being transcribed, the DNA is run past a "reading head" which reads the message as three letter words. There are altogether 64 distinct three letter words in a 4-letter alphabet, and these code (with some redundancy) for 20 distinct amnio acids plus "stop". A sophisticated molecular machine (ribsome) attaches amino acids of the kinds specified by the DNA to form a long polymer—a polypeptide chain. Now the 20 sorts of amino acids have different characteristics: some are large, some small; some are electrically charged; some are hydrophilic; some are hydrophobic. The chain folds up into a form prescribed by the amino acid sequence in a way which is not yet fully understood. The hydrophobic amino acids end up in the interior, shielded from the water inside the cell. Proteins may be globular or stringy, hard or soft, with special patterns of +/- charge on their surface, etc. They go on to self-assemble into structures and machines—and the entire process is prescribed by the sequence of letters in the DNA. [Some small proteins called "prions" can assemble into two distinct shapes which are good and bad. In the bad form they encourage good neighbors to become bad and then to assemble into fibers which interfere with the proper working of the brain, such as in the disease CJD.]

(d)

Bacteria such as *e. coli* swim through their aqueous environments (in the gut) by means of ~6 propellers, each driven by a tiny rotary motor that can go forwards or in reverse. The propellers take the form of slender, corkscrew-like structures. When all motors are rotating in the forwards sense, the corkscrews "push" and they all associated to form a single corkscrew-like bundle at the rear of the cell. From time to time the motors go into reverse. When this happens, the corkscrews change from LH to RH helices; the bundle breaks up and the cell "tumbles". When the motors resume their forward sense of rotation the bundle reforms and the cell goes off, smooth swimming as before but in a random direction in 3D space.

If the cell happens to be swimming in a direction such that the concentration of nutrients is increasing, then the cell's control system prolongs the period of smooth swimming; if the cell is going in a direction of decreasing nutrients then the smooth-swimming phase is shortened. In this way the bacterium navigates in a "biased random walk" towards the nutrients—which enable it to grow and divide. The cell travels about ten times as far as it would if it could simply steer always towards the food. But nature's apparently cumbersome strategy works well—and has been doing so ever since bacteria evolved from some more primitive life-form.

(e)

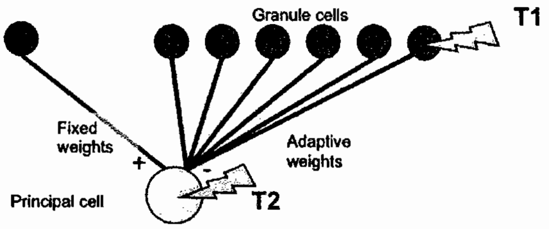
In principle, it is not difficult to imagine a protein molecule which can act as a building brick to construct a long cylindrical tube. The bricks would need to have a very precise slope and to be provided with electrically charged "sticky patches" (+, -) so as to adhere to their neighbors in the desired way. Such an arrangement would build a straight tube and all bricks would be in identical environments to their neighbors.

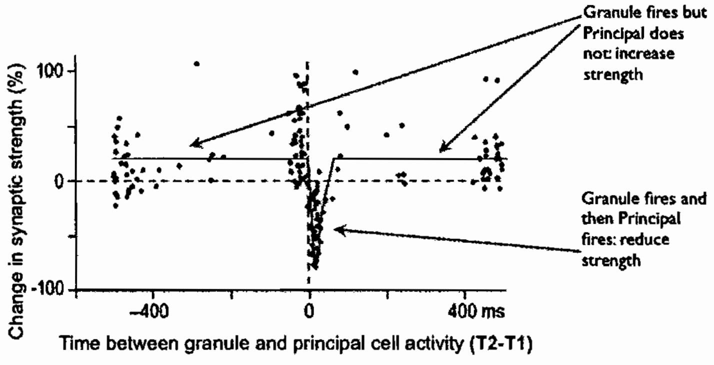
Bacterial flagellar filaments are just like this except that they are curved. A rod or tube can be curved only if the building bricks are slightly longer on one side than on the other. That could be achieved, probably, if there were two sorts of bricks, similar overall but with one sort a bit longer than the other. But it is known from chemical studies that bacterial flagellar filaments are made from a single type of building brick—the "flagellin" protein molecule.

The trick which nature devised to enable a single type of brick to build a curved tube was to make each brick in the form of a switch, so that it could exist in two different states: long and short. The way in which bricks in the two states assemble into a curved tube involves some very subtle geometry and mechanics. It turns out that if a given protein molecule can build a curved, helical corkscrew (there must also be twist as well as curvature, but that is relatively easy to provide) then the application of mechanical torque—as when the motors go into reverse—enables the filament to switch into a different helical form.

19 (i) Anti-Hebbian learning describes a particular class of learning rule by which synaptic plasticity can be controlled. These rules are based on a reversal of Hebb's postulate, and therefore can be understood as dictating reduction of the strength of synaptic connectivity between neurons following a scenario in which a neuron directly contributes to production of an action potential in another neuron. Often the synapse strengthens slightly if the action potential at a synapse does not cause an action potential post-synaptically.

(ii) Research on the weakly electric fish has demonstrated that the electrosensory lateral-line lobe (ELL) receives sensory input from electroreceptive sensory organs which utilize a self-generated electrical discharge (called an EOD; electric organ discharge) to extract information from the environment about objects in close proximity to the fish. The cerebellar-like structure is responsible for filtering the input to remove predictable components of the input. The principal cells receive input from both the sensory surface (fixed input) and from the granule cells (from all sensory modalities and efference copy). The synaptic weight between the granule and principal cells undergoes anti-hebbian modification. Therefore if the Granule fires but Principal does not the synapse will slowly increase strength whereas if Granule fires and then Principal fires the synapse will reduce strength – (see figure – students may reproduce the second figure in answer to part (i) which is fine).

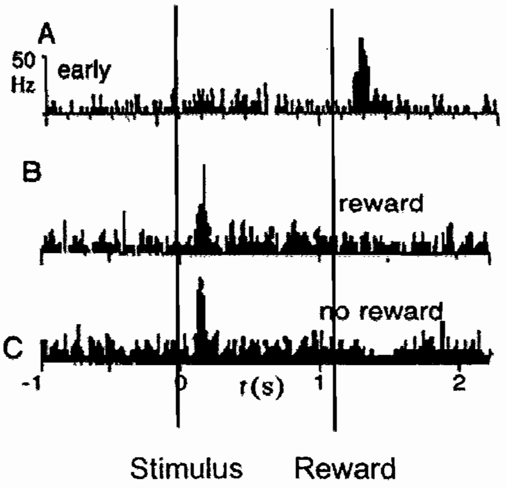




(iii) By predictively cancelling the expected sensory feedback the fish can be attuned to unexpected sensory inputs in the environment such as predators or prey.

(b)

(i) sketch shown below (this shows spiking but a rate based sketch is also fine) A. unexpected reward, B expected reward, C unexpected removal of reward



(ii)

$$\delta(t) = r_{t+1} + \gamma V(s_{t+1}) - V(s_t)$$

the reward prediction error = the current reward + discounted value of the next state minus the value of current state

(iii)

In A the reward is unexpected so =0 r_{t+1} is positive while $V(s_{t+1})= V(s_t)=0$

In B the reward is expected so after the sensory stimulus $V(s_{t+1})$ becomes positive with respect to $V(s_t)$ as $V(s_t)$ was zero but now the value increases due to the stimulus. At the reward time , the reward r_{t+1} is equal to the predicted reward gamma*$V(s_{t+1})$- $V(s_t)$

In C as in B but with the removal of reward the reward $r_t=0$ which is less to the predicted reward gamma*$V(s_{t+1})$ - $V(s_t)$ leading to delta(t) being negative

(c)

(i) The noise has a constant coefficient of variance so that the standard deviation of the noise scales with the mean level of the command

(ii) Due to redundancy in the motor system there are many ways to move to achieve a task and each involves a different sequence of motor commands which carry different forms of noise. Therefore each possible way of moving is associated with is a different distribution of kinematics parameters such as position, velocity etc. Therefore a particular trajectory can be chosen to as to control or minimise some aspect of a task such as minimising error or variability. This is the framework of optimal control.

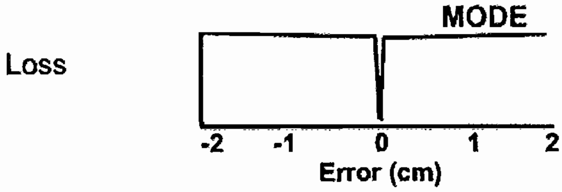
(d)

(i) the posterior is proportional to the prior * likelihood. – so any version of Bayes rule is fine such as

$$\underset{\substack{\uparrow\\ \text{Posterior}}}{\overbrace{P(\text{state}|\text{sensory input})}^{\text{Belief in state AFTER sensory input}}} = \frac{\underset{\substack{\uparrow\\ \text{Likelihood}}}{\overbrace{P(\text{sensory input}|\text{state})}^{\text{Evidence}}}\ \underset{\substack{\uparrow\\ \text{Prior}}}{\overbrace{P(\text{state})}^{\text{Belief in state BEFORE sensory input}}}}{P(\text{sensory input})}$$

(ii) they will different is there is a non flat prior such as for example in estimating speed where there is Gaussian prior on speeds so that slow speeds are more common than fast speeds. The Maximum a Posteriori (MAP) estimate will predict a slower sped than the Maximum Likelihood Estimate (MLE).

(iii) The MAP estimate will be chosen when the loss is only low for small errors – so for error to the power of a very small number or for a loss that looks like this the mode of the posterior is optimal



If the loss increased linearly with (absolute) error than for any distribution it is simple to show the median minimizes the loss.

20 (a)

The tensile stress-strain responses of soft tissue biological materials are "concave-up" as expressed by a quadratic $\sigma = k\varepsilon^2$ or exponential function $\sigma = A\left[\exp(B\varepsilon)-1\right]$. This is due to the sequential "recruitment" of initially unaligned or crimped collagen fibrils sequentially—only after straightening and alignment is each collagen fibril fully loaded in tension.

The responses are time-dependent due to viscoelastic and or poroelastic deformation, where poroelastic deformation is associated with fluid flow through a porous elastic or viscoelastic solid skeleton. Since soft tissues are highly hydrated, the poroelastic deformation is dominant in the mechanical response.

(b)

The composition is approximately 75% water, with the remainder collagen and proteoglycans in an approximately 2:1 ratio. Proteoglycans are large macromolecules consisting of sugar molecules on a protein backbone. Each sugar (glycosaminoglycan) is sulphated and has a negative charge, which binds water. The proteoglycans are aggregated into immense complexes by attachment of individual proteoglycans onto a hyaluronic acid (sugar) backbone—thus they have a three-level tree structure of sugar-protein-sugar (and are mostly sugar by weight).

Cartilage is both viscoelastic and poroelastic. The elastic modulus in compression is less than 1 MPa; the elastic modulus in tension is up to ten times larger due to direct loading of the collagen fibrils while in compression the resistance to deformation is 50% due to the repulsion (steric and charge-based) of the glycosaminoglycans within the proteoglycans. There are dramatic effects found when tissues are tested at different loading rates.

The composition of bone is approximately 50% mineral, 30% collagen and other organic molecules and 20% water. The mineral is a biological analog to hydroxyapatite. The bone is an interpenetrating phase composite with co-continuous phases, such that strain is transferred along the stiff, mineral component. Because of this, the bone mechanical response is more analogous to an engineering material (ceramic) response than to a soft tissue response (as above for cartilage). Unlike cartilage, the elastic modulus in bone is similar in tension and compression, and in the tens of GPa (10-20) when fully dense. Although some degree of poroelastic deformation does occur in bone, this is dramatically less than in cartilage due to the dominance of the mineral phase in determining the mechanical response of bone and the relatively smaller amount of water present in the tissue (20% instead of 75%). There are thus fewer rate dependent effects overall, although both the elastic modulus and strength change somewhat with loading rate. However, the most unique aspect of bone's mechanical response is that it is not ceramic-like are in its toughness, which is very large due to the reinforcing effect of the hydrated organic phase. Bone's nanocomposite structure and low density gives rise to very good specific stiffness, strength and toughness compared with even most engineering composites.

(c)

Load is $P = AE\delta/L$

Assuming springs in parallel, the deformations are identical in both components but the loads are not. Stated in the question that the lengths are the same. So the loads partition simply as $P_{component} = A_{component}E_{component}$ and

%load in each = $P_{component}/P_{total}$

Titanium component:

Cross sectional Area A (mm^2) = $\pi(25/4) = 6.25\ \pi$

AE =$6.25\ \pi * 8E_{bone} = 50\ \pi\ E_{bone}$

Bone component:

Cross sectional Area A (mm^2) = $\pi(225/4\text{-}25/4) = 50\ \pi$

AE = $50\ \pi\ E_{bone}$

Thus the loads are the same in both or each is bearing 50% of the total.

This is not ideal as bone is potentially being stress-shielded by sharing load with the implant. Bone responds strongly to mechanical loads, and when the loads are insufficient on bone, the tissue resorbs, decreasing its elastic modulus which in turn increases the proportion of load on the implant, causing a vicious cycle of bone decay.

As such improved implant designs would reduce the amount of force born by the titanium implant. This can be done in two ways: (1) reducing the cross-sectional area of the implant, (2) reducing the elastic modulus of the implant, by using a material with a modulus closer to (or even equalling) that of bone. Of course, both strategies could be employed simultaneously.

Paper 8: Section H with answers

2-1 Your company has invented a new type of hairdryer, in which hot air is blown at high speed from a linear row of small nozzles, simultaneously drying and combing the hair. In the prototype there are ten nozzles, 1 mm in diameter, spaced evenly along a line 25 mm long, and the jets of air emerge at 20 m s^{-1}. Tests have shown that the combing effect is insignificant at air speeds below 5 m s^{-1}. The body of the hairdryer is moulded in bright blue plastic with a distinctive shape, and it is proposed to market it under the name 'Blu-jet'.

(a) State the tests which an invention must satisfy in order to be patentable.

[4]

(b) Describe the structure of a typical patent document and explain why it is desirable to use more than one claim. Suggest drafts of one main claim and one dependent claim which could be used in a patent for the invention described above.

[10]

(c) Describe any other types of intellectual property, apart from patents, which would be relevant to this product. In each case identify issues which should be considered in protecting that IP.

[6]

Answer

(a) For an invention to be patentable it must satisfy four tests:

- it must be novel: i.e. never disclosed anywhere in the world in public;
- involve an inventive step: i.e. not be obvious to someone in the light of what has been done before (the 'prior art');
- have a practical application: be capable of being made or used in some kind of industry; and
- not be excluded (e.g. scientific theory or mathematical method, method of doing business, perpetual motion machine….)

(b) A typical patent contains a discussion of the state of the art before the invention was made (the 'prior art'), showing why there was a need for the invention;
a discussion of the main features of the invention and its advantages;
a full description of how the invention can be put into effect (often with diagrams);
and a set of claims.

The claims define the scope of the monopoly covered by the patent. The first (main) claim usually includes all the essential elements of the invention – it should not include non-essential elements as it should aim to be as general as possible without including prior art (because if it does, it will not be valid). If the first claim includes non-essential features, then a competitor can simply omit those features in his/her product and will not have infringed the patent. Subsequent ('dependent') claims usually add extra features to the earlier claims, narrowing the scope – the aim being to provide a fall-back in case the earlier claims are invalidated – either during the patenting process or later.

Possible draft claims:

main claim: 1. a device for drying hair in which hot air emerges from a plurality of (or 'at least two') holes or nozzles at a speed of at least 5 m s–1.

dependent claim: 2. a device as described in claim 1 in which the holes or nozzles are arranged substantially in a line

or 2. a device as described in claim 1 in which the air emerges at a speed between 5 and 30 m s-1.

or 2. a device as described in claim 1 in which the hole or nozzle diameter lies between 0.5 and 2 mm.

(The point here is that the claims should include only functional aspects of the invention, and that they should progressively narrow the scope from claim 1. So, for example, claims mentioning the colour of the device or its material would not be correct answers)

(c) Two areas of IP which are also relevant would be design and trademarking.

The design could be protected by design registration which would protect the appearance, colour, shape and material used for the hairdryer – it relates to form not function. However, small changes to the design could be introduced by a competitor to get round the protection, so careful definition of the design would be essential.

The name 'Blu-jet' would be potentially a useful trademark but it would be unlikely to be accepted as a registered trademark as it is descriptive of the product (despite the mis-spelling).

2.2 (a) List and describe the typical stages of the evolution of an industry.

[4]

(b) Explain what is meant by the following types of innovation:

(i) radical innovation;

(ii) incremental innovation;

(iii) sustaining innovation; and

(iv) disruptive innovation.

Illustrate your answer with examples of each of the four types of innovation.

[8]

(c) Discuss, using examples, why large, long-established firms often find it difficult to deal with disruptive innovations. Describe possible strategies that such firms can use for managing disruptive innovations.

[8]

Answer:

(a)

Era of ferment / disruption: From a small number of experiments emerge numerous competing approaches to addressing a particular opportunity. E.g., PC platforms, powered road transport, etc.

Dominant design / standards: The market settles down onto one or two standard / dominant designs. Once this happens, more customers will have the confidence to buy. Once there is a standard / dominant design, producers of peripherals will have the confidence to enter the market.

Incremental innovation: Once there is some agreement in the market as to what is wanted, companies will focus effort in improving cost, reliability, efficiency, etc, rather than developing completely new products.

Maturity: Once the product has reached the limits of cost effective improvements, the market may slow down until another disruption occurs and the cycle starts over.

(b)

(i) Radical innovation = significantly different changes to products, services or processes: 'do what we do differently': Bessant and Tidd (2007). Ex = petrol to electric engines for cars

(ii) Incremental innovation = Small improvements to existing products, services or processes: 'doing what we do but better' Bessant and Tidd (2007). Ex = increasing size of LCD screens

(iii) Sustaining innovation = Improving performance of existing products with reference to current customers' performance measures. Ex = improving acceleration in cars with fuel injectors

(iv) Disruptive innovation = Product performance may be initially worse but presents new value proposition. Ex = mobile phones.

(c)

Large firms struggle with disruptive innovations for reasons including:

+ they are focused on pleasing the needs of their current, average customers.
+ they have built up substantial resources to address their current markets
+ they struggle to identify and validate high potential disruptive opportunities
+ the skills required to manage sustaining and disruptive innovations are not the same

+ current products based on sustaining innovation may still have a long life; how do you ensure that you continue to milk these innovations while also managing the more uncertain disruption.
Examples of poorly managed disruptions include: Kodak and the digital imaging technologies; Microsoft and the Internet; mini-computer manufacturers and the PC.
Some possible strategies for dealing with them are as follows (from Dr Rick Mitchell, ex Technical Director of Domino Printing Sciences):

- Lie low and hope. It may not happen.
- Buy and suppress.
- Undermine by stealth: publicity, legislation etc.
- Invest and accept the cost (or use partners).
- Fight back and emphasise the differences.
- Aim to be the last survivor.
- Reinvent the company.
- Accept mortality.

2-3 (a) Describe the difference between:

(i) market pull, and

(ii) technology push.

List three ways in which inventions may arise from each of (i) and (ii) above. Give examples for each one listed.

[8]

(b) It has been said that: "*In order to develop a new product which is commercially successful, it is essential to have a clear view of exactly who it is aimed at and why they will want to buy it*". Explain how you could effectively assess potential markets for a new product.

[12]

Answer:

(a)

(i) Market pull = 'we want it'; Responding to a (clearly) articulated demand for something to be solved (though not necessarily a clearly defined way of doing it).
Inventions may arise through:
Experience in use; lead user ideas; fashion; legislation/context; luxury to common.

(ii) Technology push = 'we can do it'; 'A solution looking for a problem'. Seeking to find an application for a new invention, discovery, etc.
Inventions may arise through:
Accident; analogy and transfer; structured search; mapping, new materials

(b)

The market is defined as: the set of all actual or potential buyers of a product or service. Within each overall market, it is possible to identify clusters of customers who have similar needs, who respond to promotional material in a similar way, who have similar backgrounds or motivations. This is called market segmentation. It can be incredibly powerful and is an essential step in estimating the size and growth of a potential market opportunity. It is also a useful way of really defining the target customer for a proposed product as a precursor to detailed consumer research.
There are many are many ways of **segmenting the market**, but four generic methods:

1. Product attributes: perhaps the easiest to do is to compare and product attributes. This tends to say little about the consumers and is perhaps the weakest approach
2. Product usage: describing the ways in which the product is used
3. Users / consumer: could be based upon demographics / geography, or psychographics.
4. Benefits delivered: what benefits do the users derive from the product.

Good market segmentation will mean that each segment targets a group of customers who have similar responses to the marketing mix (product, price, place, promotion).
Consumer research can then be carried out, the process for which needs consideration of:

- Focus: new market or existing market?
- Stakeholders: internal versus external; purchase stakeholders (initiator; influencer; decider; buyer; user)
- Researchers: who will gather the data? Who to involve in the research?
- Data collection: validation / qualification of existing viewpoints or exploration of new insight / inspiration? Different tools need to be matched to different needs.