ENGINEERING TRIPOS PART IB

Monday 1 June 2009 9 to 11

Paper 1

MECHANICS

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

The answers to questions in each section should be tied together and handed in separately.

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper
Single-sided graph paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 A heavy uniform prismatic block of mass m is shown in end view in Fig. 1. It rolls about point P with angular velocity ω_1 and is just on the point of making contact at point Q.

You should assume that there is no slippage, deformation or rebound in the system.

(a) Show that the polar moment of inertia about an axis passing through centroid, G, and perpendicular to the page is given by

$$\frac{5ma^2}{9}$$ [8]

(b) Show that after making contact at Q the block rotates about that point with angular velocity given by

$$\frac{14}{23}\omega_1$$ [6]

(c) If a = 1 m, find the value of ω_1 required so that after making contact at point Q the block will continue to rotate about that point until its centre of mass just reaches a position vertically above point Q. [6]

(cont.

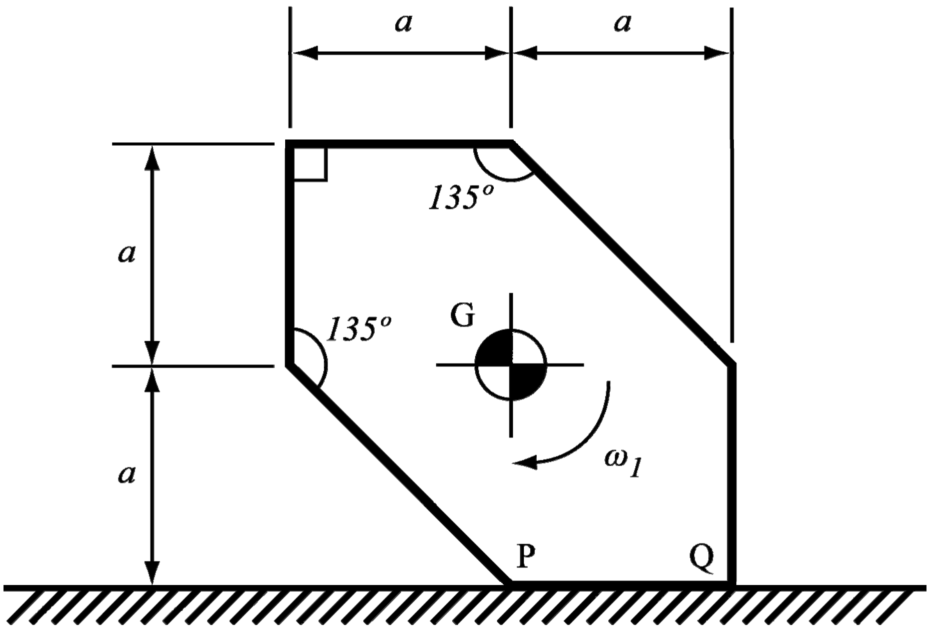

Fig. 1

(TURN OVER

2 The suspension system for the rear wheel of a mountain bike can be modelled as shown in Fig. 2. The rear axle is attached to B. Points A, D and F are all pivot points on the main frame. The light rigid rocker plate CDE rotates about the fixed point D. The shock absorber, EF, can compress and extend to absorb the bumps in the path. Friction may be neglected throughout the system.

At the instant shown, AB and CE are horizontal and angle ABC is 45º; EF is vertical, and of length L. A bump in the path has resulted in point B being pushed upwards causing bar AB to rotate with constant angular velocity ω.

(a) By constructing a velocity diagram, show that the instantaneous rate of compression of the shock absorber, EF, is given by

$$\frac{2L\omega}{3+\sqrt{3}}$$ [6]

(b) Now taking $L = 0.2$ m and $\omega = 5$ rad s^{-1}, construct an acceleration diagram to find the magnitude and direction of the acceleration of point E. [8]

(c) Calculate the angular acceleration of the shock absorber. [6]

(cont.

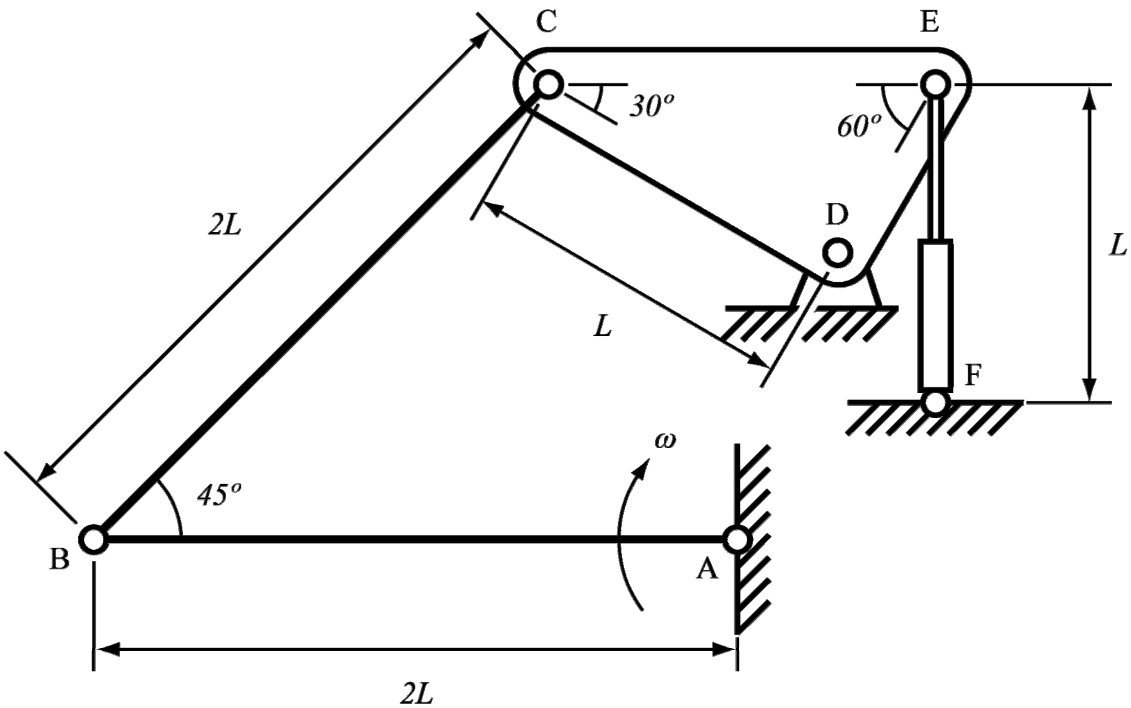

Fig. 2

(TURN OVER

3 A complex machine can be modelled as shown in Fig. 3. The main body of the machine is modelled as a uniform cube, with mass m and sides of 100 mm. The rotating components are modelled as two identical uniform annular rotors. Each rotor is mounted flush with the cube and has an axis of rotation coincident with the centre of one face of the cube. Each rotor has an external diameter of 80 mm, an internal diameter of 40 mm, and a thickness of 20 mm. The mass of each rotor is m .

The machine is lowered onto a base plate and mounted at points A, B and C (as shown in Fig. 3). Points A, B and C lie on three corners of an 80 mm square, the centre of which is coincident with the centre of the footprint of the main body of the machine.

(a) Sketch the mounted machine in plan view (i.e. from above), and identify the overall centre of mass relative to the three mounting points. [4]

(b) If the rotors are not spinning and the machine is mounted to a stationary platform, show that the reactions at the mounting points A, B and C obey the ratio

$$R_A : R_B : R_C = 1 : 2 : 1 \qquad [4]$$

(c) If m = 0.6 kg, find the polar moment of inertia of each rotor. [2]

(d) If ω_1 = 500 rad s^{-1} (constant), and the machine is yawing at a constant rate of 0.5 rad s^{-1}, find the value of ω_2 required for the reaction at point B to be zero. [10]

(cont.

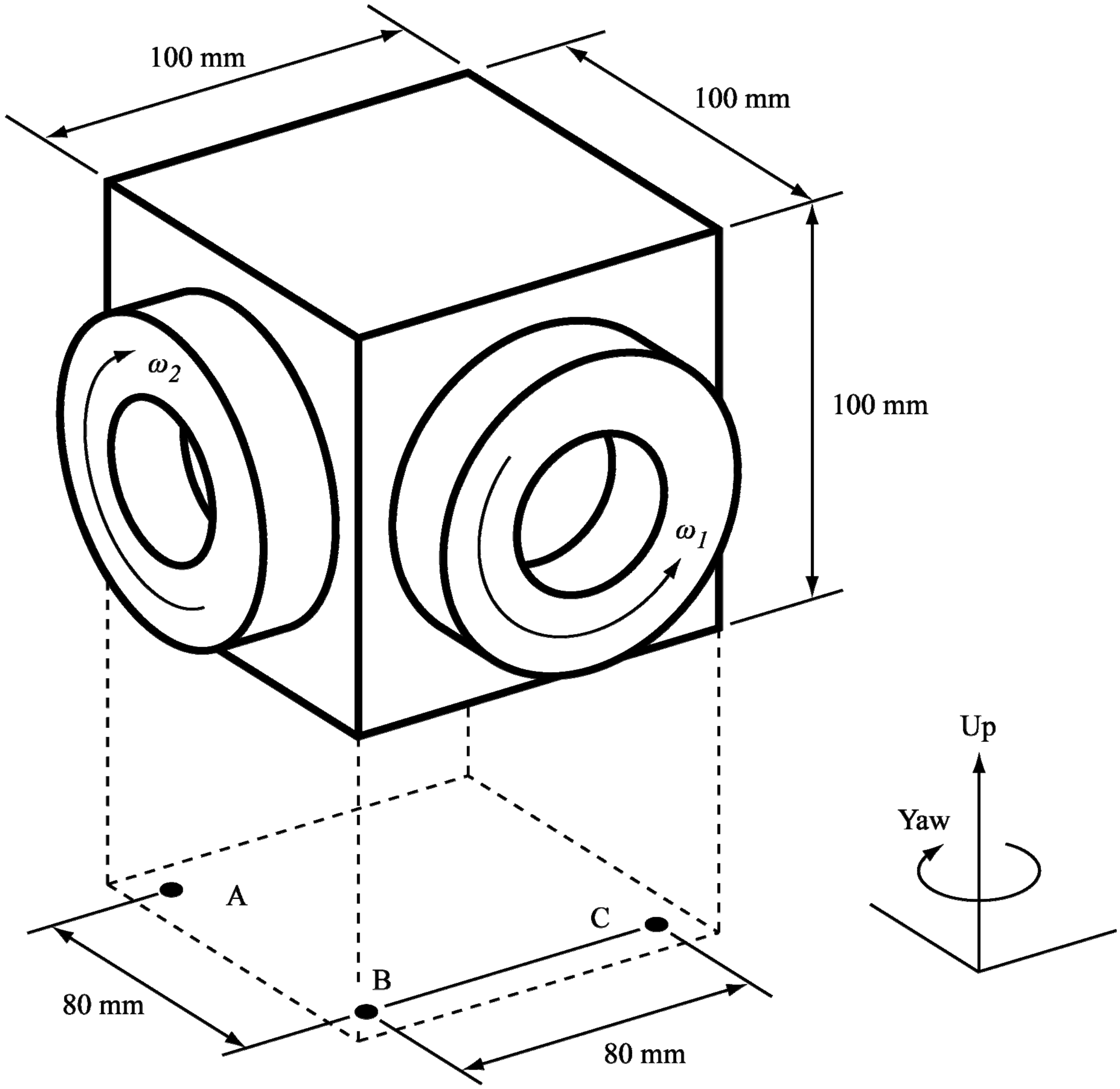

Fig. 3

(TURN OVER

SECTION B

4 A golf ball of mass m and radius r is at rest on a tee firmly pushed into the ground. The contact force between the ball and tee may be taken to act through a single point. The golfer mis-hits the ball striking it with impulse I, on a horizontal diameter, at an angle inclined at 30° to the horizontal, as shown in Fig. 4.

The ball may be modelled as a uniform solid sphere whose moment of inertia is $0.4mr^2$.

(a) Show that for the ball to roll (rather than slide) immediately after the impulse μ must be at least equal to $(5+2\sqrt{3})/7$, where μ is the coefficient of friction between the ball and tee. [10]

(b) The golfer tries again, but this time first places the ball directly on the ground. The ball is struck as before and initially moves with linear speed v_0 and angular speed ω_0; it does not bounce. The coefficient of friction between the ball and the ground is only 2/7, therefore the ball initially slides along the ground. Neglecting air resistance, show that the acceleration of the ball is constant and calculate the distance it slides before it starts to simply roll: in terms of v_0, ω_0 and the acceleration due to gravity, g. [10]

(cont.

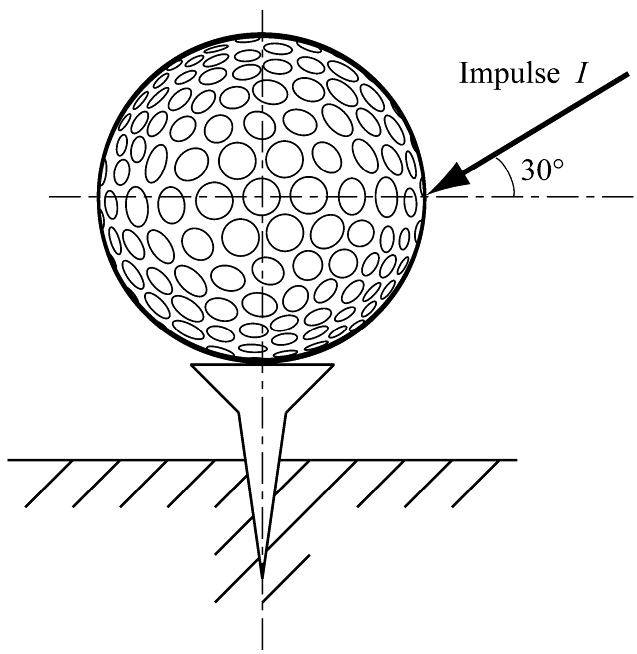

Fig. 4

(TURN OVER

5 A data storage system uses a mechanically-scanned head to record a signal onto a flat horizontal magnetic medium by the mechanism shown in Fig. 5.

The light recording head slides freely over the magnetic storage medium. The system is driven by a crank BC, of length L. At the instant shown BC is horizontal and rotating at a constant angular speed Ω about a horizontal axis at point C. Point C is 1.5 L above the level of the recording head. The head is joined to the crank by a uniform bar AB, of mass m and length $3L$.

You may assume that the pin joints at A, B and C are horizontal and frictionless and that motion only occurs in the plane of the page.

(a) For the instant shown, find the angular velocity of bar AB. [5]

(b) Show that the magnitude of the angular acceleration of AB at this instant is $\dfrac{4\Omega^2}{27\sqrt{3}}$. [5]

(c) In order for the recording head to function correctly it must remain in direct contact with the storage medium at point A. Calculate the maximum permitted value of Ω. [10]

(cont.

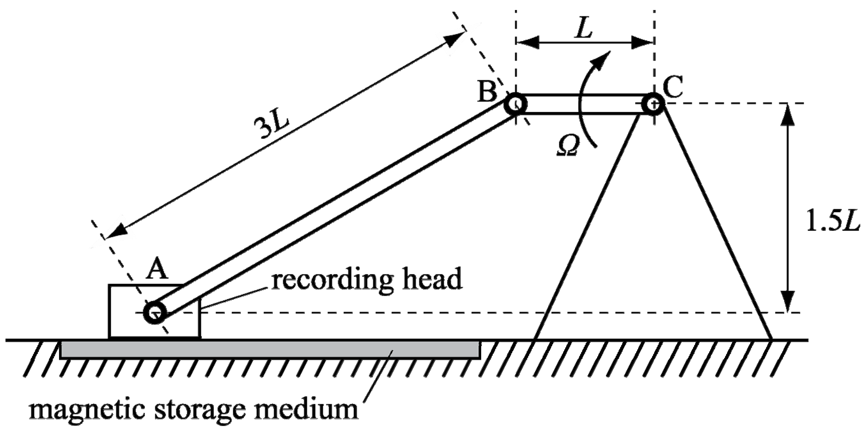

Fig. 5

(TURN OVER

6 A light shaft, 3 m in length, sits in two roller bearings, 1 m apart, equally spaced about the midpoint of the shaft.

The shaft carries three heavy discs, each 1 m in diameter, one at the midpoint of the shaft, the other two 0.3 m in from each free end.

The central disc, B, is out of balance by 0.012 kgm, the outer two, A and C, by 0.009 kgm and 0.015 kgm respectively. The discs are appropriately mounted on the shaft such the system is statically balanced.

(a) Briefly explain the difference between statically and dynamically balanced shafts. [4]

(b) Show that the out-of-balance at C must be at 233° relative to A. [4]

(c) Using a graphical method, determine the dynamic loads on each roller bearing when the shaft rotates at 8000 rpm. [6]

(d) Calculate the mass and position of the balance weights which must be attached to the rims of discs A and C to ensure dynamic balance while preserving static balance. [6]

(cont.

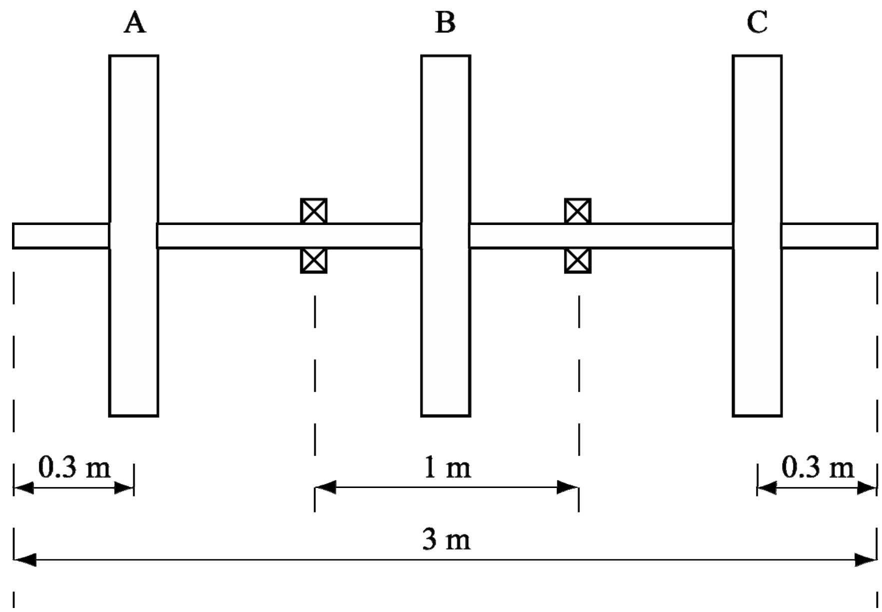

Fig. 6
(not to scale)

END OF PAPER