ENGINEERING TRIPOS PART IB

Monday 1 June 2009 2 to 4

Paper 2

STRUCTURES

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachments: Answer sheet for Q5

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Graph paper	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 (a) By considering the wedge AEF cut from a general two dimensional stress system (Fig. 1), show that the normal and shear stresses, σ_n and τ_n respectively, are given by equations that define Mohr's circle, and hence that the principal stresses and maximum shear stress are: [12]

$$\sigma_1 = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{1}{2}\sqrt{[(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}^2]}$$

$$\sigma_2 = \frac{\sigma_{xx} + \sigma_{yy}}{2} - \frac{1}{2}\sqrt{[(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}^2]}$$

$$\tau_{max} = \pm\frac{1}{2}\sqrt{[(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}^2]}$$

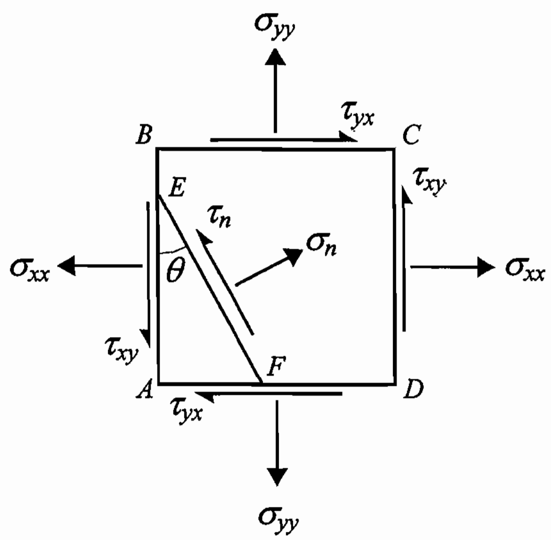

Fig. 1

(cont.

(b) The I-section beam shown in Fig. 2 is simply supported over a span of 6000 mm and is subjected to point load of 150 kN. Assuming that the self-weight of the beam is negligible:

(i) determine the magnitude and direction of the principal stresses and the maximum shear stress at point P by calculation; [5]

(ii) draw the corresponding Mohr's circle. [3]

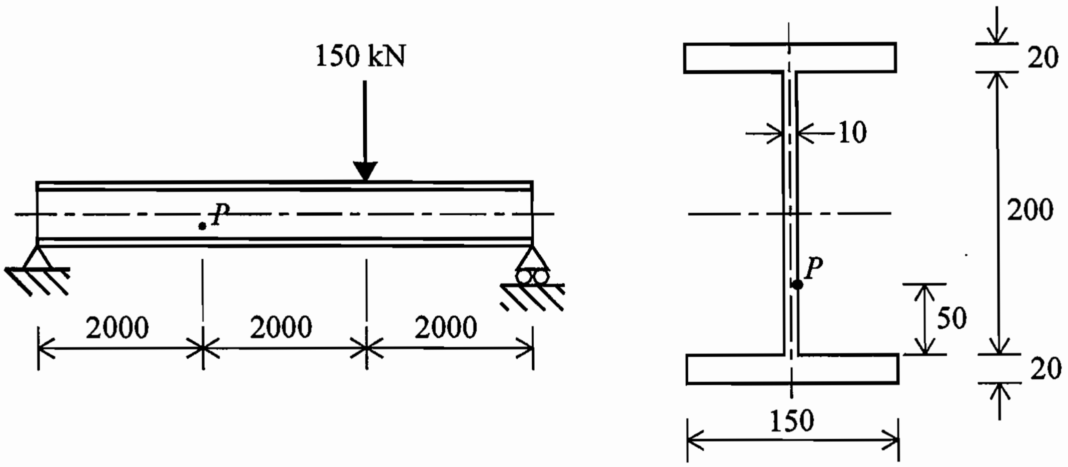

Fig. 2

2 The weightless pin-jointed structure shown in Fig. 3 has ten members I to X and is subjected to a point load W. The unloaded structure is free of stress and there is no connection between bars III and IV and between bars VIII and IX respectively.

(a) Show why there are two redundant bars. [3]

(b) Using the diagonals IV and IX as the redundant bars:

(i) find the particular solutions for bar tensions in equilibrium with the applied load W; [3]

(ii) find the possible states of self stress in the structure; [4]

(iii) find the elastic solution for the bar tensions under load W. [10]

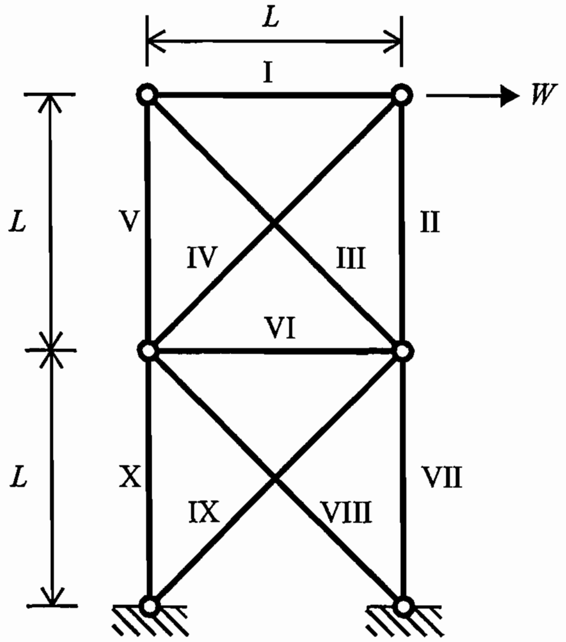

Fig. 3

3 A supporting bracket for a large cladding panel consists of a cranked thin-walled circular section (as shown in Fig. 4). The circular section is fabricated from grade S275 steel (Structures Databook, Section 10.1). The bracket has an outer diameter of 50 mm and a wall thickness of 2 mm. It is rigidly restrained at one end and is subjected to a point load P acting in the negative y-direction and a point load Q acting in the negative z-direction. Both loads act in the y-z plane through the centroid of the cross-section at O. Assume that $Q = P/4$ and that a safety factor of 2 is required.

(a) At the root of the cantilever, determine the magnitude and location of:

(i) the maximum normal stress; [5]

(ii) the maximum shear stress. [5]

(b) By considering the stresses at these two locations only, calculate the safe value of P using both the Tresca and the von Mises yield criteria. [4]

(c) By making reference to your answers to parts (a) and (b), comment briefly on the differences between the Tresca and von Mises yield criteria. [2]

(d) Calculate the displacement in the y-direction at point O when the supporting bracket is subjected to the von Mises safe working load. State the assumptions you make in this calculation. [4]

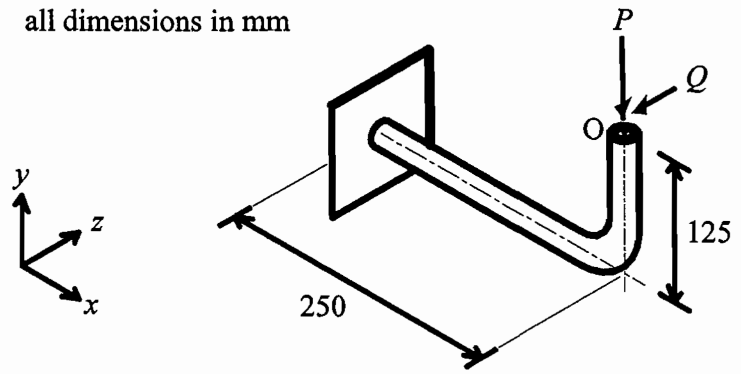

Fig. 4

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SECTION B

4 (a) Perform an *elastic* analysis to find the bending moment at the clamped support of the propped cantilever shown in Fig. 5 when subjected to a point load W. [4]

State *clearly* what assumptions you have made in this analysis. [2]

(b) If the beam in the figure has moment capacity M_p in both hogging and sagging bending, perform a *lower bound* analysis on the same structure and hence determine the maximum load that the structure can carry safely. [6]

(c) Sketch the load deflection plot for the beam, showing clearly the loads that correspond to the solutions to parts (a) and (b). [4]

(d) Without performing further calculations, describe how your solutions to the earlier parts of the question would have been modified if the right-hand support were built slightly higher than the designer intended. [4]

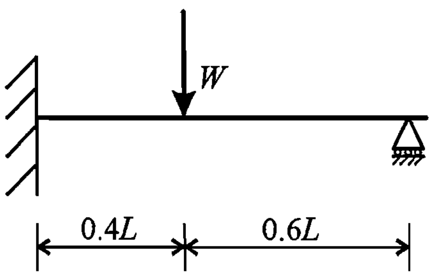

Fig. 5

5 The semi-circular clamped arch shown in Fig. 6 has fully-plastic moment capacity M_p for bending in either sense. It is loaded by independent loads V and H at C and D.

Assume that plastic hinges can only form at the equidistant points A, B, C, D and E.

A separate sheet with two copies of the arch drawing is supplied. Sufficient accuracy can be obtained by measuring dimensions from suitable diagrams drawn on these figures, which should be handed in as part of your solution.

(a) Determine the combination of V and H that would lead to collapse for a sway mechanism with hinges at A, B, D and E only. [8]

(b) Repeat the analysis for another asymmetric mechanism that allows vertical displacement at C. Plot this line, together with the result of part (a), on an interaction diagram between V and H. [8]

(c) Explain why no symmetrical collapse mechanisms can ever occur, even when $H = 0$. [4]

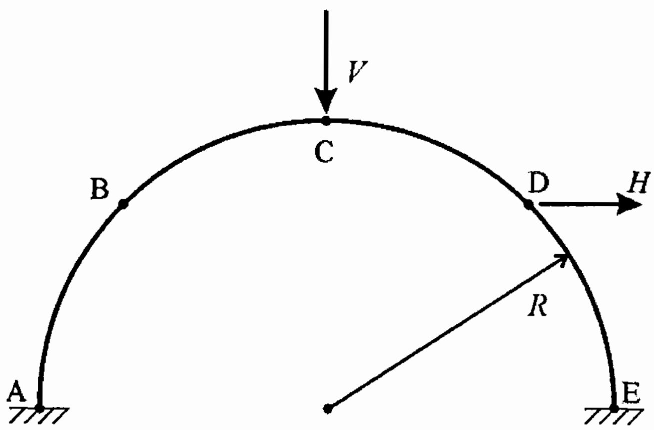

Fig. 6

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6 An aluminium extrusion press is being designed to reduce the area of a billet by a factor of two, as shown in Fig. 7. The designer wishes to investigate the variation of the inlet pressure p as a function of the length of the die L. The velocity of the material entering the die is v. Assume that there is no friction on the face of the die, and also that there is no variation in section normal to the plane of the drawing.

(a) For the slip planes shown, draw a velocity diagram and show that the absolute velocity of the material in region C is $v_C = \frac{3v}{2}\sqrt{1+\frac{H^2}{16L^2}}$. [6]

(b) Determine the variation in p as a function of L and plot the results for $0.5 \le L/H \le 1.25$. What would be the best choice for L? [10]

(c) In an alternative design, both the incoming and outgoing sections are square, so this now becomes a three-dimensional problem. Without doing any detailed calculations, explain how the procedures you have used in parts (a) and (b) would be modified. [4]

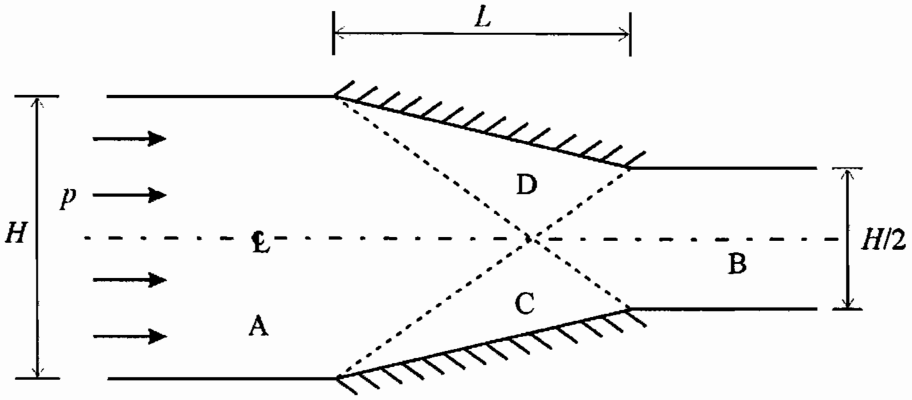

Fig. 7

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